Ultimate Limit States of Foundation on Random Unsaturated Soils

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Abstract: This paper investigates stochastically the case of a shallow foundation resting on unsaturated heterogeneous soil under rainfall infiltration. Porosity is assumed to vary spatially while other soil parameters are constant. The study employs Monte-Carlo analysis to estimate numerically probabilistic bearing capacity and rotation (represented by off-centred rotating moment) in ultimate limit states. Bearing capacity failure is induced by constant rate strain loading until a sufficient number of elements fail to form continuous shear bands. While off-centred rotating moment is the result from the same loading process due to the simultaneous effects of partial saturated condition and spatial variability of porosity. The study shows that the average of the bearing capacity varies marginally in ultimate limit states as the soil becomes more variable (i.e. increasing coefficient of variation). As the soil becomes more correlated with correlation length between 50 and 200% of the foundation width, there are no apparent changes in the mean and standard deviation of the ultimate bearing capacity. The off-centred rotating moment is maximum when the correlation length is equal to the foundation width.

Keywords: Foundation; bearing capacity; unsaturated; heterogeneity; spatial variation.

1 Introduction

Foundation design is an important topic for geotechnical engineers because foundation support is required in almost every civil engineering structure. Improper design of foundations might cause numerous problems including excessive settlement, differential settlement/rotation and bearing capacity failure. These results in reduction in safety, aesthetic value and economic losses. Spatial variability of soil properties, if not properly taken into account, can significantly contribute and/or considerably exacerbate problems with foundation design. In unsaturated soils, foundation designer faces additional challenge due to the extra complexity associated with suction associated with partial saturation condition. Change in suction due to hydrological processes (e.g., rainfall, evaporation, groundwater variation) can alter soil shear strength that governs settlement or bearing capacity failure. The problem of foundation on unsaturated heterogeneous soils is therefore of practical important particularly for shallow foundation which lies normally in unsaturated zone.

Spatial variability of soil properties is a prime factor contributing to large and non-uniform settlement of foundations. Numerous studies have studied settlement of shallow foundations on heterogeneous soils (e.g., Wu and Kraft 1967; Resendiz and Herrera 1969; Baecher and Ingra 1981; Zeitoun and Baker 1992; Paice et al. 1994 and Paice et al. 1996). Griffiths and Fenton (2001), Fenton and Griffiths (2002), Fenton et al. (2005), and Griffiths and Fenton (2009) used the random finite element method and took into account not only spatial correlation but also local averaging of soil properties. Griffiths and Fenton (2001), Popescu et al. (2005), and Griffiths and Fenton (2009) pointed out that settlements of foundations are dominated by soil regions with low stiffness and that ignoring spatial correlation can lead to unconservative predictions.

Most of studies so far are limited to saturated soils and little research has been undertaken on foundations on unsaturated soils. Some researchers have considered foundation on homogenous unsaturated soils (e.g., Xu 2004; Jahanandish et al. 2010; Vanapalli and Oh 2010). Kawai et al. (2007), among very few studies on deformation of unsaturated heterogeneous soils, studied the non-uniform settlements induced by rainfall infiltration into a compacted earth fill of variable thickness. Le et al. (2013) studied the impact of random variation of porosity on differential settlement of foundation on unsaturated soil subjected to rainfall infiltration. This study modifies the method that has been established in Le et al. (2013) to study ultimate limit states of a shallow foundation on heterogeneous unsaturated soil during a rainfall event.

2 Methods

2.1 Model

The mesh discretization, geometry and dimensions and boundary conditions of the shallow foundation model are shown in Fig. 1. The study investigates the variation of bearing capacity and off-centred rotating moment in ultimate limit states. Soil porosity is assumed to vary spatially while other soil parameters are kept constant. The foundation is assumed to behave rigidly and is prevented from rotating. A constant and uniform displacement rate of 10^{-7}m/s is imposed on section BC up to bearing capacity failure (Fig. 1). Displacements in the horizontal direction of nodes directly underneath the foundation (along BC in Fig. 1) are not restrained; therefore the

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foundation behaves as a rigid smooth structure. The use of displacement instead of stress controlled boundary condition significantly improves numerical stability of the simulation process, especially approaching failure.

