Impact of data acquisition parameters and processing techniques on S-wave velocity profiles from MASW – Examples from Trondheim, Norway

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ABSTRACT
The shear modulus is one of the most important engineering parameters, and it relates directly to the soil's shear-wave velocity. Shear-wave velocities can be derived from both laboratory measurements (e.g., bender elements) and various field methods (seismic CPT, surface wave analysis, and shear-wave reflection/refraction surveys). The major advantage of field measurements is that the soil is tested in its natural state, thus mitigating the effects of sample disturbance caused by e.g. drilling, tube insertion, extraction, transportation, storage, trimming, and reconsolidation.
In recent years, Multi-channel Analysis of Surface Waves (MASW) has become a powerful and common method for obtaining in situ shear-wave velocity data that can subsequently be used for geotechnical site characterization. Whereas this method is proven, it is important to be aware of its potential pitfalls and limitations from survey design to final interpretation of the results. Indeed, the receiver and source geometry, the time sampling, the processing methods as well as the inversion method and parameters used may influence the resulting shear-wave velocity profile. All survey design, processing and analysis parameters should be thoughtfully chosen taking into account the given site as well as the target. In addition, lateral variations (inhomogeneity, dipping layers) disturb the 1D assumption typically adopted in surface waves inversion. Therefore, it is important in engineering geophysics to investigate the reliability, potential and limitation of this prospecting method, properly take into account uncertainties and show that differences in sampling techniques and inversion may lead to differences in shear-wave velocity profiles and thus influence the site characterization.
In this study, we present data from one MASW experiment nearby Trondheim. The receiver and shot-geometrical effects are revealed and the necessity for careful survey design as well as the need for systematic data evaluation is highlighted. Shear-wave velocity profiles were inverted using two different algorithms revealing discrepancies, hence the need for appropriate inversion parameter selection.

Keywords: Surface wave analysis, *in situ* shear wave velocity, acquisition parameters, inversion parameters, soft clay

1 INTRODUCTION

Because of its direct correlation to the shear modulus and the possibility to obtain in situ measurements, the shear-wave velocity (*Vs*) is widely used in geotechnical engineering.

One of the methods used to extract in situ *Vs* is based on the characteristics of surface wave propagation and velocity dispersion. The common approach these days is the Multi-channel Analysis of Surface Waves (MASW) method (Stokoe et al., 1994).

For onshore applications, the analysis typically exploits the dispersive nature of Rayleigh waves in vertically heterogeneous media (Nazarian, 1984; Stokoe, 1988; Stokoe et al., 1994). The phase velocity dispersion results primarily from the variation of shear wave velocities with depth. An active MASW experiment includes (Socco et al., 2004):

1. Data acquisition during which the wave field related to the propagation of a perturbation generated by a controlled dynamic source is recorded by geophones or accelerometers on the ground surface (without clipping);
2. Extracting the dispersion curves from the seismic data; and
3. The inversion process, based on wave propagation in layered media, which allows establishing the S-wave profile as well as an uncertainty or error margin.

Each of these steps requires careful parametrisation.

Here, the need for adapted survey designs, objective dispersion curve extraction and careful inversion parameter selection will be discussed. Then, the latter is exemplified using data from various MASW surveys around Trondheim, Norway. Note that Rayleigh waves are only one of several surface waves that can be generated. Others are Love waves, generalized Rayleigh waves or Scholte waves. The study solely focuses on Rayleigh waves.

The wave equations have multiple solutions at given frequencies or wavenumbers, which explains the multi-modal character of surface wave dispersion.

2 THE FIELD PROCEDURE

For an MASW experiment, one typically uses a set of vertical receivers, usually geophones. These are spread (often in a linear array) on the ground surface to record the ground motion induced by the propagation of seismic waves generated by a source (see Figure 1). When using a vertical source (e.g., sledgehammer, weight drop), more than two thirds of the total generated seismic energy is emitted as surface waves, which stand out on the seismic gathers as high-amplitude events with variable waveform that propagate with low velocity. Therefore, the surface waves are the easiest seismic wave to generate onshore. Despite this, efficiently recording surface waves requires appropriate field configurations and acquisition parameters in order to reliably record the planar Rayleigh waves.

