

Accounting for Uncertainty and Variability in Geotechnical Characterization of Offshore Sites

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Abstract. This paper presents an overview on how uncertainty and variability of mechanical soil properties are dealt with in offshore site investigation and presents some ideas for utilizing the reliability tools in a more optimal manner for this purpose. Two types of problems are addressed. First, how to extract the maximum amount of information from geotechnical site investigation, which is often constrained by high costs. Second, how to establish characteristic or representative soil properties for design while taking into account the uncertainties caused by the natural variability of soil properties and the interpretation of the in situ and laboratory tests.

Keywords. geotechnical site characterisation, offshore, uncertainty, variability, foundation reliability, random field, kriging

1. Introduction

Exploitation of offshore resources, development of communication and transport corridors, fishing habitat protection, and the protection of coastal communities require knowledge of the mechanical properties of seabed sediments and improved understanding of offshore geohazards, in particular seafloor mass movements and their consequences.

A major challenge is that the costs of site investigation in the offshore environment are orders of magnitude greater than the corresponding costs on land, especially in deep waters. Today a significant part of the offshore oil and gas field development worldwide takes place in water depths of more than 500m, like in the Gulf of Mexico, Gulf of Guinea, offshore Brazil, the North Sea, offshore Australia, the Adriatic Sea, offshore China, and the Bay of Bengal.

Although the sediments in the deep-water areas are generally normally consolidated soft clay, overconsolidated clay can exist in areas where overburden has been removed by previous slides. Active sedimentation and erosion can also occur. The larger water depths require improved and innovative geotechnical and geophysical investigation techniques and procedures.

While soil profiles for specific soil types could be similar for deep and shallow water settings, there are significant differences between shallow and deep water sites. In shallower waters,

seabed soils can vary widely, and properties and experience gained at one location are not necessarily applicable at another. The scope of a soil investigation for one type of structure is not necessarily adequate for another. Extra caution is therefore necessary when dealing with unconventional soils or unconventional foundation concepts. On the other hand, the deep water environment is generally less dynamic and there is a greater dominance of fine grained sediments (clays) at deep water sites. Typically these environments are located at a greater distance to the sediment sources on land, and thus have a lower sedimentation rate.

There are, however, important exceptions. Many deep water sites are on the continental slope where there is extensive evidence for mass movements. Canyons are another morphologic characteristic of slopes that form the conduits for both mass transport and density currents, which in turn can bring sand and gravel out to the ocean basin.

Marine soil investigations include both offshore and nearshore soil investigations, which can provide very different challenges. In order to determine whether or not a soil profile is representative for an area, an evaluation of the geological setting can be useful.

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ity tools in a more optimal manner for this purpose. Two types of problems are addressed. First, how to extract the maximum amount of information from geotechnical site investigation, which is often constrained by high costs. Second, how to establish characteristic or representative soil properties for design while taking into account the uncertainties caused by the natural variability of soil properties and the interpretation of the in situ and laboratory tests.

2. Code requirements

The International Standard ISO 19901-8, Part 8, sets the premises for marine soil investigations:

"The determination of geotechnical parameters, and the assessment of geological hazards and constraints result from an integrated study of the area using geophysics, geology and geotechnical engineering. Geophysical surveys should be performed before the geotechnical investigation.

Geophysical data are acquired to develop a geological model so as to better understand depositional and other processes and features of an area. The geophysical data are also used to help interpret the stratigraphy from geotechnical boreholes, to define lateral variability across a site, and to provide guidance on optimizing the location of the proposed facilities. Incorporation of geotechnical data into the geological model gives insight into the potential impact of geological conditions on man-made facilities, such as structures, pipelines, anchors and wellheads.

Shallow geophysical investigation can provide information about soil stratigraphy and evidence of geological features, such as slumps, scarps, irregular or rough topography, mud volcanoes, mud lumps, collapse features, sand waves, slides, faults, diapirs, erosional surfaces, gas bubbles in the sediments, gas seeps, buried channels, and lateral variations in stratum thicknesses. The areal extent of shallow soil layers can sometimes be mapped if good correspondence is established between the soil boring and in situ test information and the results from the seabed surveys.

The types of equipment for performing shallow geophysical investigation:

- Echo sounders or swathe bathymetric systems (in which a series of sweeps of the

bathymetric equipment are used) define water depths and seafloor morphology.

- Sub-bottom profilers (tuned transducers) define structural features within the near-surface sediments.
- Side-scan sonar defines seafloor features and seafloor reflectivity.
- Seismic sources, such as boomers or mini-sparkers, can define the structure to deeper depths up to approximately 100 m below the seafloor and either single or tuned arrays of sparkers, air guns, water guns or sleeve-exploders can define structure to deeper depths.
- Seabed refraction equipment provides information on the stratification of the top few metres of the seabed.

