Icelandic avalanche runout models compared with topographical models used in other countries

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ABSTRACT. A statistical topographical model for the computation of runout for snow avalanches in Iceland has been derived from a recently assembled data set of long Icelandic snow avalanches. The avalanches are from hills above towns in western, northern and eastern Iceland. The model, $\alpha = 0.85\beta$, expresses the average slope of the avalanche path, α, as directly proportional to the average slope of the avalanche track, β. A similar model for a data set of avalanches collected through systematic investigations of several regions in western Norway is found to be $\alpha = 0.93\beta$. The residual standard error in α for the models is similar, $\sigma_{\Delta\alpha} = 2.2^\circ$ for the Icelandic data and $\sigma_{\Delta\alpha} = 2.1^\circ$ for the Norwegian data. The models thus indicate that avalanches in the Icelandic data set reach somewhat further than avalanches in the Norwegian data set for similar β-angles, but the relationship between α- and β-angles in the two data sets is nevertheless quite similar (*cf.* Fig. 2). Worthwhile improvements in the models were not obtained by adding intercept or curvature terms or terms corresponding to other parameters than β. Statistical models based on runout ratios were not found to be an improvement over models based on α- and β-angles.

INTRODUCTION

Empirical models for the computation of snow-avalanche runout distance are often used for estimating avalanche hazard (McClung and Lied, 1987; NGI, 1994, 1996). Frequently used models of this type are statistical models based on topographical parameters. The Norwegian α/β-model (Lied and Bakkehøi, 1980; Bakkehøi, Domaas and Lied, 1983; Lied and Toppe, 1989) relates α, the average slope of the avalanche path from the fracture line to the outer end of the avalanche deposit, to β, the average slope of the avalanche track to the foot of the slope where the slope angle declines to 10° (*cf.* Fig. 1 for graphical definitions of these variables). Several expressions of this model for different ranges of β and a few additional independent parameters, such as the starting slope θ, in addition to β have been derived (*cf.* NGI, 1994, 1996), but the simplest expression

$$\alpha = 0.96\beta - 1.4\ , \quad \sigma_{\Delta\alpha} = 2.3^\circ\ , \quad R = 0.92\ , \quad n = 206 \qquad (1)$$

from Bakkehøi, Domaas and Lied (1983) fits a data set of 206 long Norwegian avalanches almost as well as more complicated expressions. The same type of model has been used in an analysis of a data set of 80 long Austrian avalanches (Lied, Weiler, Bakkehøi and Hopf, 1995). The simplest model of that study is similar to eq. (1), *i.e.* $\alpha = 0.946\beta - 0.83$, and gives almost identical predictions over the relevant range in β.

Another statistical model based on topographical parameters describes the runout distance in terms of the runout ratio, $r = (x_{stop} - x_\beta)/(x_\beta - x_{start})$, between the horizontal distance from the β-point to the extreme runout position, on one hand, and the distance from the starting position to the β-point, on the other (*cf.* Fig. 1). According to McClung, Mears and Schaerer (1989) and McClung and Mears (1991), the runout ratio, r, may be expected to be Gumbel distributed with different statistical coefficients for different mountain ranges with different topographical characteristics. The Gumbel statistical distribution has the cumulative probability function and the probability density function

$$D(r) = e^{-e^{-(r-a)/b}}\ , \quad d(r) = D'(r) = e^{-e^{-(r-a)/b}} e^{-(r-a)/b}/b\ . \qquad (2)$$

McClung and Mears (1991) find that the statistical coefficients $a = 0.143$ and $b = 0.077$ are appropriate for a data set of 80 long avalanches from western Norway.

The α/β-model and the runout ratio model based on Gumbel statistics are intended to estimate the runout distance of "long" dry snow avalanches for the avalanche path under consideration. The meaning of "long" in this connection depends on the data set which is used as a basis for the model. The avalanches in the Norwegian avalanche data set are estimated to have a return period of approximately 100-300 years (NGI, 1994), but some of the avalanches will correspond to somewhat longer or shorter periods. The return period of the avalanches in the Icelandic data set of long avalanches considered here is not easy to estimate. The Icelandic data set is likely to be less homogeneous than the Norwegian data set because some of the avalanches are from areas where observations are relatively recent whereas others are from areas which have been populated for centuries. A rough estimate of the return period of the Icelandic data set is 100 years, but as for the Norwegian data set, one may expect some of the avalanches to correspond to longer or shorter periods than this.

According to both the above models, avalanches

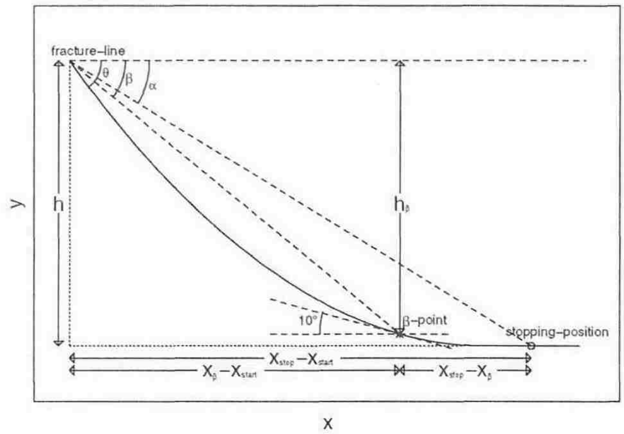

Fig. 1. Definition of geometrical parameters used to analyse extreme runout.

released from gentle slopes (low β-angles) have a tendency to travel further, *i.e.* have relatively low α-angles, compared with avalanches that are released from steeper slopes (high β-angles). In the α/β-model, this tendency is formulated as a linear relation between the angles α and β (*cf.* eq. (1)). In the runout ratio model, steep paths have comparatively low values of $(x_\beta - x_{start})$ which leads to relatively low values of $(x_{stop} - x_\beta)$ for the same runout ratio r. McClung and Mears (1991) show that the runout ratio is statistically independent of the path steepness, β, for several avalanche data sets from a number of regions in the world. This means that the negative correlation between runout distance and path steepness, which is the basis for the α/β-model, is to a large extent absorbed in the definition of the runout ratio. As a consequence, the runout of avalanches in a data set can be analysed by investigating the statistical distribution of the runout ratio rather than analysing the deviation of the observed α-angles from the linear relation expressed by eq. (1). The main difference between the α/β-model and the runout ratio model lies in the different statistical assumptions regarding the distribution of the residuals, *i.e.* the (implicit) normal distribution in the case of the α/β-model and the Gumbel distribution for the runout ratio model.

