NORGES GEOTEKNISKE INSTITUTT

Prosjekt/rapportnr.: **20091808-01-TN**

Utarbeidet av: Alf Harbitz / Carl B. Harbitz

Rapporttittel: Conceptual model for quantification of snow avalanche return periods

Prosjekttittel: InfraRisk - Konsekvenser av ekstremvær på infrastruktur

Dato/Tegninger: Dato: 2012-12-11

Distribusjon: Fri

Technical Note

NGI

Main office:
PO Box 3930 Ullevål Stadion
NO-0806 Oslo
Norway

Trondheim office:
PO Box 1230 Sluppen
NO-7462 Trondheim
Norway

T (+47) 22 02 30 00
F (+47) 22 23 04 48

BIC No. DNBANOKK
IBAN NO26 5096 0501 281
Company No.
958 254 318 MVA

ngi@ngi.no
www.ngi.no

To:	Norwegian Research Council
Attn.:	Jostein Kandal Sundet
Copy:	
Date:	11 December 2012
Revision/Rev. date:	0
Document No.:	20091808-01-TN
Project:	INFRARISK
Prepared by:	Alf Harbitz, IMR and Carl B. Harbitz, NGI
Project manager:	Regula Frauenfelder
Reviewed by:	Regula Frauenfelder

Conceptual model for quantification of snow avalanche return periods

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1 Background

This note is a contribution to the InfraRisk project (Module B) where one of the aims is to suggest an improved standard for hazard mapping. In particular, it is attempted to overcome the deficiency pointed out for 'WP B1 Hazard Mapping' stating that today's hazard maps are inappropriate to assess the risk *inside* the hazard zone (the maps define only the frequency of an expected event and most often only along a single line).

The aim of this study is to establish a conceptual model for quantification of snow avalanche return periods at any given location along the avalanche track. The conceptual model is based on the modified α/β-model presented by Harbitz et al. (2001), which is again based on the original topographical/statistical α/β-model (e.g. Bakkehøi et al. 1983; a summary description is also presented by Harbitz 1998).

In the original α/β-model, the avalanche run-out distance equation is found from regression analysis correlating the longest registered run-out distance in 206 avalanche paths to a selection of topographic parameters, finally giving the simple relation

$$\alpha = 0.96\beta - 1.4^{\circ}$$

where α is the average inclination of the total avalanche path, and β is the average inclination of the avalanche path between the starting point and the point of 10° inclination along the terrain profile, Figure 1. The standard deviation $\sigma = 2.3^{\circ}$ and the correlation coefficient $R = 0.92$.

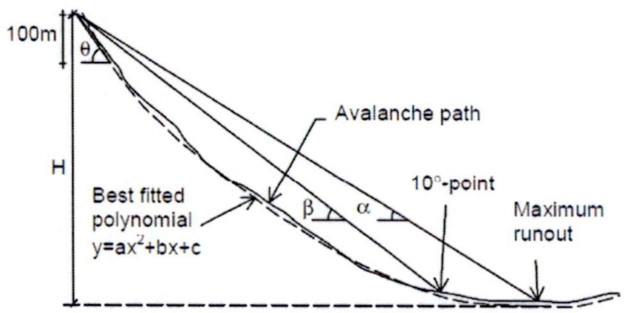

Figure 1: Definition of parameters used for the α/β-model.