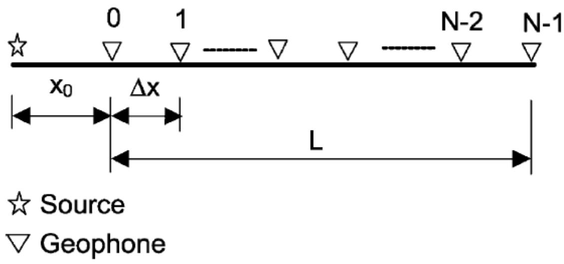

Figure 1 A scheme of MASW field setup, where x_0 is the source to 1st geophone offset, so-called near-offset, Δx is the geophone spacing, and L is the length of the receiver spread.

2.1 Importance of the near-offset

As the surface wave method requires the analysis of horizontally-travelling Rayleigh waves, it is important to avoid recording non-planar components. Surface waves only become planar after travelling a certain distance from the source. This distance is a function of wavelength (longer wavelengths require larger offsets), and thus it is site-specific (Stokoe et al., 1994; Park et al., 1999). Although the half-wavelength criterion ($x_0>\lambda_{max}$, Stokoe et al., 1994) has been adopted as a rule of thumb in this rule can be relaxed significantly (Zhang et al., 2004) and the proper near-offset distance is highly sensitive to the wavelength itself.

One also has to keep in mind that a larger value of the near-offset (x_0) and a longer receiver spread (L) will increase the likelihood of recording higher-mode surface waves.

2.2 Importance of the length of the receiver spread

The maximum investigation depth (Z_{max}) depends on the longest wavelength (λ_{max}) of the surface waves and is often used as Z_{max}=0.5λ_{max}. Also, λ_{max} is governed by the energy and area of impact of the seismic source, which may be active (e.g. sledge hammer or accelerated weight drop). The longer λ_{max}, (deeper the Z_{max}) can be achieved with greater source power and lower frequencies in the source signal. For unusually shallow investigation, a relatively light source with higher frequency content (>50 Hz) should be used. When conducting active measurements, ambient noise can be significantly reduced by vertical stacking of multiple impacts.

Typically, vertical low-frequency geophones (e.g., 4.5 Hz) are recommended and although planted geophones give the highest sensitivity, the coupling provided by a land streamer can be equally efficient. The high end of geophone frequency is not critical.

The length of the receiver spread (L) is directly related to the longest wavelength (λ_{max}) that can be analysed, which in turn determines the maximum depth of investigation (Z_{max}). Therefore, L usually has to be equal to or greater than Z_{max}.

The maximum extent of the receiver spread is also limited by body waves and higher mode surface waves which tend to dominate over the fundamental mode at far offsets and at higher frequencies (Park et al., 1998; 1999). Due to its relatively lower attenuation, the body waves (including refracted, and often guided waves) tends to dominate with offset. Because of its inherent complexity, it is difficult to assess the maximum offset simply based upon one single parameter, such as wavelength. Nonetheless, the resolution/sharpness of the dispersion curve image increases rapidly with the length of receiver spread and the number of active geophones (Park et al., 1998; 2001). When higher modes tend to take most of the energy and need to be separated from the fundamental mode, the resolution issue becomes critical. A longer receiver spread is therefore recommended, and also needed for the lower frequencies.

2.3 Importance of the geophone spacing

The receiver spacing (Δx) is related to the shortest wavelength (λ_{min}) and therefore determines the shallowest resolvable depth of investigation (Z_{min}). If the frequency content generated from the source is high enough, a short Δx is required to capture small *Vs* variation at shallow depth. Moreover, as the number of active channels increases, the ability to acquire data along a profile without roll-along is increased.

2.4 Importance of time

Finally, time sampling should be small enough to avoid aliasing, and a one-millisecond of sampling interval is commonly used with a 1 second recording time. However, a smaller time sampling would be necessary for processing of body waves, and a longer recording time mandatory in case of extremely low velocities or if a long receiver spread is used.

3 DISPERSION ANALYSIS

Extracting dispersion curves is critical in surface wave analysis, as the results are directly used in the data inversion.

3.1 Dispersion curve extraction principles

The dispersion curves are typically extracted after remapping the data into a different domain where the surface waves stand out as high-energy branches (e.g., frequency as a function of wavenumber, wavelength, slowness or phase velocity) that can be easily picked. To this end, the phase-shift method is one option, in addition to the more traditional methods like tau-P or the f-k method because it achieves higher resolution.