Both the above types of models suffer from the rather arbitrary choice of a 10° reference slope in the definition of the β-point as an independent variable in the model. The β-point may, furthermore, not be uniquely determined for avalanche paths with a complicated shape where the slope angle may become equal to 10° at several locations along the path with steeper stretches in between. Therefore, the models are most appropriate for longitudinally concave paths.

A problem with the avalanche data sets considered here is the non-random sampling of the avalanches. The selection of avalanches in a data set involves a subjective estimate of what parts of the mountain slopes in the region under investigation qualify as avalanche paths. A restrictive definition of avalanche paths will lead to a data set with more extreme avalanches. Furthermore, if the avalanches are collected from reports of damages and extreme events which have been reported to scientists or official institutions because they were unusual, then a data set of such events is obviously biased towards long and extreme events. Finally, avalanches tend to be released in avalanche cycles which affect whole regions at the same time. The longest recorded avalanches in neighbouring paths have therefore sometimes been released in the same cycle and can thus not be considered independent events. These problems have to be kept in mind when interpreting differences between regions or countries or judging the significance of model coefficients.

Another category of problems with topographical statistical models has to do with the interpretation of such models in terms of return periods or risk. The models are based on extreme events from many different avalanche paths with different frequencies of avalanche cycles. The longest avalanche from a certain avalanche path may be among 5% of the most extreme events in an extensive data set of avalanches. This does not necessarily carry directly over to a specific return period of avalanches exceeding a certain runout distance in this path or a definite estimate of the risk facing inhabitants in a specific building threatened by avalanches released in the path. The statistics of extreme avalanches do, nevertheless, display a certain consistency or regularity which has been found useful by avalanche researchers in many countries. The statistical models and the underlying data must, however, be used with due regard to the problems mentioned above.

The present paper describes the derivation of statistical topographical models for long Icelandic avalanches and compares the results with models derived for a data set of long Norwegian avalanches. Models based on both the α/β approach and on runout ratios are derived and compared.

THE ICELANDIC DATASET

A data set of Icelandic avalanches has been compiled at the Science Institute of the University of Iceland and at the Icelandic Meteorological Office. An initial version of the data set is described by Tómasson, Friðgeirsdóttir, Jónasson and Sigurðsson (1995), but the data set has since been expanded and improved from this first version. This data set currently contains 197 avalanches of which 53 are the longest known avalanche in the corresponding avalanche path. The analysis presented in this paper is based on this data set restricted to snow avalanches which are longest in their path. A few slush flows and several avalanches with uncertain path location or runout distance were omitted from the data set. Some very small avalanches compared with larger avalanches in neighbouring paths of the same hill were furthermore omitted, in case the path had only been observed for a short period. Two avalanches from Ólafsvík in western Iceland and Dýrafjörður on the North-Western Peninsula were also added to the data set. The data set of "long" Icelandic avalanches obtained in this way was examined and information about several avalanches was corrected in accordance with the current records in the written archives of the Icelandic Meteorological Office. The resulting data set contains 45 avalanches, of which 25 terminate on land and 20 terminate in the ocean. Most of the 45 avalanches are from 8 avalanche prone Icelandic villages, 10 are from Neskaupstaður, 8 from Ísafjörður, 7 from Siglufjörður, 6 from Hnífs-

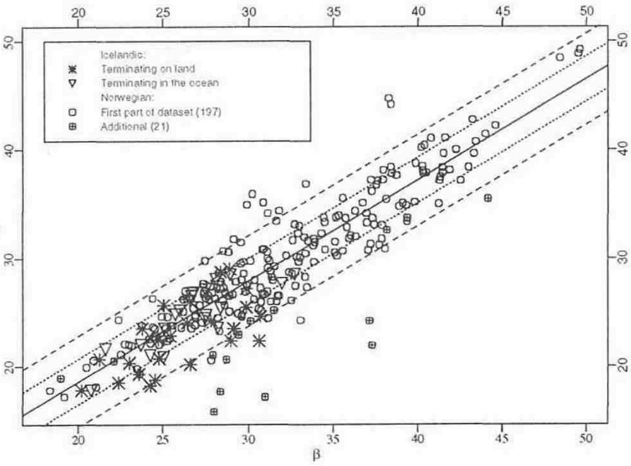

Fig. 2. α-angles plotted against β-angles for the Icelandic and Norwegian data sets. Avalanches terminating in the ocean and avalanches terminating on land are differentiated with different symbols for the Icelandic data. For the Norwegian data, separate plotting symbols are used to differentiate 21 avalanches described in separate reports from 197 avalanches catalogued in systematic investigations of whole regions. The α/β-model for Norwegian avalanches given by eq. (6) is shown as a family of parallel lines where the solid line represents eq. (6), the short-dashed lines represent runout angles corresponding to $\alpha \pm \sigma$ and the long-dashed lines represent runout angles corresponding to $\alpha \pm 2\sigma$. The α/β-model given by eq. (1) is almost the same as the model given by eq. (6).

dalur, 5 from Flateyri, 3 from Seyðisfjörður, 2 from Súðavík and 2 from Patreksfjörður. One avalanche comes from Ólafsvík and one is from Dýrafjörður. Date and location of the avalanches in the Icelandic data set are listed in the Appendix together with the corresponding α-, β- and θ-angles and comments regarding damages or other additional information.

The Norwegian avalanche data set considered here contains 218 avalanches, all of which are the longest observed avalanche in the corresponding avalanche path and none of which terminate in the ocean. The first 197 avalanches in the Norwegian data set are collected through systematic investigations of whole regions, Ørsta, Stryn, Valldal, Sunnylven, Horningdal and Strandadalen. The last 21 avalanches in the data set have been catalogued separately because they have caused damage or have for some other reason been described in separate reports.

10°-β-points for both the Icelandic and the Norwegian data sets were computed by linear interpolation of the slopes between pairs of neighbouring points in the digital path. The location of the β-point is not clearly defined for some paths where the slope may be close to 10° or fluctuate around 10° over a long distance in the lower part of the profile. Such avalanche paths in the Icelandic data set are discussed in a note in the Appendix.

Tables and graphs in Jóhannesson (1998) summarise and give a graphical overview of the statistical distribution of several topographical parameters for the Icelandic and Norwegian data sets, including the α-angle, 10°-β-angle, θ-angle, horizontal runout distance, vertical fall and the starting elevation of the avalanches.