In the modified α/β-model (Harbitz et al., 2001) the expression for the regression line is generalized to:

$$\alpha(m_s) = 0.96\beta - 1.4 + b(m_s) - W$$
$$W \sim N(0,\sigma) \text{ or } W \sim G(0,\sigma)$$

where α is the most extreme run-out angle observed in a certain track during an observational period of time $T_{obs,0}$. The general constant k replaces the value of 1.4^o of the original α/β-model. N and G denote normal and Gumbel distributions, respectively, with the same expectation value (0) and standard deviation (σ) for both distributions (the negative sign in front of W is introduced to cover a Gumbel extreme value distribution where the smaller α-values represent the most extreme events). Assuming an extreme value Gumbel distribution

$$b(m_s) = -6^{1/2} \cdot 2.3 \cdot \ln(m_s)/\pi$$

where m_s denotes the ratio between the observational period (T_{obs}) and the observational period $T_{obs,0}$ in the data behind the original α/β-model (e.g. 200 years). The term $b(m_s)$ describes a downward shift of the regression line if $m_s > 1$ (return period is longer than $T_{obs,0}$, i.e. smaller α-values), and an upward shift if $m_s < 1$ (return period is shorter than $T_{obs,0}$, i.e. larger α-values).

In Section 2 of this document, the modified α/β-model is applied to find a "correct" k-value and then a return period at the α points based on observations along the railroad Raumabanen, western Norway, Figure 2. Subsequently, in Section 3, estimators for the return period corresponding to any expectation value α (e.g. shifted to the railroad), or vice versa, are validated by the observations at Raumabanen.

2 Snow avalanche return period at the α point based on observations at the railroad Raumabanen

The original α/β-model is based on one single observation in each track, assumed to be the most extreme event over the last say 100-300 years, thereby yielding the value $k = 1.4^o$ for the regression line. The observations along Raumabanen are different:

- It is only known that the avalanches have crossed the railroad; the total run-out distance is unknown
- More than one avalanche is observed in each track

The angle from the top of the release area to the railroad is denoted the object angle, α_Ω. Observations of mass flow impacts along the railroad for a period of $T_{obs} = 63$ years (1924-1987) were assembled by Kristensen (2011) based on data provided by the railroad responsible Stig Arild Brenden, JBV (pers. comm. to Kristensen, 2011), see Table 1 and Figure 2.

Table 1: Snow avalanches registered on the railroad Raumabanen 1924-1987. The locations are displayed in Figure 2. The α-angle in the table is calculated by the original α/β-model. Source: Kristensen (2011), based on data provided by S.A. Brenden, JBV.

ID	Name of avalanche path	α_Ω angle	β angle	α angle	Registrations
1	Halsa	36.4	38.2	35.3	1976, 1981, 1982
2	Ødegård	38.6	43.6	40.5	1955, 1956, 1968, 1976, 1982
3	Romsdalshorn	49	50.8	47.4	1952, 1982
4	Joengfonna	39.5	43.7	40.6	1932, 1940, 1943, 1947, 1965, 1966, 1967, 1985
5	Grønfonna	31.1	32.5	29.8	1952, 1958, 1965, 1981
6	Gurifonna	30.3	31.1	28.4	1942, 1958
7	Fossagrovfonna	32.1	32.6	29.9	1940, 1942, 1952, 1955, 1987
8	Fossalia	32.5	32	29.3	1955, 1968, 1974
9	Kverngrova	31.8	30.7	28.1	1942, 1958, 1974

We search the return period T_α for a "serious" avalanche comparable to those reaching the railroad, i.e. assuming that avalanches reaching the railroad also follow an extreme value distribution for run-out distance in a similar manner as the "extreme" avalanches behind the original α/β-model. However, these avalanches are not denoted "extreme" since more than one avalanche is observed in each track) but rather names as "serious" avalanches. It should be noted that we assume that only one half of them actually reach the railroad (provided that the railroad is situated such that we observe one half of a family of "serious" avalanches), i.e. there is a 50% probability that the avalanche does not reach the railroad, and 50% probability that the avalanche is observed. Hence, the "observed return period" must be divided by two).

We further search the "correct" k value for this set of observations (intuitively less than 1.4° as we have several avalanches per track per 63 years, i.e. the α values are higher).