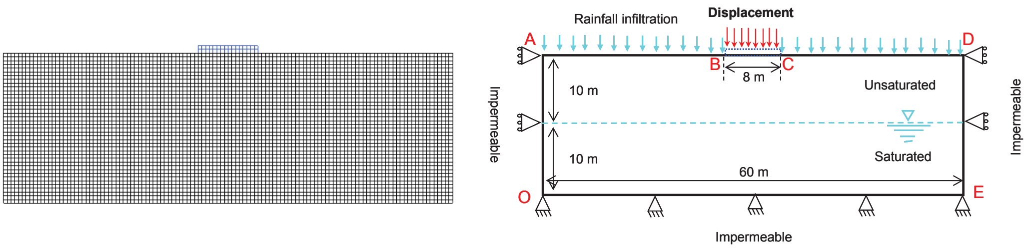

Figure 1. Mesh, geometry and boundary condition for the shallow foundation model.

2.2 Simulation process

The bearing capacity and off-centred rotating moment (referred to as "eccentricity moments" from here after) of the foundation are estimated at six times (at 0, 1, 2, 5, 10 and 15 days). These times capture the most important transitions in the pore water pressure distribution over the rainfall. Each realization involves three stages (1) apply gravity (9.81 m^2/s), (2) simulate rainfall and (3) induce failure to estimate studied variables.

The 1st stage establishes the stress distribution in the soil domain in equilibrium with gravity. Porosity heterogeneity is introduced before applying gravity to take into account the variable soil weight. Then, an analysis of the rainfall infiltration is performed without the load application between BC to establish the pore water pressure distributions. A continuous rainfall at a constant rate of 43.2 mm/day is imposed along AB and CD over 15 days. The rainfall infiltration is simulated with full coupling between the hydraulic and the mechanical behaviour to establish the stresses, strain and pore water pressure distributions at the six times of interest. At each of the six times, stresses and strains (from 2nd stage) are imposed as the initial conditions at corresponding element in stage 3 while the pore water pressure (from 2nd stage) is fixed. The constant displacement rate is applied until failure occurs.

The vertical stress of every element directly underneath the foundation (along BC in Fig. 1) is multiplied by the element width (0.5 m) and added together to calculate the total vertical reaction force. Since the displacement rate is constant over time, the reaction force increases steadily until failure. After failure, the displacement keeps increasing without further increase in the reaction force. The ultimate bearing capacity (q_u) is equal to the total reaction force (after stabilization) divided by the foundation width. The eccentricity moment (m_u) is the sum of the products between the vertical reaction force of each element directly underneath the foundation and the distance from the center of the element to the center of the foundation. This moment is caused by the asymmetrical distribution of stresses underneath the foundation. This asymmetrical stress distribution is of interest because it is resulted from the combination of partial saturation condition and soil heterogeneity.

The values of input parameters for the van Genuchten (1980) and van Genuchten and Nielsen (1985) hydraulic model and the mechanical models are detailed in Table 1. Details about the hydraulic and mechanical models can be found in Le et al. (2013, 2015, 2019). The modified Mohr-Coulomb failure criteria for unsaturated soils are used (Fredlund et al. 1978).

Table 1. Base values of soil parameters adopted in the numerical analyses.