α/β-MODELS

α- and β-angles from the Icelandic and Norwegian data sets are plotted in Figure 2 with separate symbols for Icelandic avalanches terminating on land and in the ocean. Icelandic avalanches terminating in the ocean are plotted as if they had terminated on the shoreline. The figure also shows the α/β-model for Norwegian avalanches given by eq. (6) which is derived below. It is seen that the Icelandic avalanches have lower α- and β-angles than the Norwegian avalanches, but the two data sets appear to have a similar relationship between α and β, because the Icelandic data are similar to the Norwegian data in the same range of β-angles. Thus, the lower runout angles (longer runout) of avalanches in the Icelandic data set seem to be to a large extent explained by more gentle slopes of the Icelandic avalanche tracks. The Icelandic avalanches terminating in the ocean have slightly higher α-angles than the avalanches terminating on land. A least squares line through avalanches terminating on land is approximately 1.5° lower in the middle of the range of the Icelandic avalanches than a line through avalanches terminating in the ocean (not shown). Omitting the avalanches terminating in the ocean from the analysis or treating them as if they had terminated on the shoreline will lead to a biased model because these avalanches would have reached lower α-angles if they had not reached the ocean prematurely.

It appears from Figure 2 that many of the last 21 avalanches in the Norwegian data set, which were not collected through systematic investigations of whole regions (denoted with separate symbols in the figure, designated as "Additional"), have very low α-angles. This indicates that the sampling of avalanches in this part of the data set may have lead to different statistical characteristics of these avalanches compared with the other avalanches in the data set as further discussed below.

The Icelandic data set is seen more clearly in Figure 3 which shows an expanded view of the Icelandic avalanches with different plotting symbols for different regions in Iceland. Regional runout differences are hard to analyse for the Icelandic data set due to the low number of avalanches from each region and the non-random sampling of the avalanches, but there are no clear indications of regional differences in the data set shown in Figure 3 (with the possible exception of Siglufjörður).

The Icelandic data set may be expected to be biased towards high α-angles due to avalanches that terminate in the ocean. This problem is presumably not present for the Norwegian data set, where avalanches terminating in the ocean have already been eliminated, because many Norwegian avalanche paths are well above sea level whereas most observed Icelandic avalanche paths end close to sea level. In Jóhannesson (1998), it is shown how one can find model coefficient estimates that take both avalanches that terminate on land and in the ocean into account simultaneously. For the avalanches terminating in the ocean, one computes the

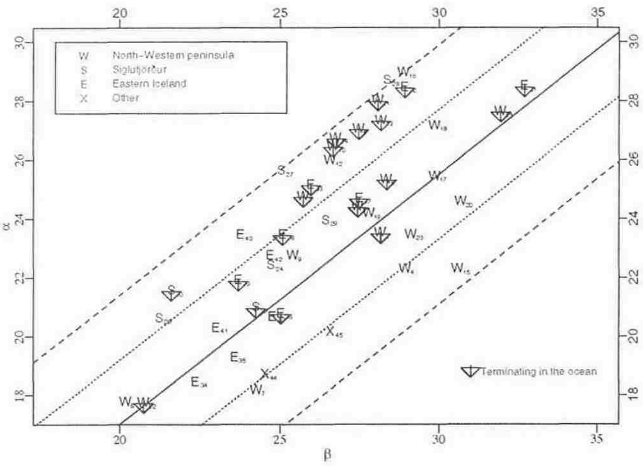

Fig. 3. α-angles plotted against β-angles for the Icelandic data set. Avalanches from different inhabited regions in Iceland are differentiated with different symbols. Avalanches from Ísafjörður, Hnífsdalur, Flateyri, Súðavík and Patreksfjörður on the North-Western Peninsula are denoted with "W", avalanches from Siglufjörður with "S", avalanches from Neskaupstaður and Seyðisfjörður in eastern Iceland with "E" and the avalanches from Ólafsvík and Dýrafjörður with "X". Subscripted numbers refer to line numbers in a table in the Appendix where the individual avalanches are listed together with dates, locations and other information. Avalanches terminating in the ocean are indicated with down-pointing arrows since all that is known about these avalanches is that they reached further, or equivalently that the runout angle α was lower, than the corresponding point on the graph. The α/β-model given by eq. (5) is shown as a family of parallel lines where the solid line represents eq. (5), the short-dashed lines represent runout angles corresponding to $\alpha \pm \sigma$ and the long-dashed lines represent runout angles corresponding to $\alpha \pm 2\sigma$. The model is not centered on the data due to avalanches that terminate in the ocean (see text).

probability of an avalanche reaching beyond the shoreline, and for the avalanches terminating on land, the probability of an avalanche reaching the observed runout. These probabilities are considered simultaneously using the maximum likelihood method. This approach reduces to the ordinary maximum likelihood estimation of model coefficients corresponding to a normal distribution of residuals when no avalanches reach the ocean. This procedure leads to the following α/β-model for the Icelandic data set shown in Figure 3

$$\alpha = 0.85\beta \ , \ \sigma_{\Delta\alpha} = 2.3^{\circ} \ , \ R = 0.71 \ , \ n = 45 \ . \qquad (3)$$

A least squares linear model without intercept for the Norwegian data set in Figure 2 is given by

$$\alpha = 0.92\beta \ , \ \sigma_{\Delta\alpha} = 3.0^{\circ} \ , \ R = 0.88 \ , \ n = 218 \ . \qquad (4)$$

The correlation coefficient, R, given in eq. (4) is computed as the square root of the relative reduction in the variance of the residuals with respect to the variance of the original data (including a subtraction of the mean in spite of the model being without an intercept term).

Figure 4 shows so-called quantile-quantile plots (qq-plots) of the residuals corresponding to the models given by the two preceding equations. The residuals for the Icelandic avalanches that terminate in the ocean are distributed randomly as described in Jóhannesson (1998). Deviations of the points in a qq-plot from a straight line indicate that the assumed statistical distribution is unable to explain the distribution of the points. Deviations from the assumed normal distribution of residuals for the Norwegian data are evident by the trend away from the straight dashed line for the most extreme avalanches (points near the lower left corner). These avalanches are not collected through systematic investigations of whole regions as mentioned above. Rather, they have been added to the data set one by one when exceptional events are reported or investigated. Figure 4 indicates that the statistical properties of these avalanches are not identical to the rest of the data set. This highlights the problems associated with the non-random sampling of avalanches in the data sets.

Based on Figure 4 and the preceding discussion it was decided to redefine the data sets so that they only contain avalanches collected by systematic investigations of whole regions and not individual events that have been reported because they drew special attention for being extreme in the first place. The Norwegian data set obtained in this way contains the first 197 avalanches in the original data set of 218 avalanches. The avalanche in Dýrafjörður in October 1995 was furthermore omitted from the Icelandic data since it comes from an uninhabited region and was reported only because it reached an unusually long runout. The other avalanches in the Icelandic data set all come from slopes above or in the immediate vicinity of Icelandic villages. Problems due to non-random sampling are of course still present in the data sets after this change, but they should be less pronounced.

