

A new method for estimation of critical speed for railway tracks on soft ground

Karin Norén-Cosgriff^a, Eric Gustav Berggren^b, Amir Massoud Kaynia^a,
Niels Norman Dam^c and Niels Mortensen^d

^aNorwegian Geotechnical Institute, NGI, Oslo, Norway; ^bEBER Dynamics AB, Falun, Sweden; ^cBanedanmark, Fredericia, Denmark; ^dnmGeo, Allerød, Denmark

ABSTRACT

This paper presents a new method, ETL (EBER Track Lab), which allows for estimation of critical speed from measurements from a running train at normal speeds. Hence, large distances can be covered in short time. The method is based on the idea that the dynamic amplification, and the change of shape of the displacement curve under the loaded axle, starts already at speeds well below the critical speed. By observing the change in the displacement curve and comparing it with the results from a theoretical model one can derive an estimate of critical speed. The method has been validated by calculations and test runs. Both the measurements and the calculations show that ETL can give a reasonable estimate of the critical speed from a measurement speed of 0.4 times the critical speed or higher. The method has been applied for inventory measurements on the main network in Denmark.

ARTICLE HISTORY

Received 8 March 2018
Revised 4 May 2018
Accepted 7 May 2018

KEYWORDS

High-speed railway; critical speed; track deflection; vibrations; measurement

1. Introduction

For high-speed trains excessive vibrations and track movement may occur when the train speed approaches the critical speed at which train speed coincides with the Rayleigh-wave velocity of the ground. The magnitude and impact of the vibration are most often severe as compared with normal train-induced vibration, and are not only worrisome for human disturbance, but also raise concerns about the running safety of the trains, degradation of the embankment and foundation soil, fatigue failure of the rails, and disruption of power supply to the trains. Vibrations due to high-speed trains may occur on very soft soil such as peat and some clays. For such soil conditions, critical speed problems may be encountered at speeds even below 200 km/h. In Denmark, there are plans to increase the speed to 200 km/h or more on large parts of the existing railway network. Since many parts of the network cross areas of soft soil, critical speed is considered an issue, and needs to be addressed in the planning phase.

Considering that the critical speed is closely related to the Rayleigh wave velocity of the ground, the following methods are used to determine the critical speed today: 1) Geophysical measurement using surface waves, e.g. MASW measurements to determine the shear wave

CONTACT Karin Norén-Cosgriff  knc@ngi.no  P.O. Box 3930 Ullevål Stadion, N-0806 Oslo, Norway Norwegian Geotechnical Institute, NGI

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velocity of the ground, or using the dispersion curves to calculate critical speed of the combined track-soil system as shown in [1]. 2) Dynamic measurements of the track, using a Track Loading Vehicle (TLV) or a variation of it, the Rolling Stiffness Measurement Vehicle (RSMV) [2]. 3) Use of correlations with data from geotechnical site investigations, most prominently CPT, and/or laboratory tests, to obtain a rough estimate of the shear wave velocity [3,4]. However, while the first two methods are either expensive and/or tie up the infrastructure for unacceptably long periods, the third method relies on data which is often not available when the purpose is screening of large sections of a railway network.

In this paper, a new method, ETL (EBER Track Lab) is introduced for estimation of structural parameters of the track. As a spin-off of this method, the critical speed on soft soils can also be detected. Since the method allows for measurement from a running train at normal speed, large distances can be covered in short time. The method has been used for inventory measurements of 1400 km track on the main network in Denmark as presented in this paper.

2. Description of the critical speed phenomenon

At low speeds, compared to the characteristic wave velocities of the medium, the ground response from a moving source is essentially quasi-static. That is, the displacement and stress fields resemble those for static condition but simply move under the load. However, as the speed of the load increases, dynamic phenomena gradually take over and dominate the response. This is reminiscent of the supersonic condition in aerodynamics. As shown by among others [5,6] the resulting vibration is different from that from normal train speeds and is often characterized by development of Mach-lines behind the train. The train speed that is equal to the characteristic surface wave velocities of the medium is called the critical speed (v_{cr}). As pointed out in [7] the mitigation measures used against critical speed problems are similar to the subgrade stiffening described for mitigation of normal train-induced vibration, but placed beneath the track, rather than in form of screens next to the track, the purpose being to increase the overall surface wave velocity.

When train speed approaches v_{cr} the track deflection increases dramatically. Correspondingly, the magnitude and impact of the vibrations become severe compared with normal train-induced vibration. Much of the competence within this area, especially in the Nordic countries, is based on the knowledge gained from Ledsgård on the west coast of southern Sweden. At Ledsgård the phenomenon was observed in 1997, shortly after the speed was increased to 200 km/h. An extensive program was initiated by the Swedish Railway Administration (Banverket) to explain the phenomenon, develop prediction and simulating tools and to find effective countermeasures [8,9]. [Figure 1](#) shows measurement of track displacement at Ledsgård before introduction of countermeasures. The figure plots the maximum upward and downward displacement amplitude as a function of the train speed. Measurements were performed up to a train speed of 200 km/h, and from the measured displacements the critical speed was estimated to 235 km/h. The peak to peak displacement of the rail was estimated to be about 25 mm at critical speed. Compared to the low-speed/static displacement of about 5 mm this indicated a dynamic amplification of 400%. The measurements also showed that the dynamic amplification starts at rather low speeds. Already at 100 km/h the