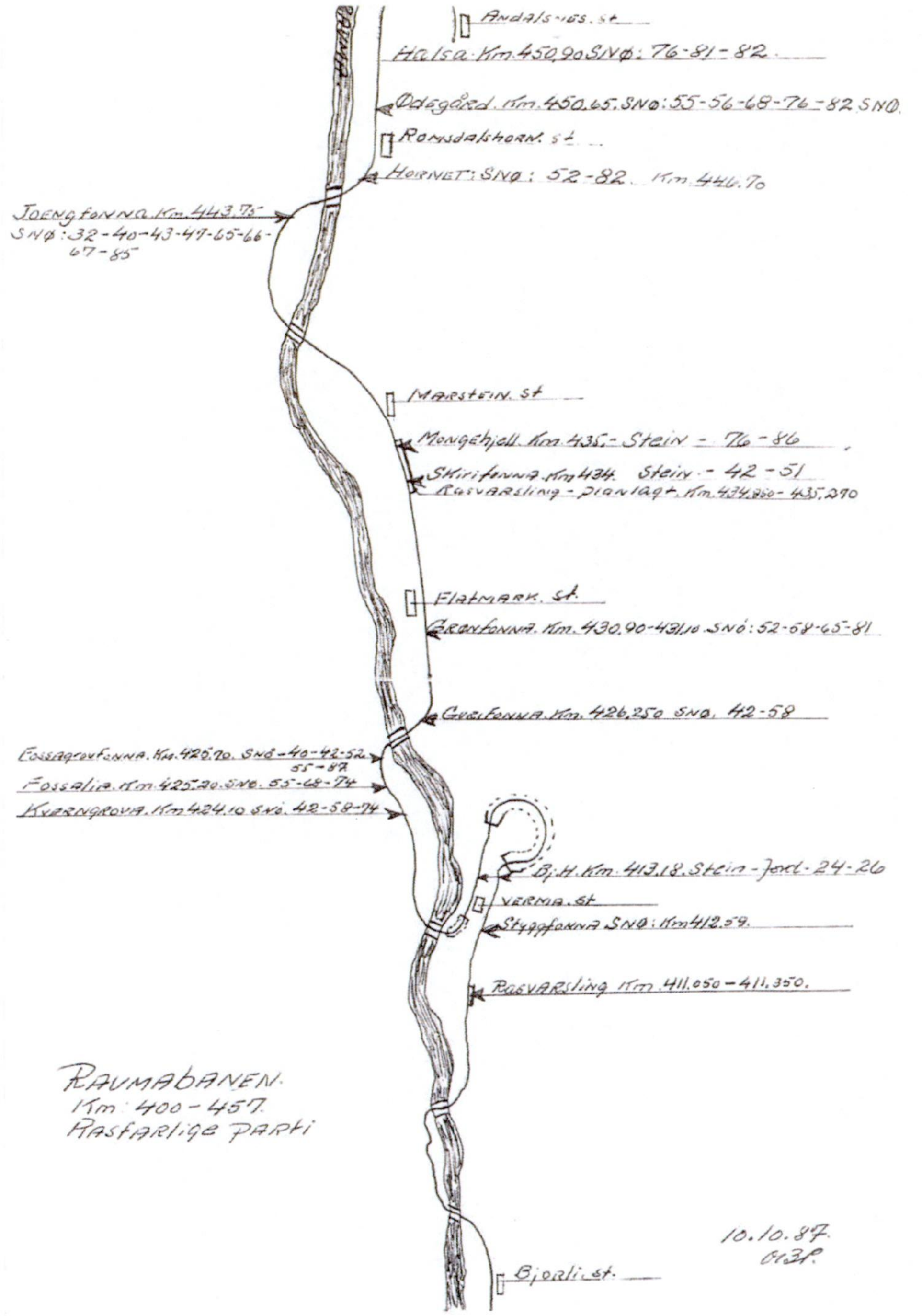

Figure 2: Observations of mass flow impacts along the railroad Raumabanen 1924-1987. Source: *JBV map, provided to Kristensen (2011) by S.A. Brenden, JBV.*

