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Small strain stiffness within logarithmic contractancy model for structured anisotropic clay

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Abstract. Stiffness of soils in the small strain region is high and it decays nonlinearly with increasing shear strains or with mobilization of shear stresses. However, the commonly used critical state based constitutive models use a simple elastic formulation at small strains that falls short in the prediction of the small strain nonlinearity and anisotropy. This paper proposes a simple way for rendering the existing constitutive models with the capability to capture the small strain behaviour of soils. This is illustrated by proposing a new model for structured anisotropic clay extending an existing model that uses the framework of logarithmic contractancy called E-SCLAY1S. The proposed model is implemented into a Finite Element program as a user-defined soil model. The model predictions are compared with experimental data for various clays. Furthermore, the effect of nonlinearity is investigated for an excavation in soft clay.

1. Introduction

For robust design of retaining wall system in the serviceability limit state, traditional design methods such as limit equilibrium or empirical and semi empirical methods are not suitable. The wall deflection predictions of retaining wall and ground settlements during excavation can most likely be inaccurate, which would have serious consequences in the surrounding structures. Ground settlement induced by excavation is more difficult to accurately predict than wall deflection as it requires proper modelling of soil behaviour at the level of small strains ([1], [2]). This is because the shear strain of soils during excavation is expected to be less than 0.1% ([3]) as shown in Figure 1. Given these issues, the use of finite element (FE) analysis with an accurate constitutive model that can capture small strain behaviour can be a solution. However, many of the existing advanced constitutive models that can predict the structure and anisotropic behaviour of soft soil fall short in their ability to capture the small strain stiffness and its degradation.

To account for the small strain stiffness behaviour, anisotropy and structure of soils in analysis and design, it is necessary to develop a proper constitutive model for possible ranges of stress and strain. One approach is to develop models based on bounding surface theory (e.g. [4], [5]) for nonlinear and inelastic from the very early stages of loadings. An alternative simple approach is the inclusion of

nonlinear elastic model into the existing linear elastic-plastic constitutive models (e.g. [6]) which account for anisotropy and structure.
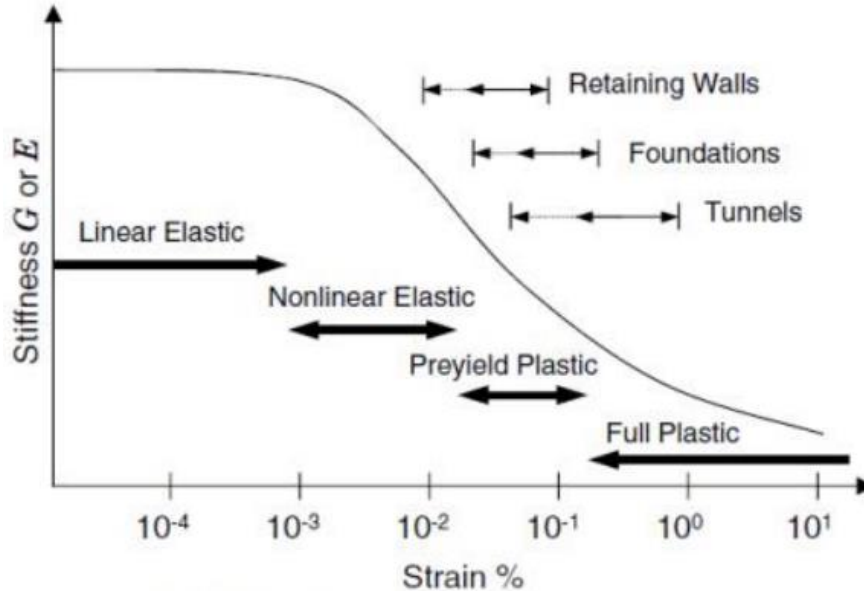

Figure 1. Characterization of reduction of stiffness of soil with typical strain ranges (after [3])

This paper presents an extended constitutive model of E-SCLAY1S ([6]) with nonlinear shear stress-strain behaviour in the small strain range without changing other characteristics of the model. The enhancements of the model are demonstrated by performing element level simulations for shear modulus reduction curves and comparing them with the reference E-SCLAY1S model. Finally, the enhancements of the model are demonstrated by performing Finite element analysis of an excavation problem. The results are compared with that of the reference model and differences and gained benefits are discussed.