The standard data processing scheme is as follows. A multi-channel field record is first decomposed via FFT into individual frequency components, and then amplitude normalization is applied to each component. Then, for a given testing phase velocity in a certain range, necessary amount of phase shifts are calculated to compensate for the time delay corresponding to a specific offset, applied to individual component, and all of them are summed to make a total energy. This is then repeated for the different frequency components. When all the energy is summed in frequency-phase velocity space, it will show a pattern of energy accumulation that represents the dispersion curves. In case of multi-modal dispersion, the behaviour of energy will appear as multiple branches in the remapped domain. Other methods have also been suggested in the literature to achieve higher resolution such as the high-resolution radon transform method (Luo et al., 2009).

3.2 Considerations on the dispersion analysis

Several aspect of the dispersion curve extraction should be addressed carefully. When applying normalization to the frequency phase velocity spectrum, the direction of normalization will introduce artefacts which can be misleading for dispersion curve picking. It is therefore necessary to keep in mind this normalization aspect when conducting the dispersion analysis, and to exclude frequencies that are not contained in the original recordings.
The maximum energy associated with the surface waves in the phase velocity or FK diagram, is not necessarily associated to the propagation of the fundamental mode. Higher modes do not only appear at high frequencies, they can also exist at low frequencies and mode jumps may take place. Also, whenever trying to extract information from deep subsurface, one has to remember the limitation introduce by the field acquisition itself, i.e., the source weight and geophone natural frequency as well as the length of the array. Additionally, the energy accumulation at low frequency is usually poor, leading to smeared maximum and ambiguities. Recording passive seismic is a one way to enhance the lower frequency content and increase the confidence in the picking.
When not only the fundamental but also higher modes are present, one may wish to increase the resolution of the frequency-phase velocity diagram to help discriminate them. Long array length is then recommended; otherwise, trace padding would be advantageous.

4 DISPERSION CURVE INVERSION

Extracting dispersion curves is critical in surface wave analysis, as the results are directly used in the data inversion

4.1 Inversion principle

Inversion or inversion modelling, in general, attempts to seek the cause to a result when the result is known, whereas predicting the result from the given cause is referred to as forward modelling. An inversion is typically non-unique, thus multiple solutions exist. Rayleigh inverse problems are ill-posed implying that a given experimental dispersion curve may correspond to more than one soil profile. From a mathematical point of view, non-uniqueness in the solution of an inverse problem is either caused by a lack of information to constrain its solution or, alternatively, because the available information content is not independent. In light of this, including a priori information is often a good strategy to enforce uniqueness in the solution of an inverse problem.

4.2 Kinematic mode inversion

Whereas the goal of a field survey and data processing in MASW is to establish the mode dispersion curves accurately, theoretical kinematic dispersion curves can be determined for a given earth model using a proper forward modelling scheme (e.g., Schwab and Knopoff, 1972). The most important issue with the inversion process is to determine the best-fit earth model among many different models as efficiently as possible. The root-mean-square (RMS) error is usually used as an indicator of the fit between the measured and theoretical dispersion curves. The final solution is taken as the 1D Vs profile with a small value of RMS error. For the optimization, one can either use a deterministic method (e.g., least-squares method; Menke, 1989; Xia et al., 1999) or a random approach (e.g., Socco and Boiero, 2008). The former type is typically the faster at the expense of the increased potential of finding a local, instead of global, minimum. Another pitfall common to both types is the risk of numerical artefacts. For example, although a solution with a smaller RMS error is numerically acceptable, it may not necessarily represent a more realistic one. The multi-modal inversion technique utilizes both the fundamental and higher-mode curves for the inversion. This is done in order to increase the accuracy of the final 1-D Vs profile by narrowing the range of solutions with 1-D Vs profiles otherwise equally well suited if only the fundamental curve was used. This method can also be used to alleviate the inherent problem with the inversion method of non-uniqueness in general.

4.3 Dispersion Image Inversion

The method of inversion includes the use of dispersion image data (the frequency-phase velocity diagram) instead of discrete dispersion curves, and does not involve the extraction of modal curves (Ryden et al., 2004; Forbriger, 2003a; 2003b). This approach eliminates the drawbacks of the modal-curve based inversion such as mode-misidentification and mode-mix problems (misidentifying higher mode curve as a fundamental curve) if data acquisition and subsequent processing are not properly performed. Dispersion curves, when misidentified, may lead to erroneous Vs profiles because of the lack of compatibility in the inversion process trying to match measured and theoretical curves.

4.4 Full-waveform Inversion

This type of inversion utilizes the raw multichannel record instead of the one processed for dispersion imaging (Forbriger, 2003b). In the process, the scheme attempts to compare the whole seismic waveforms observed with synthetic waveforms generated from a forward modelling scheme. This type of approach may be advantageous over others for the fact that it is not biased by any other kind of data processing or remapping. However, it has to take into account the attenuation and interference issues, as well as layer parametrisation, since all of these can contribute to the shaping of a seismic waveform.