There is also a noticeable discrepancy between the trend of the residuals and the line corresponding to a normal distribution in Figure 4 for the shortest avalanches in the Norwegian data set (top right corner of Figure 4). This can either be caused by a real deviation of the statistical distribution of the runout from the assumed normal distribution or it can be a consequence of the non-random sampling of the avalanches. In either case, these very short avalanches will have a small but definite effect on the estimated statistical model, including the predictions of the model for long runout distances. The most important model predictions are of course the predictions for long runout distances. It is unfortunate to have the shortest avalanches pull the estimated model away from the trend indicated by all the other observations. Therefore, it is tempting to omit from the data set the 5 shortest avalanches (the avalanches with a residual larger than 6°), which deviate most from the line in a qq-plot corresponding to a normal distribution, and recompute the model from a data set trimmed in this way. Trimming the extreme ends of a data set is a common procedure in statistical modelling (*cf.* Becker, Chambers and Wilks, 1988). In this case, the trimming eliminates some data points from the less important end of the data set leading to an improved model at the more important end corresponding to long runout distances.

The model for the modified Icelandic data set is almost

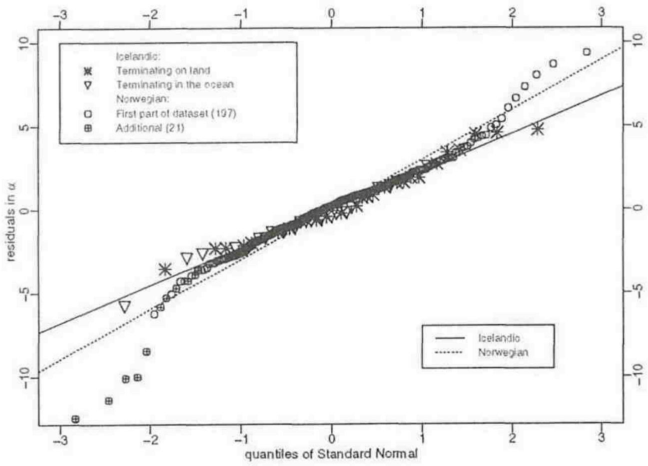

Fig. 4. qq-plots of the residuals of the models given by eqs. (3) and (4) for the Icelandic and Norwegian data sets. Avalanches terminating in the ocean and avalanches terminating on land are differentiated with different symbols for the Icelandic data. The residuals for the avalanches that terminate in the ocean are distributed randomly. For the Norwegian data, separate plotting symbols are used to differentiate 21 avalanches described in separate reports from 197 avalanches catalogued in systematic investigations of whole regions. Lines through the origin with slopes equal to $\sigma_{\Delta\alpha}$ given by eqs. (3) and (4) are also shown.

unchanged from eq. (3) and given by

$$\alpha = 0.85\beta\ , \quad \sigma_{\Delta\alpha} = 2.2^\circ\ , \quad R = 0.72\ , \quad n = 44\ . \qquad (5)$$

This model yields somewhat longer runout than a model derived from the 24 avalanches that terminate on land for which one finds $\alpha = 0.88\beta$, $\sigma_{\Delta\alpha} = 2.3^\circ$, $R = 0.68$. Therefore, the avalanches that terminate in the ocean lead to a model with longer predicted runout distances than would have been derived if these avalanches had been omitted from the analysis as one would have expected.

A model for the modified and trimmed data set of Norwegian avalanches is given by

$$\alpha = 0.93\beta\ , \quad \sigma_{\Delta\alpha} = 2.1^\circ\ , \quad R = 0.93\ , \quad n = 192\ . \qquad (6)$$

As expected, the least squares line is steeper and the residual variance is lower compared with eq. (4) because some of the most extreme avalanches have been omitted from the data set. Figure 5 shows qq-plots of the residuals corresponding to this model and the Icelandic model given by eq. (5). The residuals for the Icelandic avalanches that terminate in the ocean are distributed randomly as in Figure 4. The points in the figure are close to the estimated lines corresponding to a normal distribution of the residuals. The statistical computations for the avalanches terminating in the ocean makes it is difficult to discern deviations from the assumed distribution for the Icelandic qq-plots because the avalanches terminating in the ocean are redistributed according to the assumed normal distribution of residuals. The plot is therefore likely to be consistent with this distribution when 20 avalanches out of 44 terminate in the ocean.

The models given in eqs. (3) to (6) do not include an intercept term as the original α/β-model given by eq. (1).

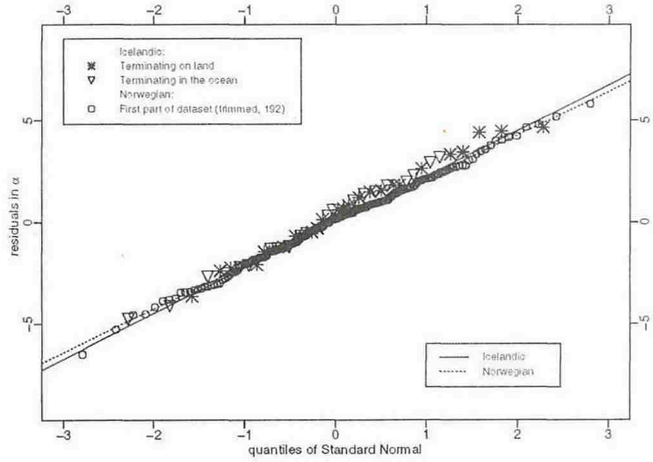

Fig. 5. qq-plots of the residuals of the models given by eqs. (5) and (6) for the modified Icelandic data set and the modified and trimmed Norwegian data sets where events which are not collected by systematic investigations of whole regions and 5 short events have been omitted. Avalanches terminating in the ocean and avalanches terminating on land are differentiated with different symbols for the Icelandic data. The residuals for the avalanches that terminate in the ocean are distributed randomly. Lines through the origin with slopes equal to $\sigma_{\Delta\alpha}$ given by eqs. (5) and (6) are also shown.

This is because such a term is insignificantly different from zero at a 10% significance level in all four cases. This was also found to be the case for data sets of avalanches from Canada, western Norway and Sierra Nevada by McClung, Mears and Schaerer (1989) (but not for a data set from Colorado). A model with an intercept term with the coefficients of eq. (1) is essentially equivalent to eq. (6) and also leads to $\sigma_{\Delta\alpha} = 2.1^\circ$ when applied to the modified and trimmed data set from which eq. (6) is derived.

RUNOUT RATIO MODELS

As discussed in the introduction, runout ratio models based on Gumbel statistics are another possibility for topographic modelling of extreme avalanches. Figure 6 shows a Weibull plot of the runout ratios for the complete Icelandic and Norwegian data sets together with lines that represent statistical models given by eq. (2) where the coefficients a and b are computed by the maximum likelihood method. The runout ratios for the Icelandic avalanches that terminate in the ocean are distributed randomly as described in Jóhannesson (1998).