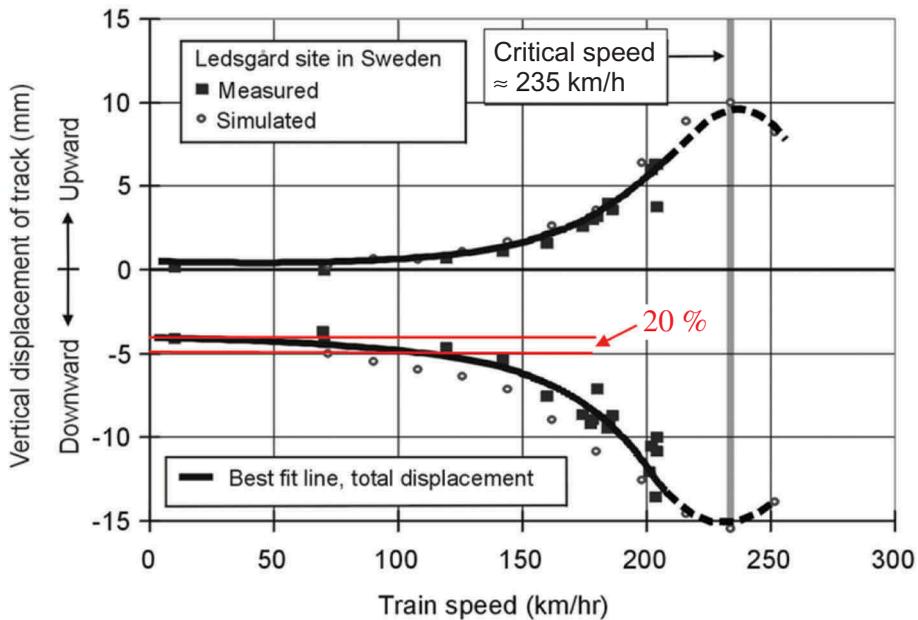


Figure 1. Displacement of track when train approaches critical speed. Measurements and simulations at Ledsgård using a Swedish X2000 train.

dynamic amplification in Ledsgård was about 20%. The experience from Ledsgård shows that in order to avoid excessive vibrations and track movement the train speed needs to be well below the critical speed. As a rule of thumb, one could use a limiting train speed of $v_{cr}/1.5$ for most soils (Poissons Ratio > 0.3) [10].

Vibrations due to high-speed trains may occur on very soft soil such as peat and soft clay. For such soil conditions, problems related to critical speed may be encountered at speeds even below 200 km/h. The critical speed approximately corresponds to the average shear wave velocity of the soil layers in the top 10–15 m representing typical wavelengths of Rayleigh waves. Typical shear wave velocities for different soils are listed in Table 1.

3. Theory behind method

The idea behind ETL for critical speed estimation is that the dynamic amplification starts already at speeds well below the critical speed and, at the same time, the shape of the displacement curve under an axle load will change. By measuring the rail deflection at several positions from the wheels along the rail, it is possible to describe the shape of the deflection

Table 1. Typical shear wave velocities in the top 10–15 m of different soil types.

Soil type	Shear wave velocity	
Till (Moraine)	400 m/s	1450 km/h
Dense sands and hard clays	250 m/s	900 km/h
Medium dense sand and medium soft clay	180 m/s	650 km/h
Loose sand and soft clays	120 m/s	430 km/h
Peat	50 m/s	180 km/h

curve accurately. The measured change in the shape of the displacement curve is then fed into a theoretical model. The unknown parameters describing the embankment and underground, are estimated by varying these parameters in the theoretical model until the best possible match is achieved between the displacement curve from the model and the measured displacement curve. The method estimates the critical speed for the actual track/ground conditions, and since the model parameters are varied until a good match are obtained for all measured sections individually, factors like track type, height of embankment, and performed ground improvements are automatically taken into consideration. After the model parameters are established, the model is used to calculate an estimate of the critical speed.

A simple but useful theoretical model is the Euler-Bernoulli beam on an elastic (Winkler-type) foundation, including also time varying parameters, described by

$$EI \frac{\partial^4 w(x, t)}{\partial x^4} + m \frac{\partial^2 w(x, t)}{\partial t^2} + c \frac{\partial w(x, t)}{\partial t} + kw(x, t) = Q\delta(x - vt) \quad (1)$$

where EI is the bending stiffness, m is the mass, c is the viscous damping, k is the stiffness, Q is the moving (wheel) load, v is the velocity of the moving load, and w is the deflection under the load. One way of solving Equation (1) analytically involves introduction of the dimensionless parameter s ,

$$s = \lambda(x - vt)$$

where

$$\lambda = \left(\frac{k}{4EI} \right)^{1/4}$$

After substitution with s , the following homogeneous ordinary differential equation is obtained:

$$\frac{d^4 w(s)}{ds^4} + 4\alpha^2 \frac{d^2 w(s)}{ds^2} - 8\alpha\beta \frac{dw(s)}{ds} + 4w(s) = 0 \quad (2)$$

where

$$\alpha = \frac{v}{2\lambda} \left(\frac{m}{EI} \right)^{1/2} \quad (3)$$

is the ratio between the actual speed and the critical speed, and

$$\beta = \frac{c}{2m} \left(\frac{m}{k} \right)^{\frac{1}{2}}$$

is the critical damping ratio.