4.5 2D Vs Inversion

This approach uses the final outputs of 1D Vs profile from current typical inversion approach as input to the second phase of the inversion based on a different forward modelling scheme. The main objective is to consider the smearing effect caused by the lateral variations during dispersion analysis as much as possible by adopting another scheme accounting for the local variation of Vs. For this purpose, the Vs structure is provided by the co-location of the previous 1D Vs output as an initial starting model to account for the local variations observed within an individual field record to update the 2D Vs model.

4.6 Considerations on the inversion parameters

Before proceeding with the inversion, one should be confident with the field data as well as with the use and evaluation of dispersion curves or wavefield transformations, considering that inversion is not objective but rather a numerical data interpretation approach.
The result of the classical fundamental mode inversion approach, is entirely subject to the correctness of the picking. Using an erroneous modal dispersion curve, the optimization adopted within the inversion will not overcome this problem.
Moreover, one has to keep in mind that the forward modelling for surface wave analysis is typically 1D. Therefore, one should be careful when applying MASW in laterally heterogeneous media.
For any type of inversion process, an initial earth model is specified to initiate the iterative inversion process. For MASW, the earth model usually consists of velocity (Vp and Vs), density, and thickness parameters. Among these four parameters, Vs has the most significant effect on the convergence of the algorithm, followed by the layer thickness. The choice of the initial model is important. If the initial model is significantly different from "true model", the inversion method used should still converge to a reliable result. An initial Vs profile could be defined by making the simple assumption that Vs at a depth z is a factor of the measured phase velocity (Stokoe et al., 1994; Park et al., 1999).
The number of individuals/models and generations to adopt has to be proportional to the algorithm effort to achieve a good solution. Parameters then have to balance the number of layers, i.e. more layers means more freedom for the system and higher computing effort, and the width of the "parameters space". The search space can be defined according to prior geological and

stratigraphic information, and to the known Vs of the most common lithological types. If a site stratigraphy is known, the layer thickness and number can be set in order to give Vs a wider range. As such, one also reduces the freedom of the system and the numbers of individuals and generations to consider.
Finally, before interpreting the velocity spectrum, one should keep in mind the original data (traces) quality: the quality of the results depends on the quality of the input and on the parameters adopted for the inversion process, which must be clearly understood in its basic founding and driving principles. Surface wave analysis is not a trick giving a solution even if data quality is low. Other signals, particularly the guided waves, can result in dispersive signals that could wrongly be read as surface waves. Besides, the different modes can interfere with each other and give misleading or wrong results.
The user should be suspicious of any unexpectedly high propagation velocity (a value about the double what one would expect) that needs to be related to guided waves whose propagation depends on the Vp but not on Vs (e.g. Robertsson et al., 1995; Roth and Holliger, 1999).

5 SELECTED EXAMPLES FROM TRONDHEIM, NORWAY

Seismic data presented in this section were acquired close by Byneset, approximately 18 km west of Trondheim, Norway. The study area is composed of thick marine clays overlaying mica-gneiss bedrock. Here, a large quick clay landslide occurred in 2012.Three MASW profiles were recorded in the vicinity of this landslide (Figure 2). All profiles were acquired using 24 10 Hz vertical geophones. Profile Sh1098 and Sh1152 were acquired at the same location with two different setups: 1 and 3 m geophone spacing, a record length of 1 and 2 s, a 5 and 10 kg sledge hammer and 1 m and 8 m near-offset respectively for Sh1098 and Sh1152. Profile Sh1094 has the same setup as for Sh1098 with 12 m near-offset.

Using the commercial WinMASW software (Dal Moro et al., 2015), the seismic traces are time windowed to remove unwanted signals, e.g., the high frequency refracted waves (first arrival), before extracting both the phase velocity and FK spectra.
The fundamental mode is then picked in phase-velocity domain (with quality control. in FK domain), and subsequently stored for inversion. The dispersion curves are then used for inversion using WinMASW and NGI in-house inversion routine based on LAYSAC forward modelling (Kaynia, 1996).

5.1 Raw data analysis

The field setup was defined according to the a priori information and limited by the available equipment. Therefore only the Rayleigh wave fundamental mode is considered here. First, the geometry setting is controlled and data quality is assessed both in time and frequency domain (Figures 3 to 11). The data quality is good with high signal-to-noise ratio and no pre-processing is required prior to surface wave analysis With the short receiver spread, the recording time of 1 s is sufficient.