Direct observation of the seafloor using a remotely operated vehicle (ROV), autonomous underwater vehicle (AUV), or manned submersible can also provide important confirmation or characterization of geological conditions."

The most important parameters for the foundation design of offshore structures and subsea installations are the thickness and the spatial extent of each soil unit and the mechanical soil properties like the shear strength parameters, the parameters describing the load deformation characteristics and the consolidation behaviour, and the submerged unit weight. Some of these parameters are measured exclusively from the tests on recovered soil samples (e.g. the soil unit weight), while other parameters are interpreted on the basis of in situ tests and laboratory tests on selected soil samples (e.g. the undrained shear strength of clayey soils).

3. Uncertainty and variability of soil properties

Uncertainty in soil parameters can be analysed and categorised in many different ways. One possible categorisation is to classify the uncertainty into aleatory uncertainty and epistemic uncertainty (Lacasse and Nadim, 1996). Aleatory uncertainty refers to the inherent variability of the physical environment and represents the natural randomness of a variable. Examples of aleatory uncertainty are the spatial variation of a soil parameter within a nominally uniform geological layer, the temporal variation in the peak

acceleration of a design earthquake with a given return period, the variation in the ocean wave height or wind force, and so on. The aleatory uncertainty, which is also called the inherent uncertainty, cannot be reduced or eliminated. Epistemic uncertainty, on the other hand, represents the uncertainty due to lack of knowledge on a variable. Epistemic uncertainty includes measurement uncertainty, statistical uncertainty (due to limited information), and model uncertainty. Statistical uncertainty is due to limited information such as limited number of observations. Measurement uncertainty is due to for example imperfections of an instrument or of a method to register a quantity. Model uncertainty is due to idealizations made in the physical formulation of the problem. Epistemic uncertainty is “artificial” and can be reduced, perhaps even eliminated, by collecting more data and information, improving the measurement method(s) or improving the calculation method(s).

A second possible categorisation refers to the method of uncertainty modelling. Objective quantification of uncertainty is based on processing (e.g. by statistical and probabilistic methods) of available data for indicators. Subjective modelling relies on the analyst’s experience (expert judgement), prior information, belief, necessity or, more frequently, a combination thereof.

A third possible categorisation of uncertainties refers to at which stage in the risk estimation process they are located, i.e. in the input parameters to the models (parameter uncertainty) or in the models (transformation uncertainty) which in turn determine the uncertainty of the output parameters. In general, parameter uncertainty is partly aleatory and partly epistemic. Transformation uncertainty is due to the approximations and simplifications inherent in empirical, semi-empirical, experimental or theoretical models used to relate model inputs to model outputs. It is essentially epistemic in nature.

4. Random field model of soil properties

By nature, soils are heterogeneous and one of the important sources of uncertainty in soil properties is their inherent spatial variability. However, because of the geological processes leading to

the formation of soil layer(s), soil properties are expected to show a spatial structure both laterally and with depth. This means that there is a greater tendency for soil properties to be similar in value at closely neighbouring points than at widely spaced points.

Uzielli et al. (2006) provided a comprehensive overview of soil variability analysis for geotechnical practice. In the past few years, random field theory has been increasingly used to model the inherent (aleatory) soil variability (Vanmarcke, 1977; Cafaro and Cherubini, 2002; Fenton and Griffiths, 2002; Dasaka and Zhang, 2012; Lloret-Cabot et al., 2014). A stationary random field is characterized by its mean, variance and spatial variation structure. The latter is often characterised by a characteristic length called the scale of fluctuation (SoF).

4.1. Scale of fluctuation

When the spatial variation in a soil property is assumed to be controlled by a random process, it can be modelled as the sum of a trend component and a residual term (Vanmarcke, 1984; DeGroot and Baecher, 1993; Phoon and Kulhawy, 1999).

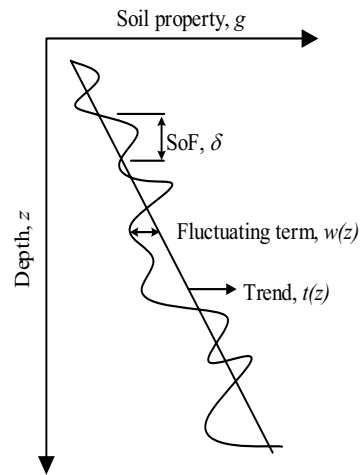


Figure 1. Illustration of the soil inherent variability.