Hydraulic model						Mechanical model					
Symbol	Units	Value	Symbol	Units	Value	Symbol	Units	Value	Symbol	Units	Value
m		0.2	k_{so}	m/s	10^{-5}	E	MPa	100	ϕ'	°	20
η		5	s_{eo}	kPa	20	v		0.3	c'	kPa	5
ϕ_o		0.333							ϕ^b	°	18

In Table 1, m is the gradient of water retention curve. Parameter η that controls the rate at which suction deviates from its reference value s_{eo} when porosity deviates from its reference value ϕ_o. k_{so} is the reference value of the saturated permeability. E is the Young's modulus while v is the Poisson's ratio. ϕ' denotes friction angle while parameter ϕ^b controlling the increase in shear strength with suction. c' denotes soil cohesion. Heterogeneity of porosity leads to spatial variability of water retention and permeability properties. This affects flow characteristics (Le et al. 2012; Le 2012). In this study, the random porosity is generated by first, generating a random field of void ratio (e). The void ratio is then converted to the porosity by the following equation $\phi=e/(1+e)$. This is convenient since void ratio varies in the positive range and is assumed to follow a log-normal distribution. Porosity is bounded between 0 and 1, thus would require a more complicated distribution function to represent. The mean void ratio is kept constant $\mu(e)= 0.5$, coefficient of variation $COV_e=\sigma(e)/\mu(e)$ between 0.2 and 0.8 and correlation length $\theta(e)$ between 4 and 16 m (i.e. 50 to 200% of foundation width).

3 Results

3.1 Failure at ultimate limit state

Fig. 2 shows a sample realisation of random porosity (converted from void ratio) and its corresponding pore water pressure, displacement and volumetric strain contours at 5 days into the rainfall. The variability of porosity results in unsymmetrical migration patterns of water underneath the foundation (Fig. 2b). This leads to the uneven suction variation between the two sides of the foundation. The resultant displacement and volumetric strain contours at 5 days of failed soil regions shows visible shear bands of large displacement directly underneath the foundation. The model predicts displacement within 40 cm prior to failure with the assumptions that there are no plastic strains prior to reaching a failure condition and then null dilantancy during shearing.

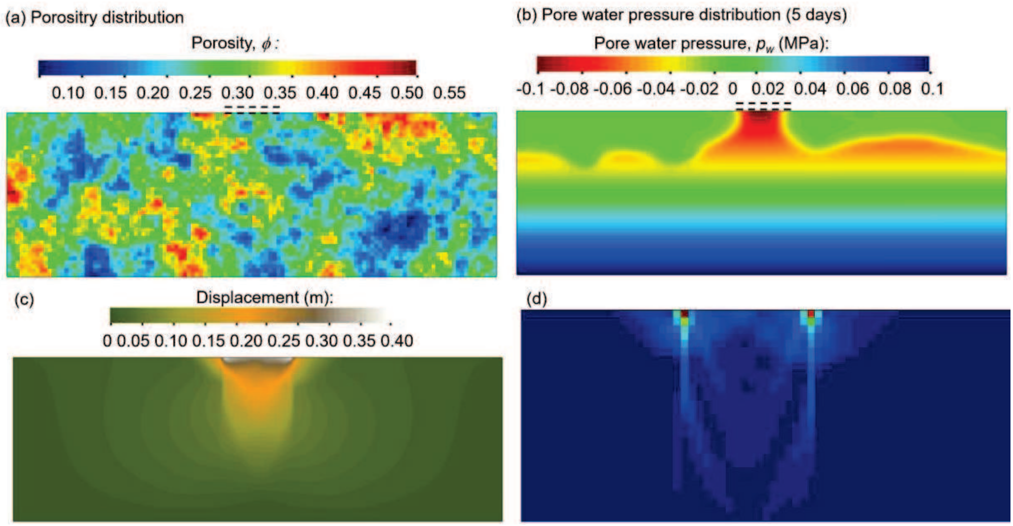

Figure 2. A sample realization of (a) random porosity and its corresponding, (b) pore water pressure contours (at 5 days), (c) displacement contour at failure, (d) volumetric strain pattern at failure (image of the failed area).