Figure 2 MASW profile location map. The blue star indicates the location of the seismic-CPTu. One can notice the remoulded surface where the quick clay landslide took place.

Looking at both data sets, one can notice that for Sh1098, the channel closest to the source (22-24. 3-1 m offset) is contaminated by near-source effects as well as direct and refracted waves. On the other hand, Sh1094 with 12 m near offset and Sh1152 with 8 m near offset, are free from near source interference and the planar assumption is met from trace 1. Due to the short spread length, the far offset are not contaminated with body wave energy, but higher modes already start to show up.

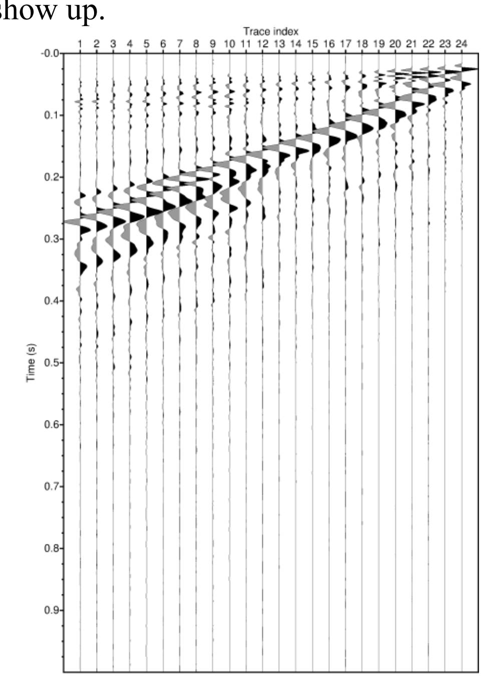

Figure 3 Sh1098, source at 1 m and 1 m geophone spacing.

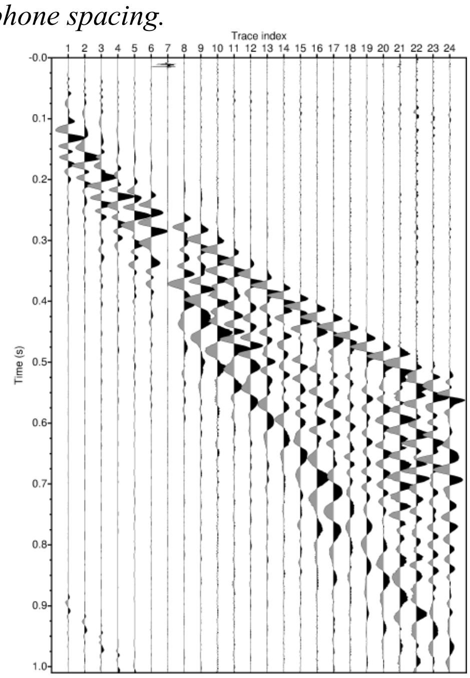

Figure 4 Sh1152, source at 8 m and 3 m geophone spacing.

5.2 Dispersion analysis

Frequency-phase velocity and frequency-wave number are computed from raw data and displayed without normalization. Guided by direct modelling and the a priori geological knowledge, the fundamental mode dispersion curve alone is picked in the frequency-phase velocity domain. The picks are then controlled in the frequency-wave number diagrams. Picking is achieved down to approximately 5 Hz in both cases, which is questionable considering the field setup. Picking is limited to around 32 Hz for Sh1098 and 25 Hz for Sh1098 and Sh1152 respectively, mainly because of the mode-mixing at higher frequencies.

Comparing the frequency-phase velocity diagrams of Sh1098 and Sh1152, one can already notice large discrepancies. Indeed, the fundamental mode energy is more widely spread for Sh1098 than for Sh1152, therefore reducing the uncertainty on the picks at low frequency. On the contrary, due to higher mode being more energetic, the fundamental mode frequency content is higher for Sh1098 than for Sh1152, but mode separation is easier for Sh1152.