Equation (2) can be solved as outlined in [11]. An analytical expression for the critical speed is obtained by setting α in Equation (3) equal to 1 resulting in:

$$v_{cr} = \left(\frac{4kEI}{m^2} \right)^{1/4} \quad (4)$$

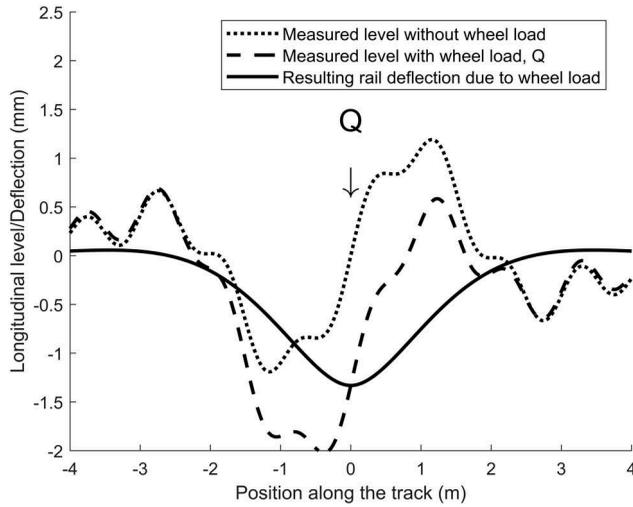


Figure 2. Principle of rail deflection measurement.

The critical velocity in this beam model is the minimum velocity at which a free wave may propagate in an infinite beam. Even though the critical speed in soil is a more complicated phenomenon, the dynamic amplification behaviour of the displacement under a moving load is of similar type.

Figures 2 and 3 illustrate the principle of the applied approach. The recorded longitudinal level of the track is a combination of the unloaded track irregularity and the deflection caused by the moving load. An example of the longitudinal level with and without a moving load Q is shown in Figure 2. The longitudinal level is measured by IMU (Inertial Measurement Unit) compensated lasers at different distances from the wheel(s) as the vehicle moves along the track. Each sensor collects longitudinal level

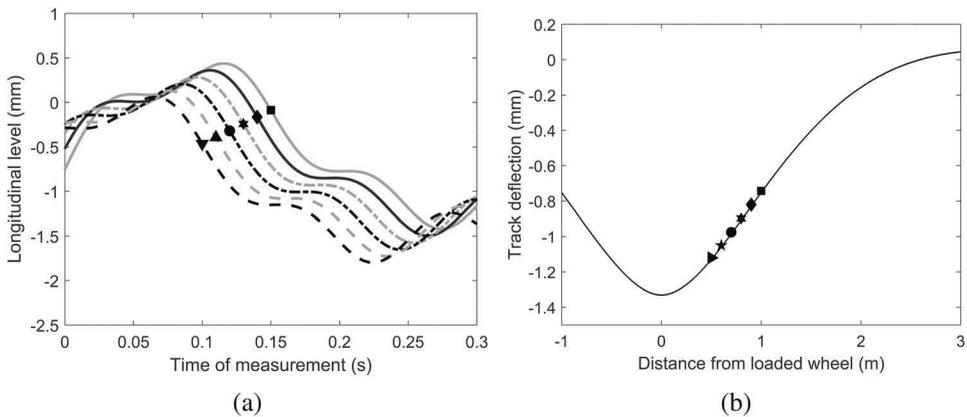


Figure 3. (a) Data from six sensors. One position in track is marked for each sensor. (b) Remaining estimate of rail deflection from six sensors and best fit model curve.

under loaded conditions, but with the important difference that the level is measured at slightly different load conditions. This is mainly due to the fact that the sensors are mounted at different distances from the wheel. In [Figure 3\(a\)](#) shows examples of a data collection from six sensors. The measured levels for all sensors at one given position along the track are marked in the figure. One of the sensors is selected as the master sensor, and its measured longitudinal level is subtracted from the longitudinal levels of the other sensors taking into consideration the difference in distance to the wheel. The remaining parts, after the subtraction, are samples of the rail deflection without the influence of the unloaded track irregularity. This is shown in [Figure 3\(b\)](#). More details of the measurements, data processing, and curve fitting procedure is found in [12].

To cover more complicated rail deflection shapes, a double beam model, or a more advanced track model may be necessary. However, a model with a limited number of unknown parameters, and where a closed-form solution is available, is a major advantage to speed up the analysis. This is because the statistical algorithms used in the analysis varies the unknown parameters and calculates many deflection shapes to choose the best match to the measurement data. Nevertheless, during the validation phase it was found that the single beam model was not sufficient to describe the phenomena. Therefore, a double beam model was applied as described in [Section 5](#).