Considering, for simplicity, only the variability along one dimension (e.g. the vertical direction, z), the spatial correlation of a soil property, $g(z)$, can be modelled as the sum of a deterministic trend, $t(z)$, and a random fluctuation, $w(z)$ (see Fig. 1):

$$g(z) = t(z) + w(z) \quad (1)$$

The deviations about the trend tend to exhibit spatial correlation. The degree of spatial correlation can be expressed through the autocovariance function:

$$c_{ij} = c(r) = E[w_i \cdot w_j] \quad (2)$$

where r is the separation distance between points i and j ($r = |z_j - z_i|$) and $E[\dots]$ is the expectation operator. The normalised form of the autocovariance function is the autocorrelation function:

$$\rho(r) = c(r) / c(0) \quad (3)$$

where $c(0)$ is the autocovariance function at $r = 0$. By definition, $c(0)$ is identical to the variance of the residuals off the trend, i.e. $c(0) = \text{Var}[w(z)]$.

If there is no noise in the measured data, the autocorrelation function approaches unity as r tends to zero. In presence of measurement noise, however, the autocorrelation function approaches a value between 0 and 1 for small values of r , depending on the magnitude of measurement noise (Baecher, 1985).

The concept of the SoF was first proposed by Vanmarcke (1977). Vanmarcke (1984) defined the SoF, δ , in terms of the autocorrelation function $\rho(r)$ as follows:

$$\delta = \int_{-\infty}^{+\infty} \rho(r) dr = 2 \int_0^{+\infty} \rho(r) dr \quad (4)$$

Within the SoF, two values of $w(z)$ will tend to be either both above or both below zero (see Fig. 1), indicating that the soil property of interest shows a relatively strong correlation within the SoF.

In practice, the SoF should be estimated from a population of observations. Various approaches for estimating the SoF have been proposed. To apply these methods, the data may need to be transformed such that the stationary assumption is valid (e.g. Campanella et al., 1987).

The SoF can be infinite when the soil is described by a fractal model (Fenton, 1999; Jaksa, 2013). Fenton (1999) suggested that there may be little difference between the finite and infinite

SoFs if appropriate finite-scale model and fractal model over the finite domain are used.

4.2. Spatial averaging and variance reduction

In many geotechnical design problems, the parameter of interest is the soil property averaged over a length, a surface or a volume. Vanmarcke (1977) showed that one of the effects of spatial averaging is to reduce the variability of the averaged parameter (e.g. shear strength) compared to the variability of the data considered separately. The reason for this reduction is the averaging of the variability over a length, surface or volume, and then only the averaged contribution to the uncertainty is of interest.

For the one-dimensional case (i.e. averaging over a length), the variance of the averaged random process can be obtained by applying a reduction factor, Γ , to the variance of the data:

$$\sigma_{x_L}^2 = \Gamma(L) \cdot \sigma_x^2 \quad (5)$$

where σ_x^2 is the variance of the parameter X at specific locations and L is the length over which the parameter is averaged.

Vanmarcke (1984) suggested that the variance reduction for most autocorrelations functions used in geotechnical engineering could be approximated by a unique curve, which results in a simple relation between the reduction factor and the distance over which the soil parameter is averaged. For practical applications, Vanmarcke (1984) suggested the following relationship for the variance reduction factor in terms of the scale of fluctuation (δ) and an averaging distance (L).

$$\Gamma(L) = 1 \text{ for } L \leq \delta, \\ \Gamma(L) = \frac{\delta}{L} \text{ for } L > \delta \quad (6)$$

Assuming an isotropic correlation structure within a plane in the 2-dimensional situation, the variance reduction factor for averaging over a rectangular area may be approximated as:

$$\Gamma(\text{Area}) \approx \frac{\delta^2}{\text{Area}} \quad (7)$$

where $Area = T_1 \cdot T_2$, T_1 and T_2 are the sides of the rectangle over which the averaging is done and δ is the scale of fluctuation. The approximation in Equation 7 is valid for $Area \gg \delta^2$.

4.3. Interpolation by kriging

In actual geotechnical problems, it would be impossible to obtain exhaustive values of data at every desired point because of practical and economical constraints. This is a major source of uncertainty at offshore sites where the costs for obtaining lots of geotechnical data over large volumes are prohibitive. Proper interpretation and interpolation of the subsurface conditions to predict the unknown values in the area or volume of interest from the data observed at known locations is thus essential for any geotechnical evaluation.

An interpolation technique that is well suited for this purpose is the "kriging" approach. Kriging is a stochastic interpolation method developed by Krige (1951) for estimating the most likely distribution of gold based on samples from a few boreholes. It accounts for the uncertainties associated with parameter being estimated and aims at minimising the variance of the estimation errors and providing the best linear unbiased prediction of the interpolated values. In the original kriging technique, the interpolated values are assumed to be governed by a Gaussian process with earlier known covariances. In other words, the method requires a knowledge of the spatial structure of soil variability.