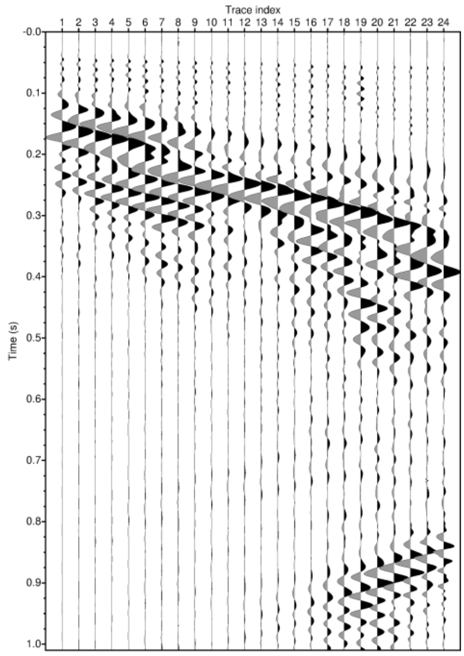

Figure 5 Sh1094, source at 12m.

5.3 Fundamental mode dispersion inversion and results

An initial model must first be defined. A 6 layer (including half space) model is chosen and the velocity and thickness constraints for the search space are defined according to the dispersion curve. The winMASW inversion is then run with 80 generations for the optimization procedure and 80 models of the population. The best model (the one with the smallest misfit) and the mean one (in this case defined according to a Marginal Posterior Probability Density computation, see Dal Moro et al., 2007) result in almost similar *Vs* model for both sites (Figures 12 and 13). If a discrepancy occurs between the two models, it would suggest that the inversion parametrisation was inappropriate. Inversion is also run using NGI in-house routines using the same initial models. The *Vs* models obtained are similar to the one obtained using winMASW inversion algorithm. Nevertheless, the retrieved depths of investigation differ significantly.

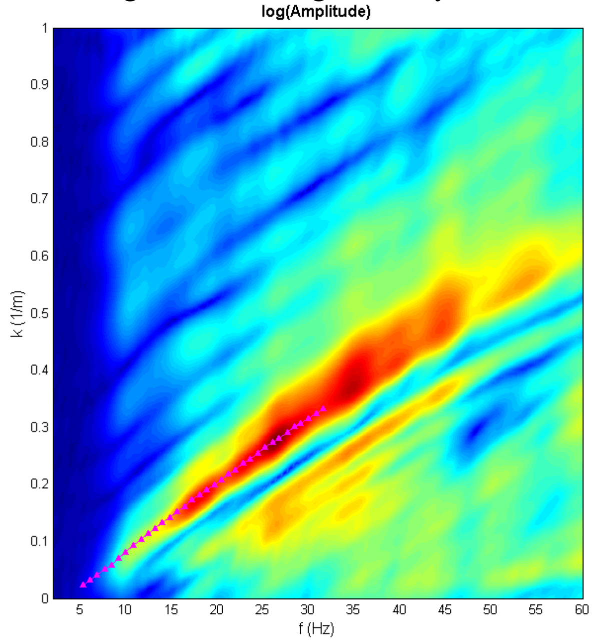

Figure 6 Sh1098, corresponding FK diagram with fundamental dispersion curve picks superimposed.

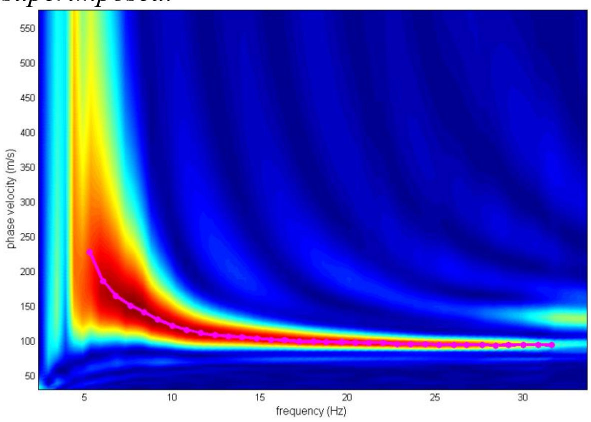

Figure 7 Sh1098, corresponding frequency - phase velocity spectrum with superimposed fundamental dispersion curve picks.

However, additional layers would have permitted the NGI inversion routine to reach equivalent depth.