Provided that the most extreme avalanche was observed during the observational period, the probability that a certain α-value is less than a given value $P(\alpha<\alpha_0)$, is - based on the distribution W - given as:

$P(\alpha<\alpha_0|\text{release}) = P(0.96\beta\text{-k-W}<\alpha_0) = 1\text{- } P(W<0.96\beta\text{-k-}\alpha_0) = \text{-}F(0.96\beta\text{-k-}\alpha_0)$

where F is the cumulative distribution of W, either $N(0,\sigma)$ or $G(0,\sigma)$. So far $k = 1.4$ based on the data behind the original α/β-model.

An updated value of k (adapted to the observations) can now be found as:

$$k = 1.4 + 2.3\Phi^{-1}\left(1 - \frac{1}{m_s}\right) \qquad \text{if } W\text{~}N(0,\sigma)\text{, or}$$

$$k = 1.4 + 2.3\sqrt{6}\ln(m_s)/\pi \qquad \text{if } W\text{~}G(0,\sigma)$$

where Φ^{-1} is the inverse standard cumulative normal distribution, $\Phi^{-1}(0) = 0$ for $m_s = 1$. However, a direct calculation of k is not possible as long as m_s is also unknown (k and m_s depend on each other). Hence, an iterative procedure to find the release period T_{rel}, $m_s = T_{rel}/T_{obs,0}$, and then the value of k based on the observations is suggested as follows:

Step 1: Choose/assume the observational period for the data behind the original α/β-model (e.g. $T_{obs,0} = 100$ years).

Step 2: Calculate T_{rel} for the railroad with $k = 1.4^o$ (this gives a first estimate of T_{rel} based on observations during T_{obs}):

$P(\alpha<\alpha_\Omega) = P(\alpha<\alpha_\Omega|\text{release})\cdot P(\text{release})$,

where the left hand side is deduced from the observations as: $P(\alpha<\alpha_\Omega)$ = (twice the number of avalanches across the railroad divided by T_{obs}) = $1/T_\Omega$.

Normal distribution:

$P(\alpha<\alpha_\Omega|\text{release}) = P(0.96\beta\text{-1.4-W}<\alpha_\Omega) = \Phi(\text{-}(0.96\beta\text{-1.4-}\alpha_\Omega/2.3))$

$T_{rel} = 1/P(\text{release}) = P(\alpha<\alpha_\Omega|\text{release})/P(\alpha<\alpha_\Omega) = \Phi(\text{-}(0.96\beta\text{-1.4-}\alpha_\Omega/2.3)T_\Omega$

Substituting α_Ω by α gives:

$T_{rel} = \Phi(0)/(1/T_\alpha) = 0.5/(1/T_\alpha)$ or $T_\alpha = 2T_{rel}$ (i.e. the obvious relation between T_α and T_{rel} for a normal distribution).

Extreme value distribution:

$$P(\alpha < \alpha_\Omega \mid \text{release}) = \mathrm{P}(0.96\,\beta - 1.4 - \mathrm{W} < \alpha_\Omega) = 1 - \exp\left[-\exp\left(-\frac{0.96\beta - 1.4 - \alpha_\Omega + 0.5772}{2.3 \cdot \sqrt{6}/\pi}\right)\right]$$

$T_{rel} = 1/\,P(\text{release}) = P(\alpha < \alpha_\Omega|\text{release})/\,P(\alpha < \alpha_\Omega) =$

$$P(\alpha < \alpha_\Omega \mid release) = \left\{1 - \exp\left[-\exp\left(-\frac{0.96\beta - 1.4 - \alpha_\Omega + 0.5772}{2.3 \cdot \sqrt{6}/\pi}\right)\right]\right\} T_\Omega$$

Again substituting α_Ω by α now gives:
$T_{rel} = P(\alpha \leq \alpha_0)/(\ 1/T_\alpha) = 0.47\ /(\ 1/T_\alpha)$ or $T_\alpha = 2.12 T_{rel}$ for the extreme value distribution.