The basic idea of kriging is to predict the value of a function at a given point by computing a weighted average of the known values of the function in the neighbourhood of the point. Figure 2 gives an example of one-dimensional data interpolation by kriging. The open red squares give the location of the data. The kriging interpolation (red curve), runs along the means of the normally distributed confidence intervals shown in grey. The dashed curve shows an additional spline function that, while smooth, can depart significantly from the values given by the means of the normally distributed variable.

Since the original work by Krige (1951), several variations of the kriging interpolation have been developed (e.g. Matheron, 1971). In the literature, one encounters terms like simple

kriging, ordinary kriging, dual kriging, universal kriging and Bayesian kriging. The latter will be discussed further in the last section of this paper.

4.4. The challenge with random field models

Random field theory has been increasingly used to model the inherent soil variability. To estimate the parameters that describe the spatial correlation, one needs a large quantity of data. The accuracy with which one can estimate scale of fluctuation depends on both the sampling intensity and extent of the sampling range. Generally, while the mean and variance can be determined conveniently, much more efforts are required to estimate the SoF of the random field (Onyejekwe and Ge, 2013).

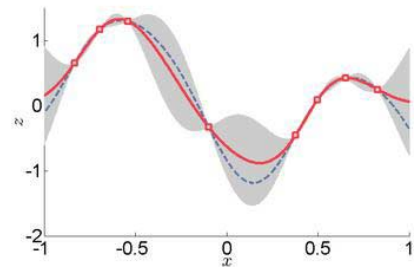


Figure 2. Example of one-dimensional data interpolation by kriging.

The quantity of data is often quite limited in geotechnical engineering. It is also largely unclear to the geotechnical profession how and where the samples should be taken in the field to ensure a reliable estimate of the SoF. The site exploration program is often planned without particular consideration of the random nature of the soil characteristics. Nie et al (2015), through numerical simulations, suggested that for an accurate estimate of the scale of fluctuation, one needs 10 measurements within one scale of fluctuation, the sampling should extend over at least 100 scales of fluctuation. The first example in the next section satisfies these requirements, but the author has encountered very few cases in practice where such quantity of data is available.

The challenge in practice is that spatial averaging, although an elegant approach, needs large quantity of data, and that there are rarely enough data to do an adequate evaluation of the autocorrelation distance and scale of fluctuation in the

horizontal direction. It is naturally possible to do spatial averaging in the vertical direction in most cases.

Neglecting the spatial variability means that the variance of a soil parameter will be overestimated, which would lead to a conservative estimate of the probability of failure in a probabilistic analysis. In a deterministic analysis, the spatial variability, and how it may impact the perception of the safety factor, is probably not considered.

5. Cone penetration testing offshore

For the last 40 years, the Cone Penetration Test (CPT) has played a key role in offshore soil investigations, mainly in connection with oil and gas development, but also for other purposes. The offshore application of CPT has been an important factor with respect to development of equipment, data processing and interpretation (Lunne 2012).

In most parts of the world, it is hardly possible to consider an offshore soil investigation without the use of the CPT, and the results are essential input for establishing the soil profile and soil parameters for foundation design. Most of the developments have been in response to requirements from the oil and gas industry, where the main challenges have been to cover deeper water and harsher and more remote areas.

There are basically two ways of pushing a cone penetrometer into the sea bottom (e.g., Zuidberg et al. 1986; Lunne 2001):

- By pushing from the sea floor until refusal, or a predetermined penetration; this has traditionally been called seabed mode.
- By drilling a borehole and pushing the penetrometer into the soil at bottom of the borehole; this is usually called down-hole mode or drilling mode.

In many situations, seabed-mode testing will be the most cost-effective solution and give the highest quality results (e.g. Peuchen 2000). Under favourable conditions, 40–50 m penetration below seabed can be achieved.

The first cone penetrometers used offshore measured the cone resistance, q_c (or the corrected cone resistance, q_t) and sleeve friction, f_s (de Ruiter, 1971). Since the early 80s, the piezocone

(or CPTU), which also measures the pore pressure, u , has been used in many offshore site investigations (de Ruiter, 1982).