4. Influence of measurement speed

As described in [Section 3](#), the idea behind the method is that the dynamic amplification starts already at speeds well below the critical speed and, at the same time, the shape of the deflection curve under a loaded axle will change. However, the train speed in the measurements needs to be high enough to induce a measurable change of shape of the deflection curve. In [Figure 4](#) responses for a double beam model are exemplified for a train speed of 216 km/h. At this speed a stiff track with a critical speed of 860 km/h deflects about 1 mm and the deflection curve is symmetric. A softer track with a critical speed of 480 km/h deflects about 5.5 mm. For this track, the simulated measurement speed corresponds to 0.45 of critical speed. The figure shows that besides an amplification of the deflection, the shape of the deflection curve is also slightly non-symmetric.

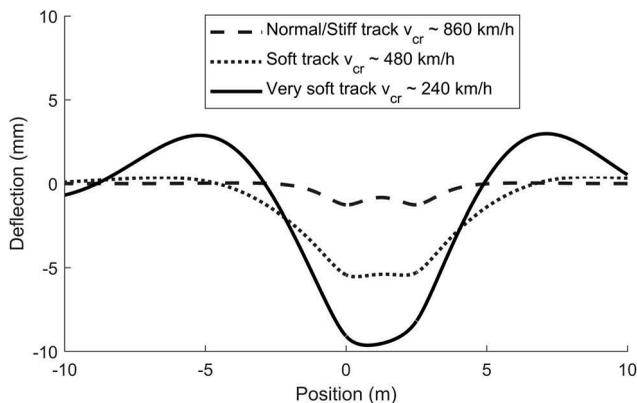


Figure 4. Deflection under bogie at train speed of 216 km/h. Calculated with a double beam model.

Finally, as an extreme case, a very soft track with a critical speed of 240 km/h is shown. For this track, the simulated measurement speed corresponds to 0.9 of critical speed and the change in shape of the deflection is clearly visible. Note that the beam-model does not capture the basic phenomena (wave propagation in the soil), but that the amplification and change in the deflection curve is clear. Theoretically, the method could give an estimate of the critical speed from a measurement speed of $0.4 \cdot v_{cr}$. However, practically, a measurement speed of $0.5 \cdot v_{cr}$ is more desirable for the method to give a reasonable estimate. Hence, to cover critical speed up to 250 km/h, measurement speed should be around 125 km/h; and to cover up to 300 km/h corresponding measurement speed should be in the range 150 km/h.

5. Validation of method through comparison with results from numerical model

The method was validated by comparison with calculations using the 3D numerical frequency-domain model, *VibTrain* [13]. In *VibTrain* the ground consists of viscoelastic soil layers over a half-space and the substructure and tracks are modelled as separate beams with elastic elements between them to represent rail/sleeper pad flexibility. The interaction between the substructure beam and the ground is accounted for by use of Green's functions for layered media (Figure 5). Estimation of the critical speed is based on finding the train speed that produces the largest displacement of the track. The software was validated against field test data from Ledsgård and for the Swedish train X2000 [8,13].

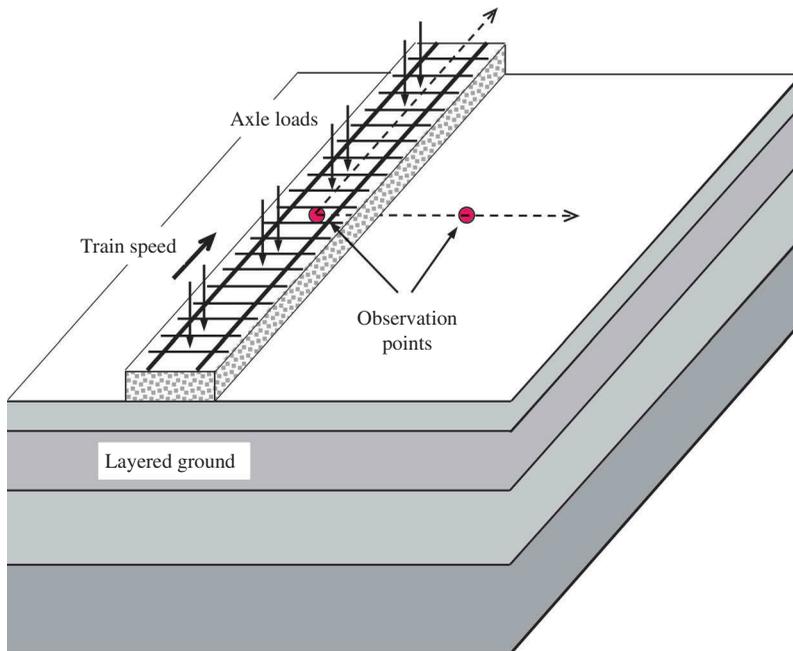


Figure 5. Key features of *VibTrain* consisting of layered ground, embankment modelled as an equivalent beam, and rails.

This model properly handles the interaction between the embankment and the soil as well as propagation of the waves in the ground. These are key issues in computing the critical speed. Critical speed is a different phenomenon in a simple beam model as used in ETL than in an advanced model as *VibTrain* that treats the ground as a layered half space. Nevertheless, amplification of track response is expected to be similar.