Step 3: Calculate $m_s = T_{rel}/\ T_{obs,0}$ and then k (based on m_s as explained above). Repeat step 2 with this new value of k.

Step 4: Repeat step 2 (with new value of k) and step 3 until convergence is obtained for k and T_{rel}.

Step 5: It is recommended to repeat steps 2-4 for various choices of $T_{obs,0}$ (e.g. 100, 200, 300 years) and perform a sensitivity analysis T_{rel} with regard to the choice of $T_{obs,0}$.

We have now obtained T_{rel} and a "correct" value of k (based on observations in certain avalanche paths), and can thus find $T_\alpha = 2.0\ T_{rel}$ or $T_\alpha = 2.12\ T_{rel}$ for the normal distribution or the extreme value distribution, respectively.

In other words, if the release area and the location of the railroad is known (defined by α_Ω), we can calculate the probability of an avalanche reaching the railroad $P(\alpha < \alpha_\Omega|\text{release})$, and multiply by the probability of release ($1/T_{rel}$) to find the probability of an avalanche impacting on the railroad.

3 Snow avalanche return period at an arbitrary point in the avalanche path – validated by observations at the railroad Raumabanen

In the previous section we estimated the avalanche frequency at the railroad or at the α-point based on the observations. We will now calculate the probability of avalanche occurrence at other points along the avalanche track, given T_Ω (observations) at a certain object point α_Ω (e.g., road or railway).

It has been assumed that the observations behind the original α/β-model originate from avalanche paths where differences in observational period and release frequency between the paths can be ignored. However, for simplicity we will in the following assume that an observational period $T_{obs,0}$ representative for the original α/β-model can be applied. We have further assumed that the distribution of the residuals in the linear α/β-model (i.e., the deviation between observed and estimated α value) is also valid for each individual path. This means that if we made observations over a series of $T_{obs,0}$ periods in one track, and for each period extracted the most extreme event, then the distribution of α values around the mean of α equals the residual distribution obtained with only one observation in each path. Such an assumption is supported by the good linear relation between the α- and β-values forming the original α/β-model, with apparently independent residuals of constant variance as functions of β. However, it should be kept in mind that certain differences between the various paths may give a significant contribution to the variance of the residual, implying that the variance for each individual path is somewhat less than for all the paths together.

A statistical extreme value model is based on observations of a variable (run-out angle α for all avalanches) and a record of the most extreme, $\alpha_{(n)}$, of n variables (n successive avalanches). With a sufficiently large n the distribution of $\alpha_{(n)}$ will follow one out of three known distributions, where the Gumbel distribution is the relevant here. When n is large enough, and we want to increase to an even larger value of n, then the Gumbel distribution will not change its character, but just be shifted more to the extreme (towards smaller α-values). If we know the location parameter in the Gumbel distribution for a certain value of n, we can easily calculate this parameter for the Gumbel distribution of any other larger value of n. We will further assume that n is proportional to the observational period.

Having assumed that the Gumbel extreme value distribution holds for the α/β-model, it will also hold for more extreme periods, e.g. for the 1000 year avalanche. The equation for $b(m_s)$ above shows how we can easily establish a model for any synthetic extreme observational period T_{obs} as long as $T_{obs,0}$ is known. It should be noted that the model can not necessarily be expanded towards shorter observational periods as the conditions for the Gumbel distribution will then be more uncertain (the n value might be too small for the extreme value model to be reasonable). Hence, the validity of our expansion to shorter periods needs to be tested against observations.

We assume that we have observations at a defined point in the avalanche path, e.g., where the path crosses the railroad Raumabanen (as stated above this time, we don't record the full run-out distance, but rather that the avalanche has at least reached the railroad). To apply the α/β-model, imagine that we shift the Gumbel distribution such that the expectation value equals α_Ω. Again, there is now a 50% probability that the avalanche does not reach the railroad, and 50% probability that the avalanche is observed (and the "observed return period" must be divided by two as explained above).