As for the α/β-modelling of the previous section, deviations from the assumed statistical distribution are evident in Figure 6 by the trend away from the straight dashed line for the most extreme avalanches in the Norwegian data (points near the top right corner). These deviations are no less pronounced for the Gumbel distribution assumed here, than for the normal distribution which is used in the previous section. This indicates that a runout ratio model based on the Gumbel distribution is no better than an α/β-model based on the normal distribution for representing the unmodified

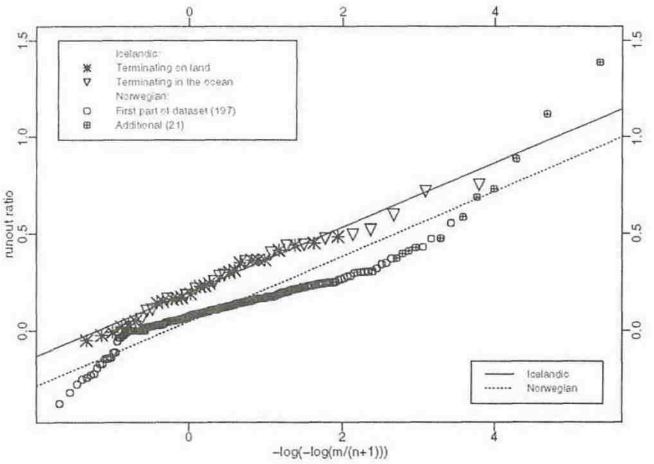

Fig. 6. Runout ratios for the complete Icelandic and Norwegian data sets plotted as a function of the reduced variate $-\log(-\log(m/(n+1)))$ (Weibull plotting positions) where n is the number of data points and m is an index of the ordered runout ratios. Avalanches terminating in the ocean and avalanches terminating on land are differentiated with different symbols for the Icelandic data, and the avalanches that terminate in the ocean are distributed randomly. For the Norwegian data, separate plotting symbols are used to differentiate 21 avalanches described in separate reports from 197 avalanches catalogued in systematic investigations of whole regions. Lines corresponding to runout ratio models based on Gumbel statistics (eq. (2)) for the complete data sets are also shown.

Norwegian data set where avalanches collected through systematic investigations of whole regions are mixed with exceptional events which have been reported or investigated individually. We therefore repeat the analysis for the same modified data sets as in the previous section where events which are not collected by systematic investigations of whole regions are omitted.

Figure 7 shows a Weibull plot for the modified data sets. The difference between the points corresponding to the Icelandic data in figures 6 and 7 is caused by the random distribution of the avalanches terminating in the ocean and indicates the variations that can arise in the computations. Compared with Figure 6, the shape of the Norwegian data set is closer to being linear for the most extreme events. Deviations from the assumed statistical distribution are however evident for the shortest events and this applies to somewhat more points than for the α/β-modelling in the previous section where similar deviations are also found for the shortest events (*cf.* Fig. 4). The break in the distribution of the points near the lower left corner of the figure pulls the estimated maximum likelihood line down in order to improve predictions of the model for these points (because large negative deviations are very unlikely for a Gumbel distribution). As a consequence, the derived model (short dashed line) fits the data poorly, especially for long runout distances. As discussed in the previous section, we are primarily interested in model predictions for long runout distances. It is again unfortunate to have the shortest avalanches pull the estimated model away from the trend indicated by all the other observations as seen in Figure 7. We

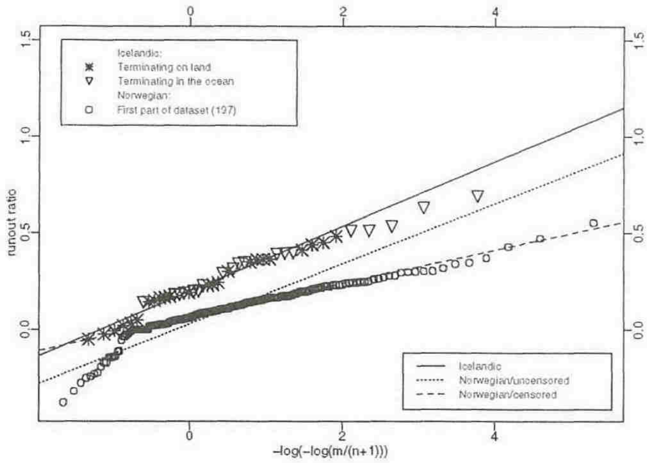

Fig. 7. Runout ratios for the modified Icelandic and Norwegian data sets where events which are not collected by systematic investigations of whole regions have been omitted (see the legend of Figure 6 for explanation). The short dashed line corresponds to a model derived from all data-points in the modified Norwegian data set. The long dashed line corresponds to a model derived from a censored Norwegian data set with a reduce variate $-\log(-\log(m/(n+1)))$ greater than zero (see text).

therefore compute a model that best fits the runout data beyond the low end break in the trend of the data points. This can be done by censoring the data as done by McClung and Mears (1991) who fit a line to data points beyond a certain lower limit in order to eliminate the effect of the shortest avalanches on the model. Here we will censor the data by finding the maximum likelihood estimate of the model coefficients based on data-points beyond a certain limit assuming that the remaining data-points are below this limit. The long dashed line in Figure 7 shows this model when the limit corresponds to the reduced variate equal to 0 as used by McClung and Mears (1991). Other choices for this limit lead to so small changes in the model that the different lines can hardly be distinguished on a plot and are therefore not shown.

The coefficients of the models shown in Figure 7 are given in the following table.

Table 1: Runout ratio models based on Gumbel statistics for the modified Icelandic and Norwegian data sets. The latter number in the "number of observations" column gives the number of observations after censoring. The table gives the coefficients a and b in the Gumbel distribution defined by eq. (2).

Data	Number of obs.	a	b
Iceland, land and sea	44/44	0.20	0.17
Iceland, land only	24/24	0.14	0.15
Norway, censored	197/125	0.065	0.087
Norway, uncensored	197/197	0.03	0.16

The second line in the table gives a model derived for the Icelandic avalanches that terminate on land (not shown in

Figure 7). This model yields shorter runout distances than the model derived from the combined data set and shows that the avalanches terminating in the ocean correspond to longer runout distances than avalanches terminating on land according to this type of model. This was also found to be the case in the α/β-modelling of the previous section.