Today, the offshore soil investigation industry generally uses a rather limited number of different cone types, all of which adhere to the EN-ISO 22476-1 Standard with respect to geometry and size. In order to find out if cones used by different organizations give similar results, a series of tests were performed at NGI's soft clay test site in Onsøy, about 100 km south of Oslo, with a number of cone penetrometers, including at least four piezocones (CPT with pore pressure measurement, CPTU) that are typically used offshore. The clay at the Onsøy site is spatially very uniform and ideal for such studies (Lunne et al., 2003). The tests showed that there are no significant differences in the corrected cone resistance (q_t) and the pore pressure (u_2) as long as the cones are properly saturated. However, the measured sleeve friction varied significantly as shown in Figure 3, where typical results of CPTs carried out using cones operated in offshore soil investigations are included. With the present large variations in f_s values, it is not possible to utilize this measurement to its full potential in terms of interpretation of results, as for instance advocated by Robertson (2009). It is, therefore, important for the profession to better understand the reasons for the large variations in the sleeve friction (f_s) readings, and to develop specifications that lead to more uniform practice.

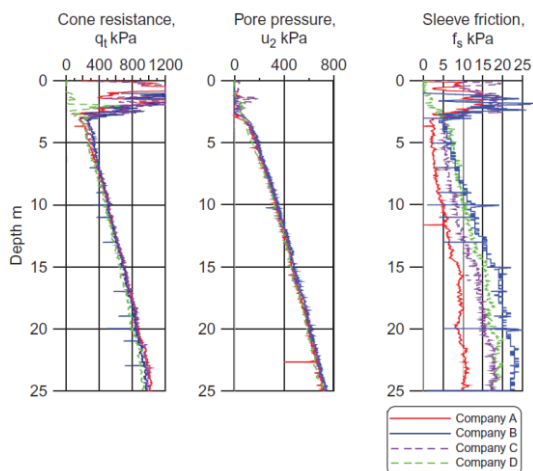


Figure 3. CPTU results in Onsøy clay with four penetrometers (Lunne 2012).

Almost all estimates of the scale of fluctuation of soil properties in the vertical direction at offshore sites found in literature are based on the analysis of CPT results (mainly the cone resistance). However, it is important to keep in mind that that key engineering soil parameters, like the undrained shear strength in clay layers, or relative density in sand layers, are estimated from the measured cone resistance, the excess pore pressure and to less extent the sleeve friction of CPT (or CPTU) using empirical and theoretical relationships. The transformation uncertainty introduced by these relationships is in most cases more dominant than the variability (aleatory uncertainty) of the CPT or CPTU measurements, or the variability of the undrained shear strength of friction angle.

6. Example applications

6.1. CDP1 Platform in the North Sea

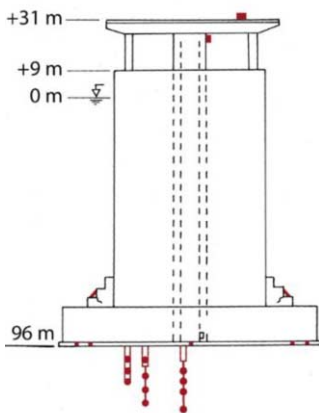


Figure 4. CDP1 platform in the North Sea.

The CDP1 gravity base offshore platform in the North Sea (Figure 4) is founded on very dense sand and a thin layer of variable overconsolidated clay extending down to 16 or 17m.

The structure has a circular annular raft base of 101 m in diameter. The operation deck is about 130m above the seabed.

The structure had a long history of erosion (due to lack of foundation skirts at the base) and other mishaps, as reported in Lacasse et al. 1991. A large number of cone penetration tests (CPT) were run in 1975 and again in 1983 to verify the foundation stability.

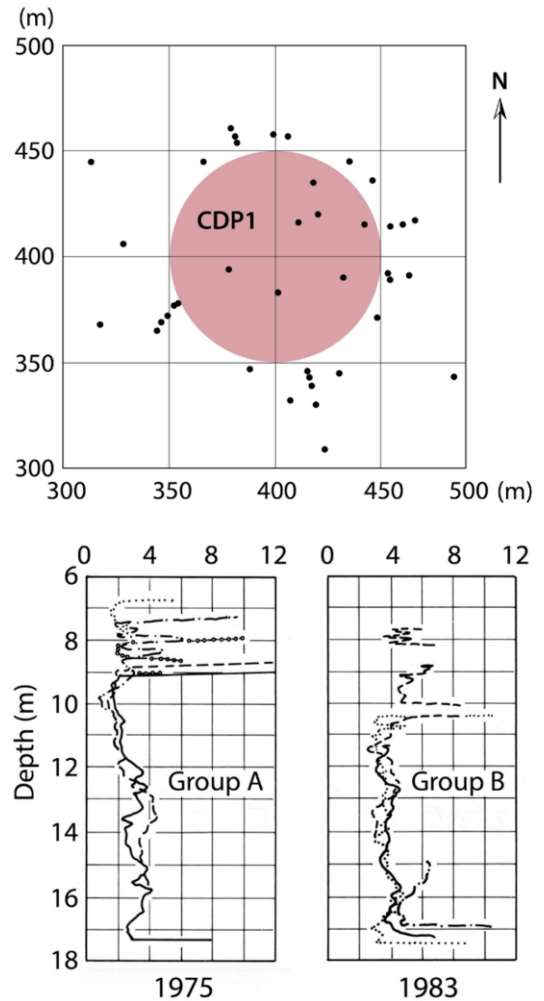


Figure 5. CPTs run at CDP1 Platform site.