The theoretical validation of ETL was performed by simulated test runs at the Ledsgård site. Displacements under the bogies were calculated using *VibTrain* to simulate the measurement car, IMV200, passing the Ledsgård site at different speeds. The calculated displacements at positions where the sensors can be placed were then used as input to a simulated test run with ETL. An uncertainty of 0.1 mm was added to the calculated input data to account for measurement uncertainty in the sensors. This uncertainty value was chosen from known reproducibility of longitudinal level for the track geometry measurement car, IMV200, with a similar set of sensors. Figure 6 shows a comparison between the calculated deflection using a single beam model in ETL and the results using the rigorous 3D track-ground interaction model, *VibTrain*. The following model parameters were used in the single beam model, $k = 1 \text{ kN/mm}$, $EI = 12.22 \text{ kNm}^2$, $m = 1000 \text{ kg}$, $c = 20 \text{ kNs/m}$. As can be seen from the figure, *VibTrain* captures the correct behaviour, where only the deflection under each bogie is distinct, whilst the single beam model produces a distinct deflection pattern under each wheel. This shows that the single beam model, with physically reasonable values of the model parameters, is not sufficient to reproduce the deflection curve correctly.

Since the single beam model was not sufficient, a double-beam model was applied. In the double beam model, the upper beam, with its parameters EI_1 and m_1 , represents the rail. The rail pads are represented by stiffness and damping, k_1 and c_1 . All these parameters are known, and can be set to fixed values, see Table 2. The second beam, with its parameters EI_2 and m_2 , represents the embankment. The second beam is resting on an elastic foundation represented by a Winkler bed, described by the parameters k_2 and c_2 . These parameters are unknown, but can be found by fitting the deflection calculated by the model to the measured deflection. A full displacement pattern can be produced from the model even though only a few points are measured.

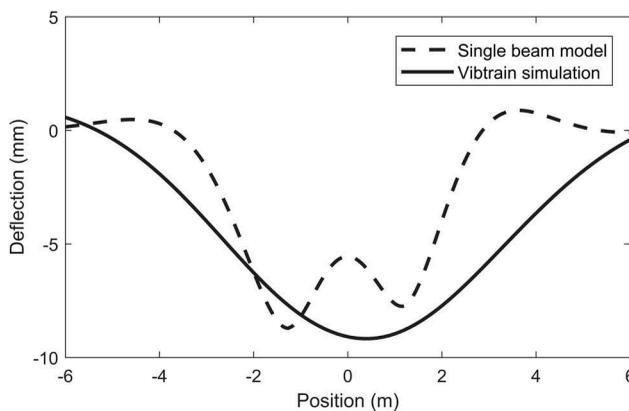


Figure 6. Comparison of results with single beam model and *VibTrain*-simulations at Ledsgård for a train speed of 125 km/h.

Table 2. Beam parameters.

Item		Value	Unit
Rail bending stiffness (UIC60)	EI_1	6293	MNm ²
Rail mass	m_1	60	kg/m
Pad stiffness	k_1	184	kN/mm/m
Pad damping	c_1	20	kNs/m

Figure 7 shows a comparison between the calculated deflection using the double beam model and the results using *VibTrain*. As can be seen from the figure the double beam model reproduces the deflection curve much better than the single beam model in Figure 6. The following double beam model parameters have been used to produce the results shown in Figure 7, $k_2 = 3.3$ kN/mm, $EI_2 = 6.2$ kNm², $m_2 = 1887$ kg, $c_2 = 1.2$ Ns/mm². Estimated critical speed with the double beam model is 250 km/h, versus the actual value of 235 km/h. The results indicates that the ETL method estimates the critical speed with reasonable accuracy for an inventory method.

6. Validation of method through full scale test runs

More comprehensive tests were performed as full scale test runs on tracks with known or suspected high-speed problems in Sweden. The tests were conducted at the end of March and beginning of April 2016. Two sites were tested more extensively, performing several test runs at different speed and in both directions.

To evaluate the results, four possible outcomes of the test runs were identified, as follows:

- (1) correctly identify areas where critical speed is a problem;
- (2) correctly indicate no problem for areas where there is no problem;
- (3) falsely indicate problem with critical speed in areas where there is no problem;
- (4) falsely indicate no problem for areas where there is problem.

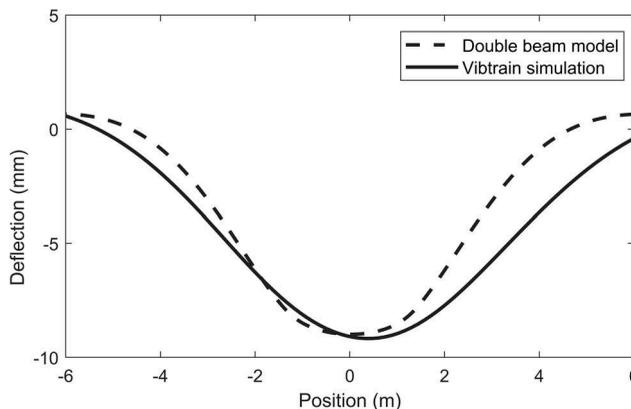


Figure 7. Comparison of results with double beam model and *VibTrain* results at Ledsgård for a train speed of 125 km/h.

Outcomes (1)–(3) are acceptable for an inventory method, also if the amount of false positive results has to be limited for the method to be successful. False positive results will require some extra evaluation, either by looking deeper into measurement data to find out if something is wrong, or by performing site investigation. However, false negative results is more severe since problem areas may be missed. One reason for false negative is too low measurement speed. Hence, it is important to point out that even though the measurements do not indicate any critical speed problems, the results can only be trusted for critical speeds up to about twice the measurement speed.