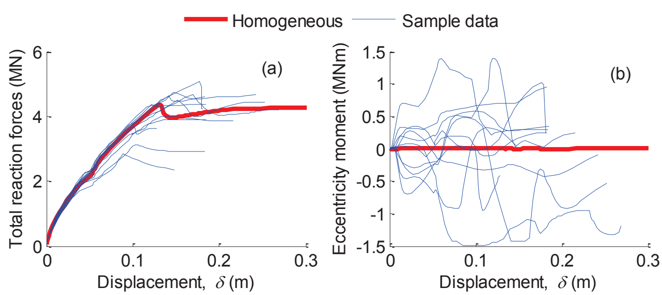

Figure 3. Evolution of (a) Total reaction forces at elements directly underneath the foundation and (b) Eccentricity moment with displacement at 10 days ($\mu(e) = 0.5$, $COV_e= 0.8$ and $\theta(e) = 8$ m), 10 realizations.

Figure 3 illustrates the variations of the total reaction forces and the eccentricity moments of 10 sample realizations. The profiles for an associated homogeneous domain are also shown for comparison purpose. Initially, the reaction forces in each realisation increases steadily with increasing displacement. Once the failure mechanism is activated, the reaction forces fluctuates around a stable value (Fig. 3a). For a homogeneous soil domain, the eccentricity moment is zero over the whole loading period. Conversely, the moment profiles for heterogeneous soils fluctuate significantly with increasing displacement (Fig. 3b). This highlights that the non-zero values of eccentricity moment are caused by the soil heterogeneity.

3.2 Probabilistic results

3.2.1 Probability distribution function

About 50 to 100 realizations are conducted for each set of results. The distributional parameters of the studied variables ($\mu(q_u)$, $\sigma(q_u)$, $\mu(m_u)$ and σ (m_u)) generally fluctuate within a relatively small range after about 50 realizations. The results for both ultimate bearing capacities and eccentricity moments are found to be reasonably captured by the normal distribution functions as can be seen in Fig. 4.

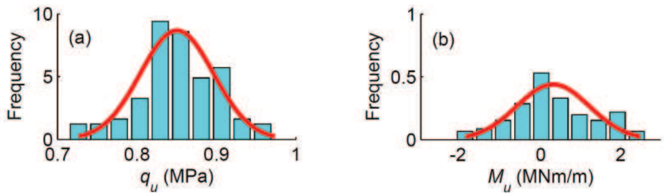

Figure 4. Frequency histogram of (a) q_u (b) m_u at 2 days ($\mu(e)$ = 0.5, COV_e = 0.4 and $\theta(e)$ = 12 m).

3.2.2 Ultimate bearing capacity (q_u)

The model predicts that the mean of the ultimate bearing capacity $\mu(q_u)$ reduces monotonically over the rainfall within the investigated range for COV_e (Fig. 5). This reduction is caused by the suction loss early in the rainfall and the water table elevation near the end of the rainfall. The $\mu(q_u)$ varies marginally between different values of COV_e and are very close to the q_u of the homogeneous soil domain (i.e. having spatially constant e=0.5). This is somewhat different from the results from Fenton and Griffiths (2003) for bearing capacity of saturated soils using an elastic perfectly plastic model with Mohr-Coulomb failure criteria. The study showed that the mean bearing capacity decreased with increasing coefficient of variation of shear strength parameters. Fenton and Griffiths (2003)'s study was however conducted over a significantly larger range of *COV* (from 0.1 to 5.0). In addition, cohesion and friction angle have more direct and stronger impacts on the soil strength than void ratio/porosity. Over a much smaller range of *COV* (from 0.2 to 0.8) and for a less influential random variable (i.e., porosity), the results indicate that there are practically no differences with respect to $\mu(q_u)$.