Figure 5 shows some of the cone penetration tests where the weaker clay layer was "definitely

found" (Group A) and "definitely not found" (Group B).

Figure 6 shows the estimated depth to the weaker clay layer by kriging in form of contours (Nadim, 1988). The depth of the top of the clay layer beneath the platform, based on the spatially averaged values, is at least at a depth of 9m.

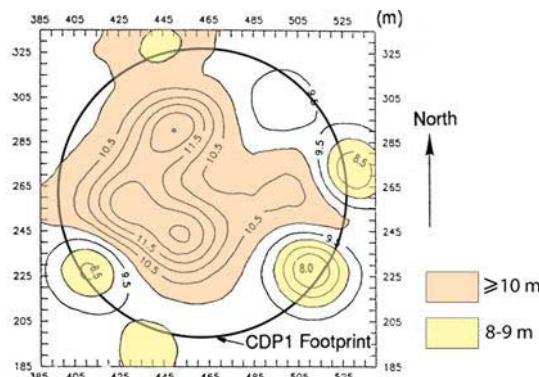


Figure 6. Depth to top of weaker clay layer close to footprint of CDP1 platform.

Horizontal sliding at the top of the clay layer was the most critical failure mode (Fig.7). The elevation of the top of the weaker clay layer beneath the foundation was therefore determinant for the resistance of the structure to wave loading.

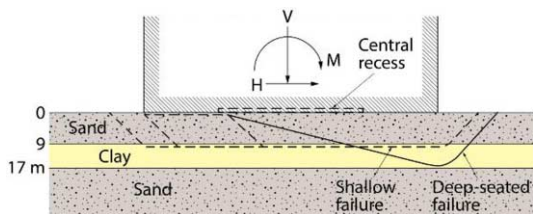


Figure 7. Deep-seated and shallow foundation failure mechanisms, depending on the depth of clay layer.

Establishing a satisfactory factor of safety was required for the continued safe operation of the platform. It was originally believed and assumed that the depth to the top of the clay layer was at 7m. With the revised environmental loads, a clay at this depth would have led to unacceptable margin of safety for the foundation.

The accurate estimation of the depth to the clay layer below the footprint of the platform allowed the geotechnical engineers to assess the foundation stability under the design loads with much higher degree of confidence than before.

6.2. Probabilistic design of offshore piles

Liu et al. (2015) presented an example of the effects of spatial variability of soil properties on reliability-based design of offshore piles. The uncertainty in soil properties affects the design of geotechnical structures, such as offshore piled jackets. A complete geostatistical characterization of a soil property requires that the mean, variance and the scale of fluctuation of the property be obtained.

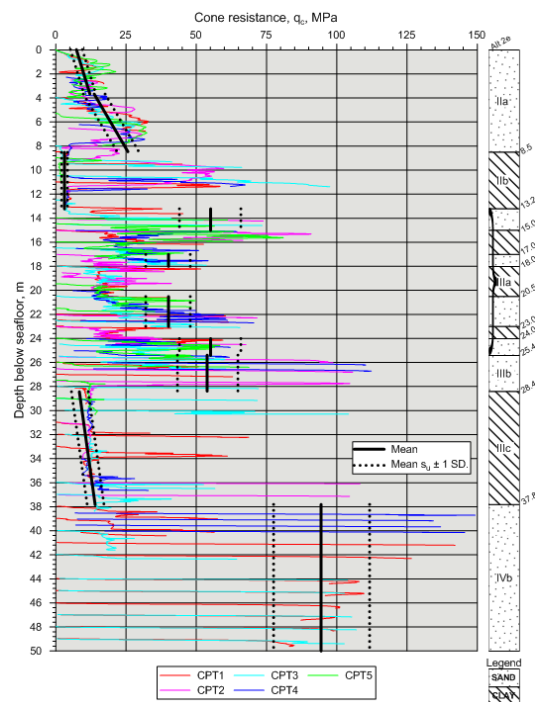


Figure 8. Cone penetration resistance with mean \pm 1 standard deviation (SD).