The test runs in Sweden showed that the method gives clear indications of low critical speed for all measured places with known high-speed problems (True positive). However, it may exist additional unknown problem areas despite no indication (False Negative). For very large portion of the network, the method as expected did not show any problems (True Negative). Some indications of problems were probably related to track geometry (False Positive). A couple of areas for improvement of the method were found which were resolved before the measurement campaign in Denmark.

At Ledsgård more extensive measurements were performed with several test runs at different speed. There are three tracks at Ledsgård, two for normal traffic and one station track. The west track was mitigated with lime-cement columns in 2000, and the vibrations were considerably reduced. The east track was not mitigated, although the high-speed phenomena was detected there as well, but it was not as severe as for the west track. The worst part according to previous investigations is at km 24 + 265. The critical speed for the west track was estimated to 235 km/h at this position before mitigation. Since the west track is now mitigated, the test runs in 2016 were performed mainly on the east track. The critical speed for the east track has never been determined, but it is expected to be somewhat higher than 235 km/h since the problems were less pronounced for the east than for the west track before the west track was mitigated.

Figure 8 shows the results from the test runs at Ledsgård's east track in 2016. The indication of low critical speed between km 24.22 – 24.31 coincides with an area of soft soil well known from previous investigations (although estimated critical speed may differ slightly from the true critical speed). The dots in the figure indicate that the lasers

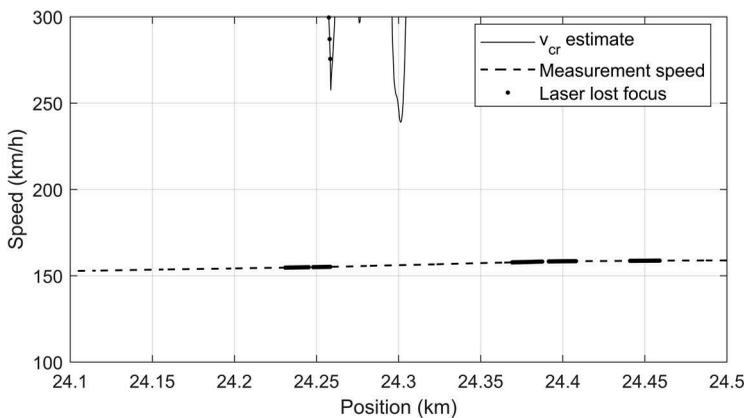


Figure 8. Critical speed measurements with ETL at Ledsgård.

in the system have lost focus of the rail, and the estimation has a much higher uncertainty.

The test measurements at Ledsgård showed that the critical speed phenomena was picked up at a measurement speed of 120 km/h ($0.5 \cdot v_{cr}$). At a measurement speed of 80 km/h, i.e. $0.34 \cdot v_{cr}$, the phenomenon was not detectable. This is in accordance with the expectations as described in chapter 4. However, as discussed above, the critical speed for the east track may be somewhat higher than 235 km/h, which makes these results conservative.

7. Inventory of main network in Denmark

Inventory measurements of the Danish network were performed in the period 19 – 20 November 2016. In the measurement campaign, 1400 km of track was measured, see [Figure 9](#). Most sections were measured at speeds that give useful evaluation of critical speeds up to about 250–300 km/h.

The measurements were performed from the track geometry measurement car IMV200. Besides the normal track geometry and ETL measurements, ground penetration radar (GPR) measurements were performed for all test sections. All test sections were also videotaped for easy identification of possible hot-spots.

The ETL results from all test sections were evaluated to find suspect areas where the collected data indicated low critical speed. Example of evaluation results are shown in [Figure 10](#). Note that the absolute values of critical speed can be estimated up to about twice the train speed only, i.e. here up to about 300 km/h. Hence the method should be considered as a means for finding suspect areas for low critical speed along large sections, rather than giving the actual critical speed values.

For the suspect areas identified by ETL, the GPR measurements were evaluated for more information with the intention of arriving at a short list of suspected hot-spots for critical speed problem. Based on this priority list, further evaluation of suspect areas with planned increased speed were performed using available geotechnical reports and geological maps. This resulted in 13 hot-spot areas covering about 900 m track. All hot-spot areas are located at or close to areas with fresh water deposits with pockets of peat, mud, clay or meltwater sand. In general, dips in the critical speed estimates coincide with soil types that are associated with softer ground, such as peat, mud etc. Based on these findings, additional in-situ investigations of the identified suspect areas, for example, with the help of MASW measurements, were recommended.