Table 1 shows that the model for the Icelandic avalanches (first line of the table) yields substantially longer runout distances than the models for the Norwegian avalanches as is also clearly seen in Figure 7. McClung and Mears (1991) derived runout ratio models based on Gumbel statistics for four different regions in the world, western Norway, Coastal Alaska, Colorado Rockies and Sierra Nevada. The model for the Icelandic avalanches in Table 1 yields longer runout than their models for avalanches from western Norway and Coastal Alaska, but shorter than their models for avalanches from the Colorado Rockies and Sierra Nevada. Their model coefficients for western Norway are $a = 0.143$ and $b = 0.077$ for a data set of 80 long avalanches as mentioned in the introduction. This yields somewhat longer runout than the model based on the censored Norwegian data set in the third line of Table 1. The difference, which is between 0.04 and 0.08 in the relevant range of the reduced variate, indicates the magnitude of the differences which can arise from the non-random sampling of avalanches from the same geographical region in these data sets.

COMPARISON OF α/β AND RUNOUT RATIO MODELS

Two questions need to be considered when comparing the α/β-models and the runout ratio models which have been derived in the preceding sections. The first question relates to the explanatory power of the models. Topographical statistical models are valuable because they explain a part of the variability of the observed runout distance of avalanches in terms of topographical parameters. Which type of model explains more of this variability? The answer to this question depends partly on the quantity which is used to measure the runout distance, *e.g.* the α-angle in the case of the α/β-model and the runout ratio in the case of the runout ratio model. The other question is, which of the assumed statistical distributions, the Gumbel distribution or the normal distribution is better suitable for describing the random part of the distribution of avalanche runout?

A comparison of the models in terms of a quantity, which is used to derive one of the models, is not totally fair to the other model because then the coefficients in one of the models, but not the other, have been chosen so that the variability of this quantity as small as possible. A comparison of the models in terms of the variability of the predicted runout ratio is therefore unfair to the α/β-model. Such a comparison can be made by computing for each avalanche the runout ratio corresponding to the predicted α-point and subtracting it from the runout ratio of the actual stopping position. These differences can be considered residuals of the α/β-model in terms of runout ratios. The sum of the squares of these residuals can therefore be compared with the variance of the original runout ratios. As discussed in the introduction, there is no significant correlation between β and the runout ratio. Therefore, one would expect the sum of squares of these residuals corresponding to the α/β-model to be higher than the variance of the original runout ratios, especially if the runout ratio formalism represents the geometry of the avalanche path better than the α- and β-angles as indicated by McClung, Mears and Schaerer (1989).

When the comparison described above is carried out for the modified and trimmed data set of Norwegian avalanches one finds that the sum of squares of the residual runout ratios predicted by the α/β-model defined by eq. (6) is 5% *smaller* than the variance of the original runout ratios. Furthermore, this reduction is statistically significant since there is a correlation at less than a 1% significance level between the original runout ratios and the runout-ratios predicted by the α/β-model. This occurs in spite of the fact that this comparison is favourable to the runout ratio model as mentioned above. Other sub-sets of the data yield similar results. When we consider the sub-set consisting of the 125 points of the Norwegian data set which remain after the censoring described above (*cf.* Fig. 7), we find that the sum of squares of the residual runout ratios predicted by an optimal α/β-model for this data set is almost 10% smaller than the variance of the original runout ratios. This occurs in spite of the excellent fit of this data set to the assumed Gumbel statistical distribution of runout ratios which is seen in Figure 7. In a similar comparison for the avalanches in the Icelandic data set, which terminate on land, it is found that the sum of squares of the residual runout ratios predicted by an optimal α/β-model for this data set is also 5-10% smaller than the variance of the original runout ratios.

A comparison of the models in terms of predicted α-angles yields somewhat larger relative differences in favour of the α/β-models. This is to be expected since such a comparison is in principle unfavourable to the runout ratio models.

As indicated above, the main advantage of topographical statistical models is that they narrow the random part of the distribution of avalanche runout by explaining a part of the variability in the runout in terms of topographical parameters. The advantage of considering a data set of avalanches in terms of an α/β-model over analysing the runout in terms of the original α-angles is that the variance of the modelled residuals in the α-angles is much smaller than the variance of the original α-angles. The importance of this narrowing of the distribution of residuals does not depend on the assumed statistical distribution of the residuals. It is not very useful to achieve an excellent agreement with an assumed statistical distribution of residuals, if this leads to an unnecessarily wide distribution of the residuals. In that case, a part of the variability in the runout, which can be explained by topographical parameters, remains a part of the unexplained random variability. The above results of the comparison of the models indicate that a geometrical description of avalanche paths and the runout of avalanches in terms of runout ratios is slightly inferior to such a description in terms of α- and β-angles. This conclusion may depend on the data sets considered here, but it appears to apply to both the Icelandic and Norwegian data sets.

It is not easy to judge which of the assumed statistical distributions, the Gumbel distribution, in the case of the runout ratio model, or the (implicitly assumed) normal distribution, in the case of the α/β-model, is better suitable for describing the random part of the distribution of avalanche runout. Figures 4 to 7 show that both statistical distributions encounter similar problems with the events in the Norwegian data set which are not collected by systematic investigations of whole regions. The figures also show that both distributions have problems in accounting for the distribution of very short avalanches in the Norwegian data set and this appears to apply to more avalanches for the Gumbel distribution than for the normal distribution (compare Fig. 7 with Fig. 4). Near the more important long runout end of the distributions it is not easy to conclude that one distribution is superior to the other (compare the top right corner of Fig. 7 with the lower left corner of Fig. 5). Note, that the avalanches in the Icelandic data set that terminate in the ocean make it very difficult to draw any firm conclusions regarding the suitability of the assumed statistical distribution from figures 5 and 7, as discussed near the end of the above section about α/β-models.

α/β MODELS WITH ADDITIONAL EXPLANATORY VARIABLES

It is possible to use other formulations in the expression of α in terms of β than the simple linear relationship of eqs. (3) to (6). In Jóhannesson (1998), it is found that worthwhile improvements in the models are not obtained by adding an intercept term, or curvature terms in β, or terms corresponding to other parameters than β, such as the starting slope, θ, the height of the avalanche track, h_β, or the curvature of the avalanche track between the starting position and the β-point, y''. These results are largely equivalent to the results of previous workers that have analysed long Norwegian avalanches (*cf.* Lied and Bakkehøi, 1980; Bakkehøi, Domaas and Lied, 1983; McClung, Mears and Schaerer, 1989). Various combinations of possible additional terms are tabulated in Lied and Bakkehøi (1980) and Bakkehøi, Domaas and Lied (1983) and discussed in the reports NGI (1994 and 1996). The lack of agreement between the tabulated expressions in these references indicates that the variations in the underlying data sets in each case play a major role in the estimated model coefficients and it is doubtful whether they represent worthwhile improvements in the model. This may be appreciated by noting that a linear model in β without an intercept term explains $R^2 = 87\%$ of the variance of the original runout angles for the modified and trimmed data set of Norwegian avalanches (*cf.* eq. (6)). The various additional terms given in tables 2 and 3 in Jóhannesson (1998) lead to less than 1% additional reduction in the variance in each case relative to the variance of the original runout angles.