From the results of piezocone penetration tests (CPTU) at the location of one piled jacket offshore (Fig. 8), the vertical scale of fluctuation of the cone resistance at different depths was determined using random field theory. The vertical scales of fluctuation computed from the cone tip resistance profiles varied for 18 to 39cm in the different soil units.

The effect of accounting for spatial averaging on the reliability-based design of the axial capacity of piles was illustrated through a reliability analysis of the axial pile capacity with calculation of annual probability of failure. The results of the reliability analyses showed that

accounting for spatial averaging in the vertical direction would reduce the calculated annual failure probability of the pile under axial loading by a factor of 2 to 3.

7. Characteristic soil properties

The reliability of offshore installations under severe environmental loading depends on the uncertainties in the parameters in the analyses. The selection of the characteristic values to use in deterministic analyses is often a source of uncertainty, and can be very subjective. The values can also vary significantly from one engineer to the other. It is essential to define in no ambiguous terms the characteristic value to use for design. The definition should try to minimize the degree of subjectivity in the selection of characteristic values. This definition is most significant for the selection of the characteristic strength parameters, for both deep and shallow foundations, in both clay and sand.

7.1. Code definitions

The draft international standard ISO/DIS 19901-4 provides the following definition and guidelines for the characteristic soil property:

"The characteristic value is the main representative value assigned to a basic variable associated with a prescribed probability of not being violated by unfavourable values during some reference period.

The principles and guidelines for selecting characteristic values of soil properties should be in line with the partial factors format or partial factor design (PFD) approach. Soil stratification and estimation of characteristic values for soil properties should consider:

- the assumptions made in the calculation model;
- the spatial variability of soil within each stratum;
- the amount and quality of site investigations and possible environmental influences, including insufficient data and imprecise knowledge;
- a priori knowledge such as geological information and physically credible values;
- measurable physical quantities that correspond to, and are representative of, the population of the properties considered in the calculation model;
- appropriate factors or functions, to convert the properties obtained from test specimens, in situ tests and other methods to properties corresponding to the assumptions made in the calculation model;
- measurement error, conversion factor uncertainty and statistical uncertainty; and
- variance reduction by appropriate methods (if relevant)."

The definition of the characteristic soil properties in other codes also leaves a lot to the judgement of the geotechnical engineer in defining the design soil profile, i.e. they are qualitative and descriptive. For example, the German standard DIN 4020 has the following clause on how to select characteristic values of soil properties

"The characteristic values shall be selected in such a way that the calculations in which they are used yield conservative results."

In Eurocodes, sub-clause 2.4.5.2 of EN 1997:1: 2004(E) states that:

"If statistical methods are used, the characteristic values should be derived such that the calculated probability of a worse value governing the occurrence of a limit state is not greater than 5%".

This sub-clause has caused more confusion than clarification. Some engineers interpret the 95% confidence level to apply to all data, while others interpret it to apply to the estimated mean value of the data.

DNV (2012) in its recommended guideline DNV-RP-C207 provides an excellent treatment on how to describe statistically soil data. There is also ongoing activity in API and ISO offshore geotechnical committees to come up with a less ambiguous and person-dependent definition of characteristic soil properties.

8. Emerging method: Bayesian kriging

An approach that has great potential for combining the results of offshore geophysical and geotechnical site investigation data is Bayesian kriging. Omre (1987) introduced the concept of

Bayesian kriging. In Bayesian kriging, the user makes an a-priori qualified guess regarding the spatial trend function for the random field. Omre and Halvorsen (1989) provided a detailed derivation of the resulting equation system for a Bayesian predictor of the expected value of the random field $Z(x_0)$, where x_0 is an arbitrary location.

Omre (1987) provided an illustrative example of the Bayesian kriging method, with five observations spread along a line (Fig. 9). The qualified guess to the expected function $\mu_z(\cdot)$ is also given in the figure. The variance of z is assumed to be 3.0 and the autocorrelation function is assumed to be spherical with $r_0 = 3.0$, i.e.

$$\rho(r) = 1 - 1.5 \frac{r}{r_0} + 0.5 \left(\frac{r}{r_0} \right)^3 \quad (7)$$

Four cases were considered:

- I. When the expected function is assumed to be known exactly. This corresponds to subtracting a predefined drift from all observations and performing ordinary kriging on the residuals.
- II. When the qualified guess is assumed to be associated with an uncertainty which has variance proportional to the value of the expected function.

- III. When the qualified guess is assumed to be similar to Case II, but the variance is also increasing with increasing value of x .
- IV. When the ordinary kriging procedure is performed and the variance of z is equal to 6.0.

The Bayesian kriging procedure was performed on the first three cases, while ordinary kriging was performed in the last case.