8. Effect of embedded soft soil layers

As shown in [Table 1](#) soils such as peat, mud, clay or meltwater sand have low shear wave velocities and correspondingly low critical speeds. Additionally, layer of softer soils interbedded in harder soils affect the shear wave velocity and the critical speed. To validate the ETL findings *VibTrain* was used to calculate the effect on the critical speed of a peat layer in sand, which is a typical condition in the identified hot-spot areas in Denmark. Two soft soil profiles representing two generic loose sandy sites containing a layer of peat were selected for the simulations. In addition, a sandy site (without any peat layer) was used as reference for evaluating the effect of the peat layers on the

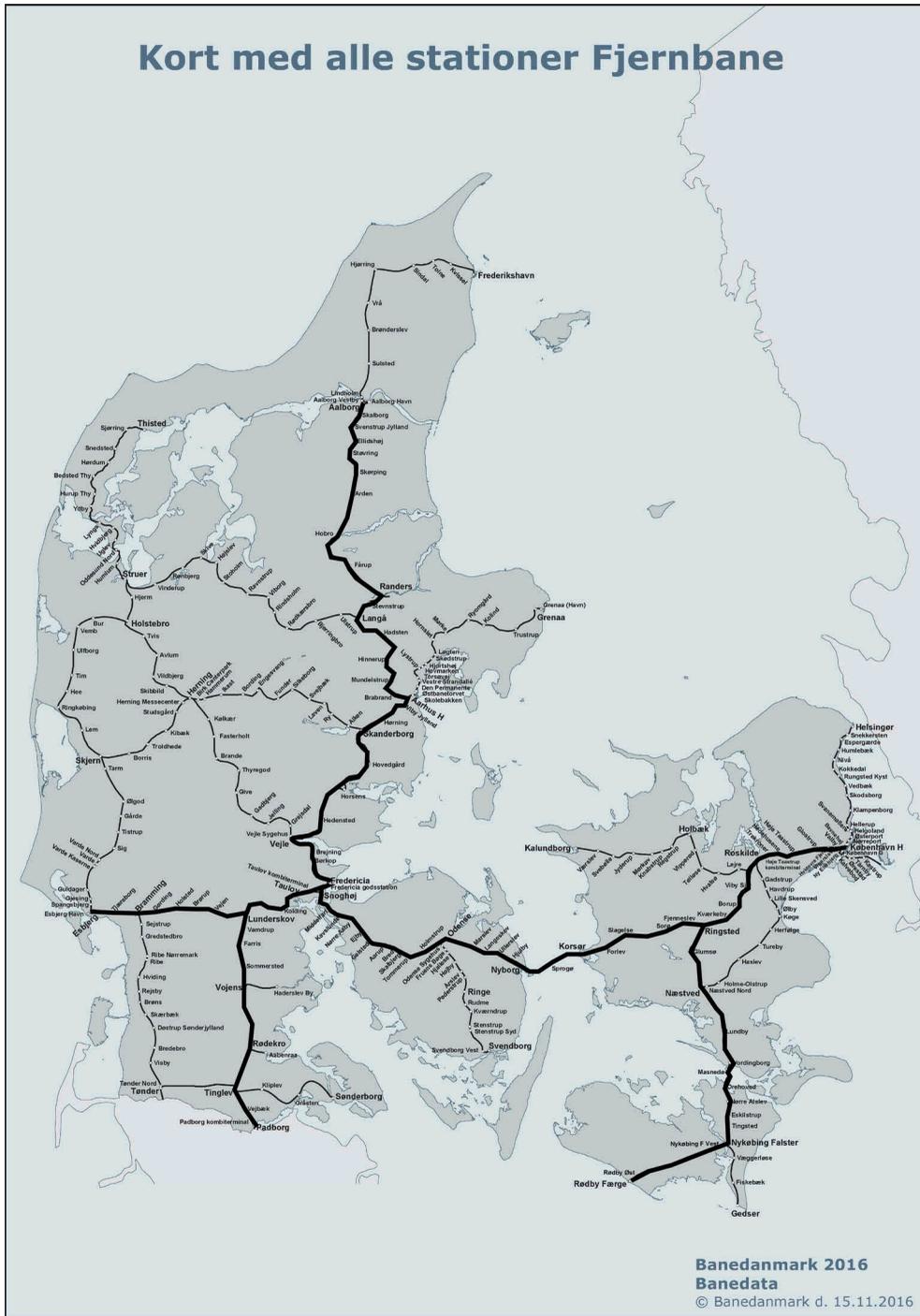


Figure 9. Measured sections of the main network in Denmark [14].

results. The soft soil profiles are characterized by a surface sand layer over a layer of peat resting on sand. In the first profile, there is a three meter peat layer at three meter

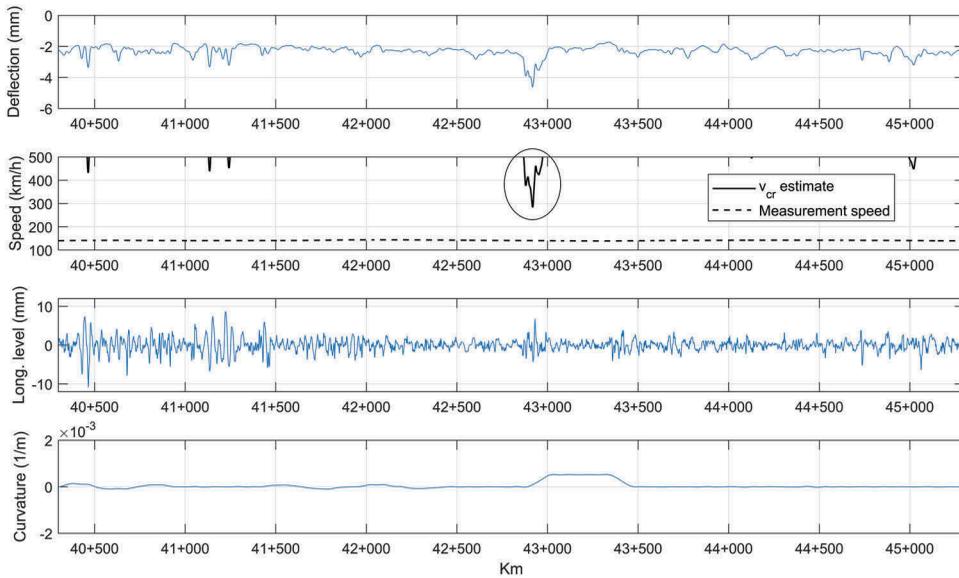


Figure 10. Example of results from the ETL evaluation. Suspected hot-spot is marked with an ellipse.