With the observations from a defined point in the terrain we can apply the α/β-model to find an estimator, $\hat{T}_{ret}$, for the return period of the category of (extreme) avalanches with (known) expectation value α_Ω. This value is subsequently compared to the observed return period T_{ret}. Or vice versa; we can estimate α_Ω by using estimated return period, and compare to the true value of α_Ω. Mathematically the two estimators are expressed as follows:

$$\hat{T}_{ret} = \exp\left(\frac{\pi}{\sqrt{6}\sigma}(0.96\beta - \alpha_\Omega - 1.4)\right) \cdot T_{obs,0}$$

$$\hat{\alpha}_\Omega = 0.96\beta - 1.4 - \frac{\sqrt{6}\sigma}{\pi}\ln\left(\frac{T_{ret}}{T_{obs,0}}\right)$$

The first estimator reveals how a return period can be estimated from the original α/β-model (with return period $T_{obs,0}$ and $k = 1.4^{o}$) for any value of α_Ω. The second estimator reveals the opposite, i.e., how an expectation value α_Ω can be estimated from the standard α/β-model for any return period. In general, the return period for a specified α_Ω or the expectation value α_Ω for a specified return period (e.g., 1000 years) can be calculated for any avalanche path described by the α/β-model. *Hence, with a given (Gumbel) distribution for a given β angle in a particular avalanche path, we can find the probability of avalanche impact for all α angles in this path. The condition is only that we know the return period in one point, e.g., α_Ω (from the observations).*

The return period is calculated in two different ways (the factor 2 is explained above):

$$T_{ret,1} = \frac{(\#\,skred)/2}{63}$$

$$T_{ret,2} = \frac{1}{2(n_s - 1)}\sum_{j=1}^{n_s-1}(yr_{j+1} - yr_j)$$

where the latter calculation is the average time interval between two subsequent avalanches. For simplicity we apply the average $T_{ret} = (T_{ret,1} + T_{ret,2})/2$ as the return period for each avalanche path. The results with observational period $T_{obs,0}$ = 200 years are listed in Table 2. The agreement between estimated and true values is not convincing.

Just for an illustration, the observational period of the original model was after trial and error set to $T_{obs,0}$ = 27 years (see Table 3). Now, the agreement between the conceptual model and the observations improved significantly. However, 27 years is an unrealistically short observational period for the original dataset (the real value is estimated to 100-300 years).

With observations like the ones for Raumabanen, we have direct estimates for return periods at the α_Ω points. Here the conceptual α/β-model can be validated. In the examples above the results are not convincing with $T_{obs,0}$ = 200 years. However, this does not imply that our interpretation of the model needs to be wrong. There are several possible sources of error. In addition to the two listed above (measuring avalanches crossing the railroad rather than measuring the full run-out distance; more than one avalanche measured in each path), the standard deviation ($\sigma = 2.3^o$) may reflect not only possible variations of the 200 years avalanche in a separate path, but also variation between the individual paths or different observational return periods $T_{obs,0}$ (independent of β). All these possible sources increase the standard deviation. From the equation for $b(m_s)$ above, it can be seen that if the standard deviation is reduced to $\sigma \approx 1^o$ (still with $T_{obs,0}$ = 200 years) the results will be approximately as good as with an observational period of 27 years (with $\sigma = 2.3^o$). In this way the data can even be used to re-estimate the standard deviation σ for an arbitrary path.

Altogether, our model is not necessarily bad in spite of the weak agreement with the observations and it is certainly worth pursuing these ideas further. It should also be stated that presently there are actually no other good alternatives.