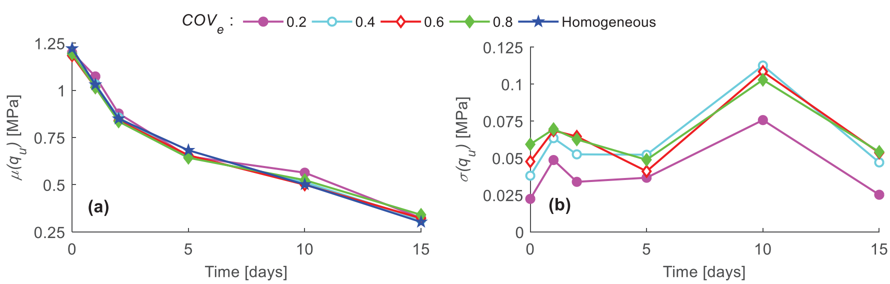

Figure 5. Variation of (a) mean $\mu(q_u)$ and (b) standard deviation $\sigma(q_u)$ of the ultimate bearing capacity with coefficient of variation COV_e.

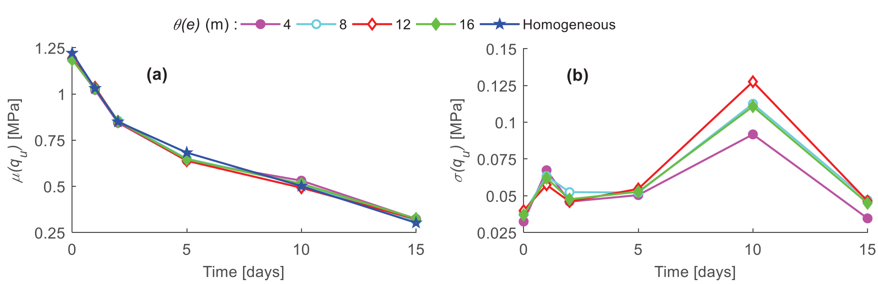

Figure 6: Variation of (a) mean $\mu(q_u)$ and (b) standard deviation $\sigma(q_u)$ of the ultimate bearing capacity with with coefficient of variation COV_e.

The $\sigma(q_u)$ appears to increase over the first day of rainfall, then decreases slightly from day 1 to 5 (Fig. 5). The $\sigma(q_u)$ reaches a shaft peak at 10 days of continuous rainfall for any COV_e within the range considered (Fig. 5). This is because the q_u at 10 days fluctuates between extreme values depending on pore pressure distribution underneath the foundation. The q_u can be significantly low corresponding to realizations in which the soil domain has reached or is close to reaching complete saturation with the water table at ground surface. On the contrary, the q_u can be high corresponding to realizations in which large areas directly underneath the foundation are still unsaturated at 10 days due to the existence of a large area of low permeability. At 15 days, almost all the realization have become saturated regardless of porosity distribution. The $\sigma(q_u)$ therefore drops considerably.

The variation patterns and peaks of $\mu(q_u)$ and $\sigma(q_u)$ for different $\theta(e)$ over the rainfall confirm those observed for different COV_e (Fig. 6). There are no substantial differences between different $\sigma(q_u)$ within the range investigated $\theta(e)$, except at 10 days when the $\sigma(q_u)$ appears to be slightly larger at intermediate $\theta(e)$ = 8-12 m.

3.2.3 *Eccentricity moments (m_u)*

Eccentricity moments can be either positive if rotating counter clockwise or negative otherwise. However the absolute magnitudes are of practical interest and hence the distributional parameters for absolute eccentricity moments are presented (Figs. 7 and 8). The $\mu(|m_u|)$ and $\sigma(|m_u|)$ exhibit similar increasing trends with increasing COV_e due to increasing possible variation in the suction distribution between the two soil regions symmetrical through the centre line of the foundation (Fig. 7). On the other hand, the $\mu(|m_u|)$ and $\sigma(|m_u|)$ peak at the correlation length equal to the foundation width (i.e., 8 m) for the range of $\theta(e)$ from 4 to 16 m (Fig. 8). These results agree well with the results for differential settlements of foundation on heterogeneous unsaturated soils (Le et al. 2013). The tendencies are also consistent with the hypothesis proposed by Griffiths and Fenton (2001), Fenton and Griffith (2003), and Breysse (2005) that the correlation length close to the foundation width is the "worst case scenario" with respect to foundation problem.