depth. The second profile is a five meter peat layer at one meter depth. The third profile is the reference profile consisting of sand only.

The most important parameter in the dynamic response of the soil is the shear modulus, or alternatively the shear wave velocity. These parameters were estimated using the empirical equation of Hardin [15] for sand by using the void ratio estimated from the unit weight and water content. For the peat layer, realistic shear wave velocities based on experience were used. Figure 11 shows the shear wave velocity in the two soft soil profiles together with the reference sandy site. The calculated maximum track response as a function of train speed for the three different profiles is shown in Figure 12. The analyses show that while the calculated critical speed for the reference sand profile is about 400 km/h, it drops to 270 km/h by inclusion of a 3 m peat layer, and even further down to just 170 km/h for a 5 m peat layer. These examples clearly demonstrate the role of soft embedded layers on the critical speed.

9. Conclusions

When the train speed approaches the critical speed at which train speed coincides with the Rayleigh-wave velocity of the ground, excessive track movement may occur. A new method for inventory measurements of critical speed, ETL, has been described and demonstrated. The method is based on the idea that this dynamic amplification, and the change of shape of the displacement curve under the loaded axle, starts already at speeds well below the critical speed. By observing the change in the displacement curve, ETL allows for estimation of critical speed from measurements from a running train at

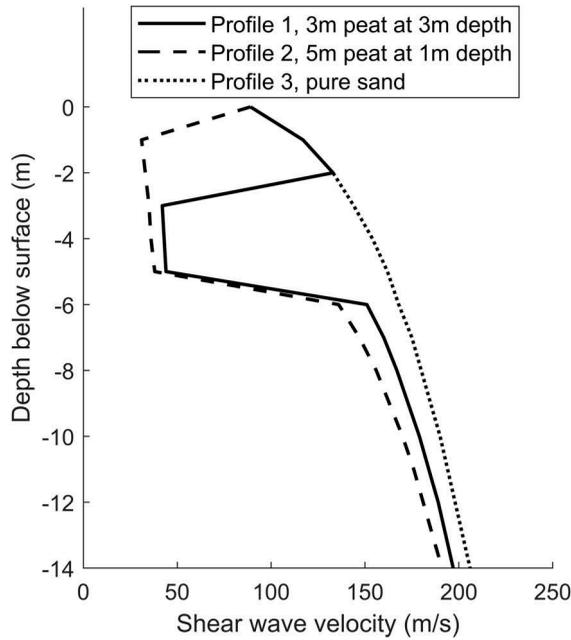


Figure 11. Soil profiles used in numerical simulations.

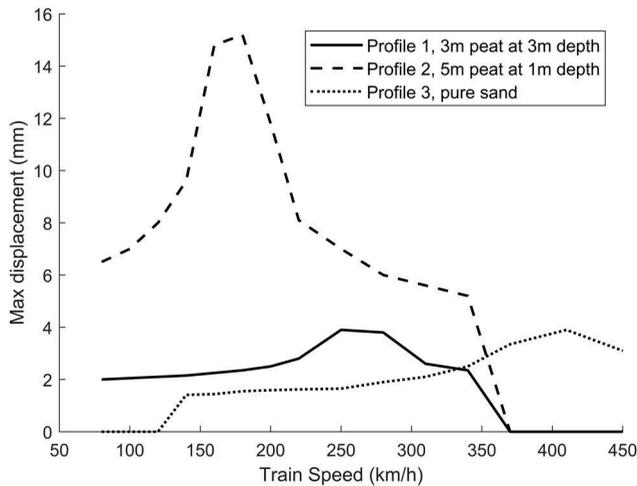


Figure 12. Maximum track response as function of train speed for different soil profiles.

normal speeds. Hence, large distances can be monitored in short time. Validations by calculations and test runs show that the method could give a reasonable estimate of the critical speed from a measurement speed of about 0.5 times the critical speed.

Despite promising results, the method still requires refinement, and at this stage contains some uncertainties. The method is only intended for inventory measurements, and it needs to be combined with supplementary methods, such as multichannel analysis of surface waves (MASW), for further investigations of the detected hot-spots.

Acknowledgments

This study has been funded by Banedanmark with partial support from the research project DESTination-Rail (Decision Support Tool for Rail Infrastructure Managers), funded by the European Commission, Grant Agreement 636285 (H2020-MG-2014-2015). Part of the work was performed by RoadScanners OY, who performed and evaluating the GPR measurements during the measurement campaign in Denmark.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by the Horizon 2020 Framework Programme [636285] and Banedanmark.

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