Other choices than α- and β-angles for the dependent and independent variables of the model are also discussed in Jóhannesson (1998). It is found that using the unscaled horizontal length of the avalanche, l, or the length scaled with the vertical fall of the avalanche, $l/h = \cot(\alpha)$, instead of the α-angle does not lead to an improvement in the model. Similarly, it is found that using the scaled distance to the β-point, $l_\beta/h_\beta = \cot(\beta)$, instead of the β-angle does not improve the model.

One may ask whether the choice of the slope of 10° in the definition of the β-point is the most effective definition of the β-point. This question was considered by computing the 15°-β-points for both the Icelandic and the Norwegian data sets. The residual error of simple α/β-models without intercept based on the 15°-β-points was in all cases considerably higher than the residual error corresponding to the original 10°-β-points. The use of the 15°-β-point lead to an approximately 40% increase in the residual variance for the Icelandic data, and an approximately 10% increase for the Norwegian data.

DISCUSSION

The above considerations lead us to the conclusion that the α/β-models given by eqs. (5) and (6) should be chosen for the Icelandic and Norwegian data sets considered here. These models are without intercept or curvature terms and they do not contain terms corresponding to other variables than β.

There is a substantial difference in the coefficients multiplying β between the model for the Icelandic avalanches given by eq. (5) and the model for the Norwegian avalanches given by eq. (6). This difference is significant at a 5% significance level and indicates that avalanches in the Icelandic data set reach further than avalanches in the modified Norwegian data set for similar β-angles. α/β-models derived by McClung, Mears and Schaerer (1989) for avalanches from Colorado and Sierra Nevada yield longer runout than the model derived here for Icelandic avalanches. Their models for avalanches from Western Norway and Canada, on the other hand, yield shorter runout.

The avalanches that reach the ocean in the Icelandic data set have an effect on the estimated model given by eq. (5) so that it yields a longer runout than a model derived from the avalanches terminating on land. A model based only on the Icelandic avalanches that terminate on land does, however, also lead to longer runout than the model based on the modified Norwegian data set. Many of the 21 avalanches in the Norwegian data set, which are not collected by systematic investigations of whole regions and which are omitted in the modified data set, reach very long runouts, apparently longer than any of the Icelandic avalanches (*cf.* Fig. 2). It is therefore not the case that Icelandic avalanches reach further than Norwegian avalanches in general. Rather, we can only conclude that for the specific avalanches which have been collected by systematic investigations of whole regions in the Norwegian and Icelandic data sets, the Icelandic avalanches seem to reach significantly further than Norwegian avalanches from similarly steep slopes.

Although the α/β-models and the runout ratio models are highly related, there is a small difference between the two types of models in the way avalanche runout is measured. The deviation from the best fit α/β-line may be considered a measure of avalanche runout for the α/β-model whereas the runout ratio itself is a measure of the runout for runout ratio models. The runout ratio depends only on horizontal distances and it is for example independent of any variations in path geometry below the β-point. Therefore, a path that is approximately level or even upsloping beyond the β-point is essentially equivalent to a gently sloping path with a slope slightly below 10° for a long distance beyond the β-point. Avalanches reaching the same runout distance in such paths will therefore have the same runout ratio, but an avalanche in a path that becomes level or slopes upward near the end will be considered more extreme than an avalanche in a gently downsloping path according to an α/β-model. Examination of the Icelandic and Norwegian data sets reveals that some of the more extreme avalanches fall in gentle paths where the lower part of the path has a slope near 10° over a long distance. A good measure of avalanche runout should include the tendency of such paths to produce long avalanches. Since the runout ratio does not have this property to the same degree as the deviation from a best fit α/β-line, this indicates that the runout ratio is an inferior measure of avalanche runout. The conclusion of the previous section about runout ratio models, that the distribution of runout ratios is somewhat wider than the distribution of residuals corresponding to an α/β-model, indicates that this difference does have a small but noticeable effect on the performance of the models. It also indicates that some improvement may perhaps be achieved in topographical statistical models by using a more elaborate description of the avalanche path.

There is a substantial difference in the predicted proportion of very long avalanches, say avalanches corresponding to runout angles below $\alpha - \sigma$ or $\alpha - 2\sigma$ or runout ratios above $a + 2b$, between the Gumbel and normal distributions due to the fact that the Gumbel distribution has a much thicker high end tail than the normal distribution. The effect of this difference is especially marked for the Icelandic data set where the avalanches that terminate in the ocean have an effect on the model coefficients through their estimated runout, which is itself computed in accordance with the estimated coefficients. The thick high end tail of the Gumbel distribution leads to a high likelihood of long runout distances for the avalanches that terminate in the ocean and this again leads to coefficient estimates that predict long runout.

It is difficult to differentiate between the two different statistical distributions on the basis of the Norwegian and Icelandic data sets (*cf.* figures 5 and 7), but it is clear from qq-plots of runout ratios and Weibull plots of deviations from α/β-lines (not shown) that runout ratios cannot be well modelled by a normal distribution nor can the α/β-deviations be well modelled by a Gumbel distribution. Although there is no clear theoretical reason for preferring one of the distributions to the other, the Gumbel distribution does have various advantages for analysing extreme events (*cf.* McClung, Mears and Schaerer, 1989; McClung and Mears, 1991). The observation that the runout ratio models seem to have a higher residual variance for both data sets indicates that a part of the variability of avalanche runout, which is in fact caused by topography, is not explained by the runout ratio models. This part of the variability, which is explained by the α/β-models and not by the runout ratio models, seems to lead to a relatively thick tail in the distribution of the residuals of the runout ratio models. This may partly explain that the Gumbel statistical distribution fits runout ratios better than the normal distribution. If this is the case, then the Gumbel statistical distribution fits the runout ratios well due to what must be considered a flaw in the runout ratio as a measure of avalanche runout and one would be inclined to prefer the normal distribution. It is important to be able to differentiate between the distributions because they lead to substantial differences in the estimated relative proportion of very long avalanches, especially for the Icelandic data, but this requires further analysis of the data.

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APPENDIX: The Icelandic data set

The following table lists the snow-avalanches in the Icelandic data set. The last column in the table indicates whether the avalanche terminated in the ocean (Y) or on land (N).