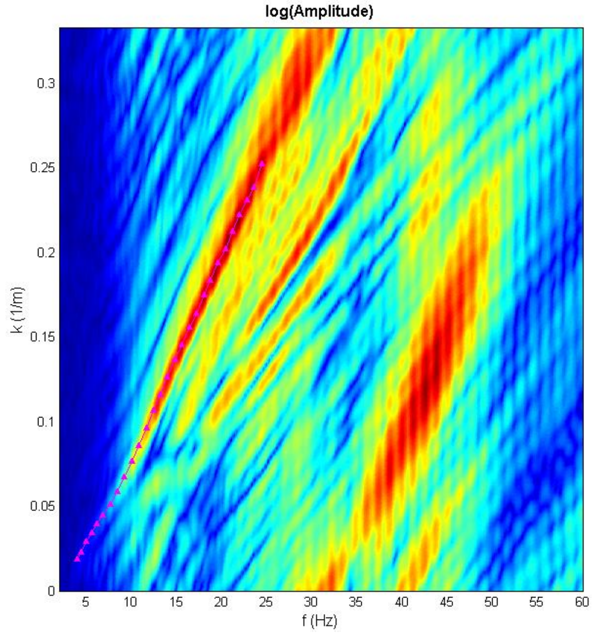

Figure 8 Sh1152, corresponding FK diagram with fundamental dispersion curve picks superimposed.

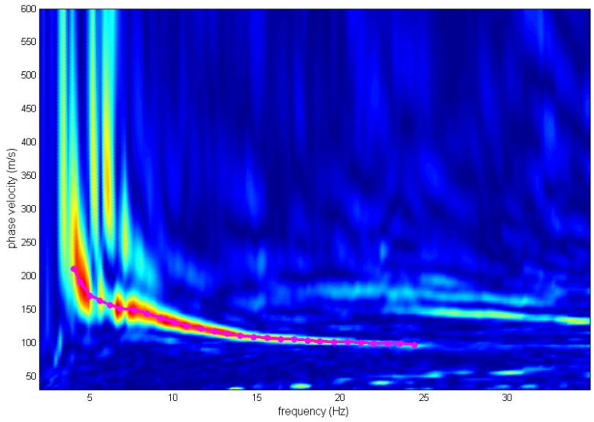

Figure 9 Sh1152, corresponding frequency - phase velocity spectrum with superimposed fundamental dispersion curve picks.

Comparison with the seismic cone penetration testing (SCPT) and a crosshole test (Eide 2015) is also available at Sh1094 (Figure 13). Even if the SCPT lack in coherence with depth, the comparison with the *Vs* derived from the MASW process shows good agreement, validating the inversion results.

Boreholes in the vicinity of the MASW profiles indicate that the bedrock should be deeper than 40 m, therefore, the high *Vs* found at depth in the winMASW results might be erroneous.

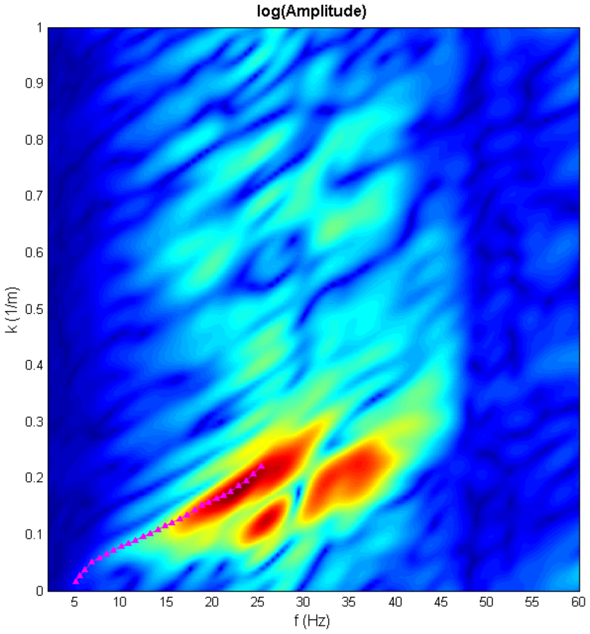

Figure 10 Sh1094, corresponding FK diagram with fundamental dispersion curve picks superimposed.

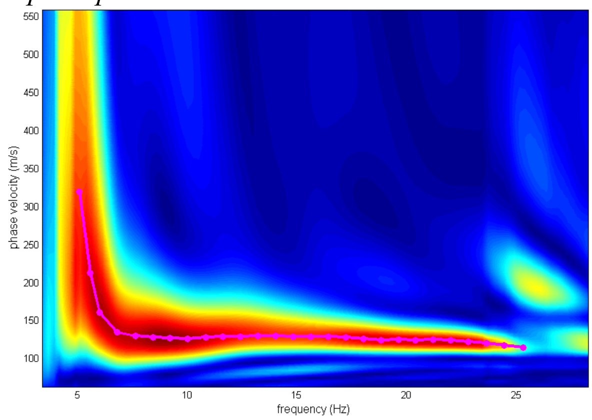

Figure 11 Sh1094, corresponding frequency - phase velocity spectrum with superimposed fundamental dispersion curve picks.