Table 2: Estimated values $\hat{\alpha}_{\Omega}$ and $\hat{T}_{ret}$ for α_{Ω} and T_{ret}, respectively, based on observational return period $T_{obs,0}$ = 200 years.

$\hat{\alpha}_{\Omega}$ (deg.)	α_{Ω} (deg.)	$\hat{\alpha}_{\Omega}$ - Ω (deg.)	$\hat{T}_{ret}$ (years)	T_{ret} (years)	$\hat{T}_{ret}$ - T_{ret} (years)
41.6	36.4	5.2	106.6	6.0	100.6
47.1	38.6	8.5	563.0	4.8	558.2
52.0	49.0	3.0	80.5	15.4	65.1
47.6	39.5	8.1	359.6	3.9	355.7
36.0	31.1	4.9	96.9	6.4	90.5
33.5	30.3	3.2	71.5	11.9	59.6
36.2	32.1	4.1	58.5	6.1	52.4
35.2	32.5	2.7	34.0	7.7	26.3
33.6	31.8	1.8	25.0	9.3	15.8

Table 3: Estimated values $\hat{\alpha}_{\Omega}$ and $\hat{T}_{ret}$ for α_{Ω} and T_{ret}, respectively, based on observational return period $T_{obs,0}$ = 27 years.

$\hat{\alpha}_{\Omega}$ (deg.)	α_{Ω} (deg.)	$\hat{\alpha}_{\Omega}$ - α_{Ω} (deg.)	$\hat{T}_{ret}$ (years)	T_{ret} (years)	$\hat{T}_{ret}$ - T_{ret} (years)
38.0	36.4	1.6	14.4	6.0	8.4
43.5	38.6	4.9	76.0	4.8	71.2
48.4	49.0	-0.6	10.9	15.4	-4.5
44.0	39.5	4.5	48.5	3.9	44.7
32.4	31.1	1.3	13.1	6.4	6.7
29.9	30.3	-0.4	9.7	11.9	-2.2
32.6	32.1	0.5	7.9	6.1	1.8
31.6	32.5	-0.9	4.6	7.6	-3.0
30.0	31.8	-1.8	3.4	9.3	-5.9

4 Acknowledgements

Lene L. Kristensen is thanked for processing the observational data (Table 1) and for valuable discussions during her master studies, which motivated for the present report.

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Kontroll- og referanseside/ *Review and reference page*

NGI

Dokumentinformasjon/*Document information*			
Dokumenttittel/*Document title* Conceptual model for quantification of snow avalanche return periods			**Dokumentnr./*Document No.*** 20091808-01-TN
Dokumenttype/*Type of document* Technical Note	**Distribusjon/*Distribution*** Unlimited		**Dato/*Date*** 2012-12-11
			Rev.nr.&dato/*Rev.No&date.* 0
Oppdragsgiver/*Client* InfraRisk project			
Emneord/*Keywords*			

Stedfesting/*Geographical information*	
Land, fylke/*Country, County*	**Havområde/*Offshore area***
Kommune/*Municipality*	**Feltnavn/*Field name***
Sted/*Location*	**Sted/*Location***
Kartblad/*Map*	**Felt, blokknr./*Field, Block No.***
UTM-koordinater/*UTM-coordinates*	

Dokumentkontroll/*Document control*

Kvalitetssikring i henhold til/*Quality assurance according to* NS-EN ISO9001

Rev./ *Rev.*	Revisjonsgrunnlag/*Reason for revision*	Egen-kontroll/ *Self review* av/*by:*		Sidemanns-kontroll/ *Colleague review* av/*by:*		Uavhengig kontroll/ *Independent review* av/*by:*		Tverrfaglig kontroll/ *Inter-disciplinary review* av/*by:*	
0	Original document	CH	CH	RF	RF				

Dokument godkjent for utsendelse/ *Document approved for release*	**Dato/*Date*** 2012-12-11	**Sign. Prosjektleder/*Project Manager*** Regula Frauenfelder R. Frauenfelder