2. Modelling of small strain behaviour

2.1. Literature review
Various empirical formulations have been proposed based on the laboratory test results to express the small strain dependency of the stiffness (e.g. [7]). In literature, there exist various proposed formulations that aim at capturing the non-linear decay of secant shear modulus, G_s, with shear strain in the small strain regime. Several of those are in the form

$$\hat{G}_s = \frac{1}{1+a\left(\frac{\gamma}{\gamma_{ref}}\right)^{\alpha_\gamma}} \tag{1}$$

where $\hat{G}_s = \frac{G_s}{G_0}$, G_0 is the initial shear stress, γ is the shear strain, γ_{ref} is a reference shear strain, and a and γ_{ref} are model parameters, ([8]). Early proposition by [9], for example, was $a = 1$, $\alpha_\gamma = 1$ and $\gamma_{ref} = \tau_{max}/G_0$, where τ_{max} is the maximum shear stress. [10] considered $a = 1$, $\alpha_\gamma = 1$ and $\gamma_{ref} = \gamma_{0.5}$. [11] modified the [9] equation considering $a = 0.385$, $\alpha = 1$ and $\gamma_{ref} = \gamma_{0.7}$. The reference shear strains $\gamma_{0.5}$ and $\gamma_{0.7}$ are shear strains at which the stiffness has decayed by 50% and 30% respectively. In [12], $\gamma_{0.7}$ is determined at a slightly modified point, where the stiffness has degraded 27.8%. When a and α_γ are fixed, only two parameters, namely γ_{ref} and G_0 are required to define the stiffness

degradation curve. The degradation rule in Eq. (1) lacks a true limit in the small strain regime and decays to zero as the sear strains increase to infinity. For using it as an overlay for enhancement of elastoplastic models, an abrupt cut off is required to limit its play in the defined small strain regime. The cut-off criterial can be either in stiffness or in strain.

A small strain degradation rule was introduced into hypoplastic models by [13] proposing the Intergranular Strain concept. The Intergranular Strain formulation, which was primarily aimed at the enhancement of the performance of the hypoplastic models in the small strain regime, has also been adopted into elastoplastic models [8]. It is highly nonlinear and requires some 5 parameters to calibrate. However, most parameters are usually put to a default value in practice.

The concept of kinematic region for modelling stress-strain behaviour within the small strain region (SSR) was put forward by [14] and further developed by [15] by introducing a yield surface to represent the SSR. The SSR is surrounded by a smaller region where the soil response is linear elastic. Following this concept, kinematic hardening multiple surface plasticity models (e.g. [16], [17]) and bounding surface plasticity models (e.g. [4], [5], [18], [19]) have been proposed to describe the stress-strain behaviour within the SSR. These models are mathematically complex and determination of input parameters from standard laboratory tests can be difficult if not impossible.

3. Small strain stiffness in E-SCLAY1S constitutive model

3.1. The E-SCLAY1S model

The recently proposed E-SCLAY1S model ([6]) is an extension of the S-CLAY1S ([20]) model, which accounts for anisotropy and destructuration. Anisotropy of plastic behaviour is represented through an inclined and distorted yield surface and a rotational hardening law to model the development or erasure of fabric anisotropy during plastic straining; while interparticle bonding and degradation of bonds (structure) is reproduced using intrinsic and natural yield surfaces ([21]) and a hardening law describing destructuration as a function of plastic straining. For the sake of simplicity, the mathematical formulation is presented in the following in triaxial stress space, which can be used only to model the response of cross-anisotropic samples (cut vertically from the soil deposit) subjected to oedometer or triaxial loading.

The yield surface of the E-SCLAY1S model (see Fig. 2), which introduces an additional degree of freedom in the shape of the yield surface using the framework of logarithmic contractancy ([22]) is given as:

$$f_y = \left(1 + \frac{|\eta - \alpha|^{n_L}}{M^{n_L} - \alpha^{n_L}}\right)^{\Psi} - \frac{p'_m}{p'} = 0 \qquad (2)$$

where Ψ is an intermediate parameter and is given as

$$\Psi = \frac{(M - \alpha)}{n_L M}\left[1 + \frac{M^{n_L} - \alpha^{n_L}}{(M - \alpha)^{n_L}}\right] \qquad (3)$$

and n_L is a new parameter (contractancy parameter) that controls the shape of the yield surface. The subscript "L" refers to logarithmic contractancy, following the notation by [22]. M is the slope of the critical state line, α defines the orientation of the yield surface, p' is the mean effective stress and p'_m defines the size of the yield surface. The E-SCLAY1S preserves the hierarchical development of S-CLAY1S, as E-SCLAY1S reduces to S-CLAY1S for n_L=2, i.e. Eq. (3) reduces as Ψ=1.