Figure 10 shows the corresponding estimates in the range of x from 1 to 20. The following characteristics may be noted:

- All kriging estimates are exact, i.e. they run through the observations.
- The Bayesian kriging procedure (Cases I through III) provides estimates which tend toward the qualified guess, corrected by a constant, in the areas without observation. The ordinary kriging estimates level out on a constant value equal to the mean of the five observations.
- Uncertainties associated with the qualified guess have some influence on the estimates, although the influence is limited in this particular example. Larger uncertainty entails relatively larger weight to nearby observations.

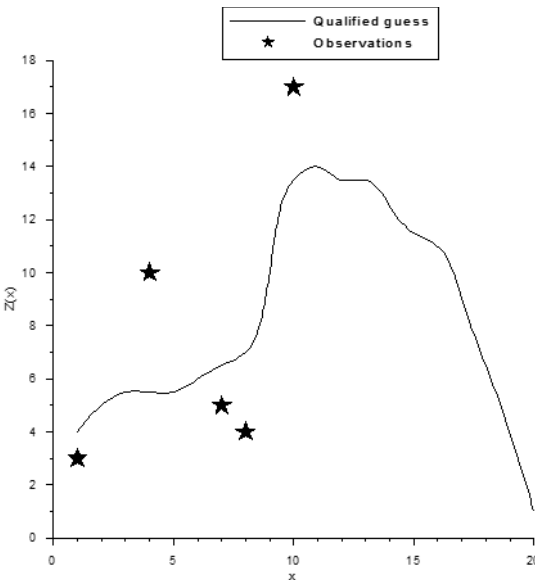


Figure 9. Observations and qualified guess in example given by Omre (1987).

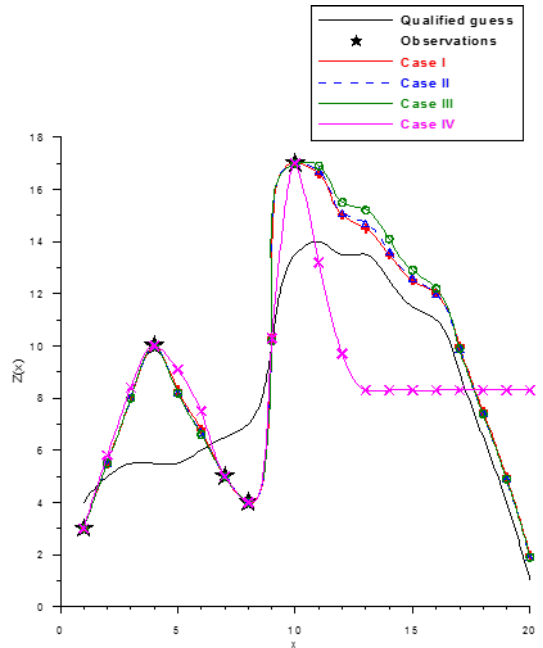


Figure 10. Observations, qualified guess and kriging interpolations in example given by Omre (1987).

Bayesian kriging is used in reservoir engineering for simulation of 3-dimensional reservoir geometry and geomechanical properties. It has not yet been applied in offshore geotechnical site characterisation. The author believes that Bayesian kriging provides a consistent theoretical framework for combining the results of geophysical surveys (which cover a very large area) with geotechnical site investigations.

9. Concluding remarks

Statistics help reduce the uncertainty in the parameters (and will therefore contribute to reducing the calculated probability of failure).

In the case of the CDP1 platform, the spatial averaging and interpolation by kriging helped verify that the stability was adequate and allowed the continued operation of the platform.

Introducing spatial averaging will reduce the uncertainty in the soil parameters. However, the reduction is probably not large compared to the other uncertainties in the analysis:

- The spatial structure of soil properties and the spatial averaging effects are real. It is very difficult in practice to establish the spatial structure of the soil properties in the offshore environment, especially in the lateral direction, because of the scarcity of data.
- Much of the uncertainty in the mechanical soil properties needed for geotechnical design is due to transformation uncertainty from the conversion of cone resistance to strength and the epistemic uncertainty due to the paucity of site-specific data, rather than due to the natural spatial variability. The epistemic uncertainty is not reduced by spatial averaging.

Neglecting the spatial averaging effects means that one overestimates the variance of the parameter of interest (conservatism).

The Bayesian kriging method described in Section 8 had a great potential for combining the results of different data source. The approach is used in reservoir engineering, but it has not yet been applied to offshore geotechnical site characterisation. The author believes that Bayesian kriging provides a consistent theoretical framework for combining the results of geophysical

surveys (which cover a very large area) with the results of geotechnical site investigations.

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