All the curves peak at 2 days suggesting that at this time, out of the six times of interest, the variation of suction is maximum between the two sides of the soil domain. As rainfall progresses, the differences between two sides reduce and hence the reduction in $\mu(|m_u|)$ and $\sigma(|m_u|)$ at 5 days compared with 2 days. Note that the exact time for the true peak might however lie somewhere between 1 to 5 days but can only be detected with consideration of many more points.

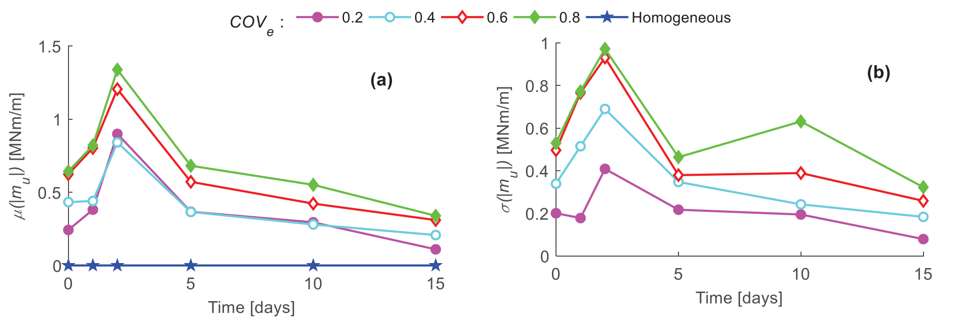

Figure 7. Variation of (a) mean $\mu(|m_u|)$ and (b) standard deviation $\sigma(|m_u|)$ of the absolute ultimate eccentricity moment with COV_e.

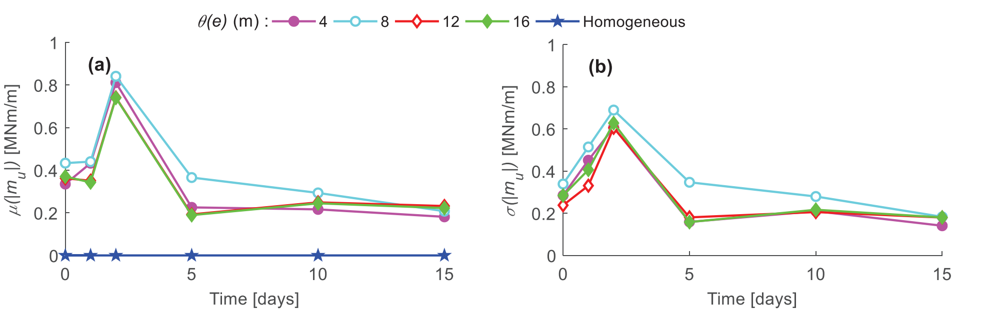

Figure 8. Variation of (a) mean $\mu(|m_u|)$ and (b) standard deviation $\sigma(|m_u|)$ of the absolute ultimate eccentricity moment with $\theta(e)$.

4 Conclusions

The results show that the means of ultimate bearing capacity monotonically decrease, while the standard deviations fluctuate over the simulated 15-day rainfall event. The model predicts a prominent peak in standard deviation of ultimate bearing capacity at 10 days before dropping to a low value at 15 days. The means and standard deviations of eccentricity moment tend to increase initially and reach prominent peaks at 2 days, then fluctuate within a narrow range from day 2 onwards. As the soil becomes more variable (i.e. increasing coefficient of variation of void ratio), the means of bearing capacity only vary marginally in ultimate limit states while the standard deviations do not show a clear pattern of variation. Both the means and the standard deviations of the eccentricity moment increase with more variable soil.

As the soil becomes correlated over a longer distance (i.e. increasing correlation length within 50 to 200% of foundation width), there are no apparent changes in the means and standard deviations of ultimate bearing capacity. In terms of eccentricity moment, the "worst" correlation length is equal to the foundation width.

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