6 CONCLUSIONS

The shear wave velocity of soils is an important parameter in geotechnical design. It is, for example, widely used for seismic design criteria in the Eurocode 8. Uncertainties in the choice of Vs can have large consequences on project economy and design methods in practice. One of the main method used nowadays to gather Vs data is the MASW technique. It is of upper most importance that geophysicists and geotechnical engineers be aware that even this method can lead in uncertainties in the choice of Vs. Care is necessary when planning field surveys and when analysing

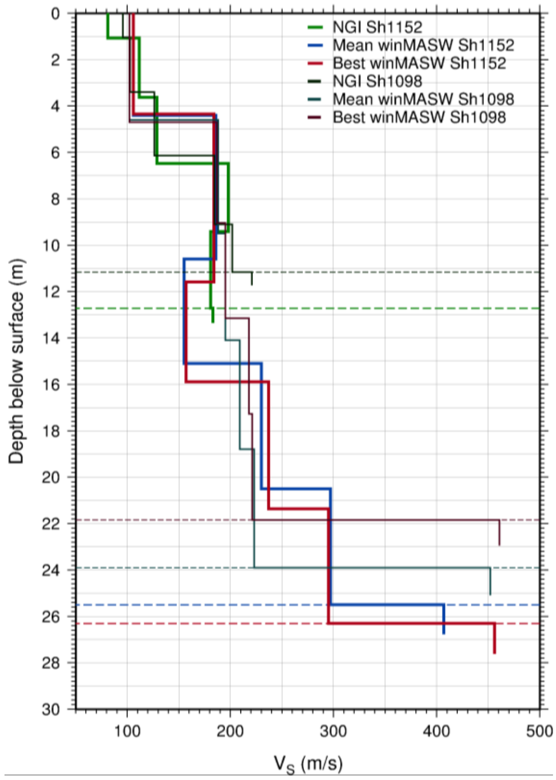

Figure 12 Sh1098 and Sh1152 inversion results.

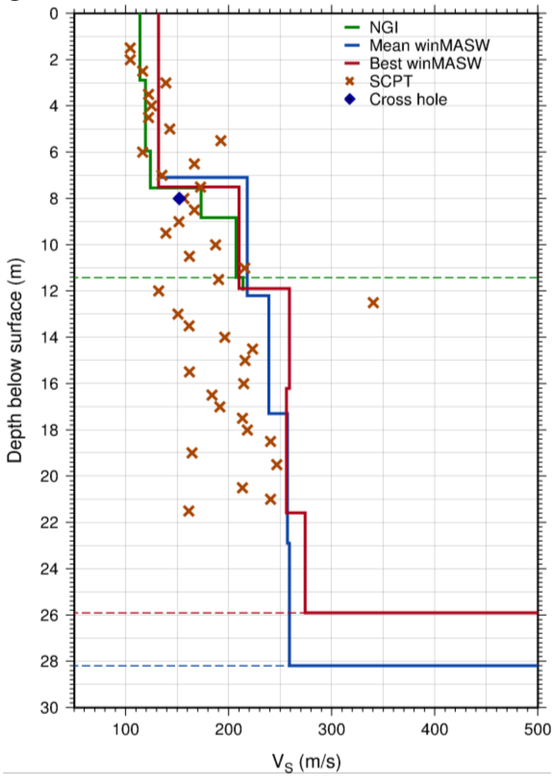

Figure 13 Sh1094 inversion results and comparison with seismic-CPTu and cross-hole.

such data. The present study showed that MASW field acquisition setup directly impact the dispersion analysis results. The near-offset distance, the geophone spacing, the array length, and the source frequency content directly affect the dispersion

diagram. Stable dispersion spectra with good resolution can be obtained when the optimal acquisition parameters are employed. A target oriented field setup is therefore mandatory for proper dispersion analysis, and, to estimate the optimum acquisition parameters, it is necessary to obtain information on the approximate velocity range.
Once a good dispersion results has been achieved, dispersion analysis of fundamental and higher modes should be conducted with good care, keeping in mind the field setup and limitations. In order to avoid introducing subjectivity through dispersion curve picking, velocity spectrum or raw data inversion should be preferred. Then, cautious inversion parametrisation is crucial to avoid falling in local minima. Finally critical interpretation of the resulting *Vs* model will make a successful MASW analysis.

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