Nr.	Date	Location	Path	α	β	θ	Sea
1	1906/1907[1]	Patreksfjörður	Vatnskrókur	24[12]	27	45	Y
2	1921[1]	Patreksfjörður	Urðir	24[12]	28	57	Y
3	11.02.1974	Flateyri	3 gullies in Eyrarfjall	25	26	38	Y
4	11.02.1974	Flateyri	Innra-Bæjargil	22	29	41	N
5	18.01.1995	Flateyri	Litlahryggsgil	27[12]	27	36	Y
6	17.03.1995	Flateyri	Miðhryggsgil	25	28	35	Y
7	26.10.1995[2]	Flateyri	Skollahvilft	18	24	39	N
8	05.04.1994[3]	Ísafjörður	Seljalandsdalur, Tunguskógur	18	20	36	N
9	24.03.1947	Ísafjörður	Seljalandshlíð, the farm Seljaland	23	25	39	N
10	24.03.1947	Ísafjörður	Seljalandshlíð, the farm Karlsá	26[12]	27	38	Y
11	12.02.1974	Ísafjörður	Seljalandshlíð, gully west of Hrafnagil	28[12]	28	44	Y
12	18.01.1995	Ísafjörður	Seljalandshlíð, Hrafnagil	26[12]	27	40	N
13	17.01.1995	Ísafjörður	Seljalandshlíð, Steiniðjugil	27[12]	28	39	Y
14	10.02.1974	Ísafjörður	Eyrarhlíð, eastern part of Gleiðarhj.	27	27	36	Y
15	1960-1965	Ísafjörður	Kubbi, Holtahverfi	22	31	41	N
16	30.12.1983	Hnífsdalur	Bakkahyrna, outer part	29[12]	29	30	N
17	19.02.1916	Hnífsdalur	Bakkahyrna, Bakkagil	25	30	38	N
18	1890	Hnífsdalur	Búðarfjall, Hraunsgil	27	30	38	N
19	24.03.1947	Hnífsdalur	Búðarfjall, Hraunsgil	24	28	43	N
20	24.03.1947	Hnífsdalur	Búðarhyrna, Traðargil	25	31	41	N
21	24.03.1947[4]	Hnífsdalur	Búðarhyrna, Búðargil	28	32	45	Y
22	16.01.1995	Súðavík	Traðargil	18	21	26	Y
23	16.01.1995[5]	Súðavík	Súðarvíkurhlíð	23	29	38	N
24	1966	Siglufjörður	Ytra-Skjaldargil	22	25	36	N
25	1936-1938[6]	Siglufjörður	Jörundarskál	21	24	39	Y
26	1938/1939[7]	Siglufjörður	Ytra-Strengsgil	21[12]	21	37	N
27	14.02.1971	Siglufjörður	Fífladalagil	26	25	29	N
28	23.11.1938	Siglufjörður	Hafnarhyrna, the farm Seljaland	29	28	36	N
29	26.12.1963	Siglufjörður	Hvanneyrarbrún/Gróuskarðshnjúkur	24	26	29	N
30	14/15.02.1971	Siglufjörður	Gróuskarðshn., north of Hvanneyrarsk.	22	22	30	Y
31	18.02.1885[8]	Seyðisfjörður	From Jókugil to Hlaupgjá	29	33	43	Y
32	19.03.1946	Seyðisfjörður	Flatafjall	28	29	41	Y
33	19.03.1995	Seyðisfjörður	Nautabás	25	26	33	Y
34	27.02.1990	Neskaupstaður	Gunnólfsskarð	19	22	37	N
35	feb/mar 1936	Neskaupstaður	Innri-Sultarbotnagjá	19[12]	24	38	N
36	26.02.1885[9]	Neskaupstaður	Ytri-Sultarbotnagjá	21	25	34	Y
37	20.12.1974[10]	Neskaupstaður	Bræðslugjár	25	27	34	Y
38	20.12.1974[11]	Neskaupstaður	Miðstrandargil	23	25	31	Y
39	jan/feb 1894	Neskaupstaður	Tröllagil	22	24	35	Y
40	27/28.12.1974	Neskaupstaður	Urðarbotn	23[12]	24	33	N
41	24.01.1894	Neskaupstaður	Drangagil	20[12]	23	39	N
42	19.12.1974	Neskaupstaður	Nesgil	23	25	33	N
43	19.12.1974	Neskaupstaður	Bakkagil	21	25	34	N
44	20.03.1995	Ólafsvík	Tvísteinahlíð	19	25	35	N
45	23-26.10.1995	Dýrafjörður	Gully, northern side of the valley	20	27	38	N

1 Here it is assumed the avalanches in 1906/1907 and 1921 in Patreksfjörður reached into a pond where the present harbour is located, but how long into the pond is not specified. The avalanches are marked as terminating in the ocean although they in fact only reached this pond near sea level.

2 The avalanche from Skollahvilft on 26.10.1995 killed 20 people and caused extensive damage in the village of Flateyri.

3 The Seljalandsdalur avalanche on 05.04.1994 killed one person and damaged summer houses in Tunguskógur to the west of the town of Ísafjörður.

4 Several other long avalanches from Búðargil in Hnífsdalur are reported. An avalanche on 18.02.1910 killed 20 people in the village of Hnífsdalur. Avalanches in 1673, 1910 and 1916 reached the ocean.

5 The avalanche from Súðavíkurhlíð on 16.01.1995 killed 14 people and caused extensive damage in the village of Súðavík.

6 The avalanche from Jörundarskál in 1936-38 is reported to have reached over Siglufjörður on ice. An avalanche on 19.12.1973 also reached the ocean.

7 An avalanche from Ytra-Strengsgil on 12.04.1919 almost reached the ocean similar to the avalanche in 1938/39.

8 The avalanche from the mountain Bjólfur on 18.02.1885 killed 24 people and caused extensive damage in the town of Seyðisfjörður.

9 The avalanche from Ytri-Sultarbotnagjá on 26.02.1885 killed 3 people near the farm Naustahvammur to the west of the town of Neskaupstaður.

10 The avalanche from the gullies Bræðslugjár on 20.12.1974 killed 5 people and caused extensive damage in the town of Neskaupstaður.

11 The avalanche from the gully Miðstrandargil on 20.12.1974 killed 7 people and caused extensive damage in the town of Neskaupstaður.

12 The $10°$-β-point is not clearly defined for several profiles in the data set where the slope may be close to $10°$ or fluctuate around $10°$ over a long distance in the lower part of the profile. For avalanches nr. 1, 2, 5, 10, 11, 12 and 13 the β-point was chosen at the lower end of a range of the profile where the slope fluctuates around $10°$. For the rest of avalanches which refer to this footnote, *i.e.* nr. 16, 26, 35, 40 and 41, the slope of the profile is close to $10°$ over a long distance around the β-point so that its location is rather uncertain.