As in the S-CLAY1S model, the effect of bonding in the E-SCLAY1S model is described by an intrinsic yield surface ([21]), which has the same shape and inclination of the natural yield surface but with a smaller size. The size of the intrinsic yield surface p'_{mi}, which is related to the size p'_m of the natural yield surface, is linked to the state parameter χ (i.e. current amount of bonding) as

$$p'_m = (1 + \chi)p'_{mi} \qquad (4)$$

The E-SCLAY1S model predicts an elastic behaviour within the yield surface similar to the critical state Cam clay types model. The elastic shear modulus is expressed in terms of mean effective stress (p'), Poisson's ratio (v), e is void ratio and the slope of the swelling line (κ) as

$$G = \frac{3}{2} \frac{(1-2v)}{(1+v)} \frac{(1+e)}{\kappa} p' \qquad (5)$$

The above formulation is mean effective stress and void ratio dependent but not shear strain dependent. However, it is well known that the shear modulus of soils is dependent on the amplitude of shear strain even at small strains.

The E-SCLAY1S model adopts three hardening rules from S-CLAY1S, namely isotropic hardening, rotational hardening and degradation of bonds rule. More details of the model formulation and implementation into a FE code can be found in [6].
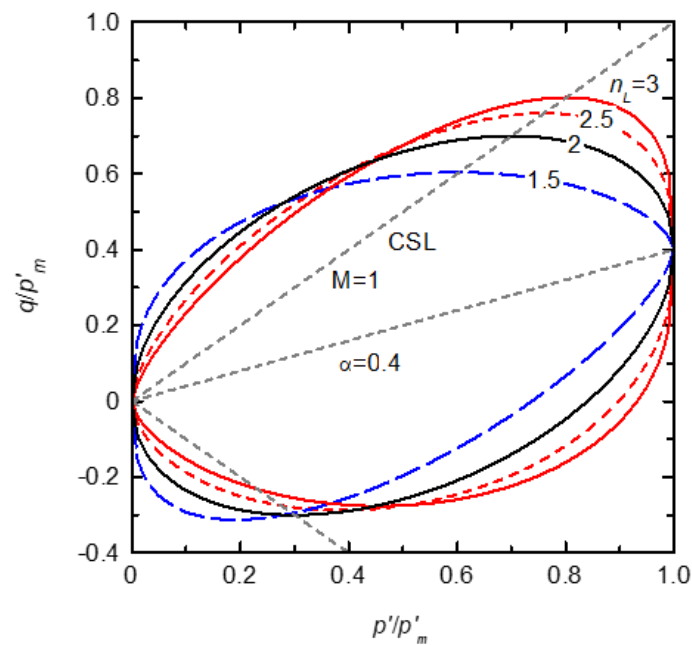

Figure 2. Different shapes of the yield surface for E-SCLAY1S.

3.2. Small strain stiffness formulation

For cohesive soils, several empirical equations have been proposed by considering the dependency of the stiffness on the strain. In this study, the secant shear modulus reduction with strain is expressed following [24] as:

$$G = G_0 \left[1 - \frac{\langle \varepsilon_q - \varepsilon_s \rangle}{A + B \langle \varepsilon_q - \varepsilon_s \rangle} \right] \qquad (6)$$

where ε_q = shear strain, ε_s = limiting shear strain and A and B are model input parameters. Below the limiting strain ε_s, which is set to a predefined value of 10^{-5}, G will be equal to G_0. $\langle \ \rangle$ are Macaulay brackets and $\langle \varepsilon_q - \varepsilon_s \rangle = \varepsilon_q - \varepsilon_s$ for $(\varepsilon_q - \varepsilon_s) > 0$ and $\langle \varepsilon_q - \varepsilon_s \rangle = 0$ for $(\varepsilon_q - \varepsilon_s) < 0$.

One of the advantages of Eq. (7), in addition to its simplicity, is that the secant shear modulus has a true limit and does not require abrupt and arbitrary cut-off when used as an overlay on elastoplastic models. The limit is, $\lim\limits_{\varepsilon_q - \varepsilon_s \to \infty} \frac{G}{G_0} = 1 - 1/B$. The level of secant shear stiffness to which the small strain overlay

is active and beyond which the elastoplastic model is adequate can then be decided using the parameter B. It should be stated here that an abrupt cut-off was needed when the hyperbolic model due to Hardin and Drnevich (and its derivatives) ([9], [23]) was used as an overlay on elastoplastic models. [13] managed this in their intergranular strain formulation but the evolution rule treaded off simplicity.

Figure 3 shows the effect of A and B on the G/G_0 vs ε_q behaviour. Figure 4 shows the influence of A and B in undrained single element tests compared to the original E-SCLAY1S model formulation. These simulations were strain controlled and volume was conserved for undrained condition by applying $d\varepsilon_1 = -2d\varepsilon_2 = 2d\varepsilon_3$. The sample was assumed over-consolidated (OCR=2) and isotropic state in the beginning and undrained shearing starts from $p' = 50$ kPa. The model parameter are summarized in Table 1.
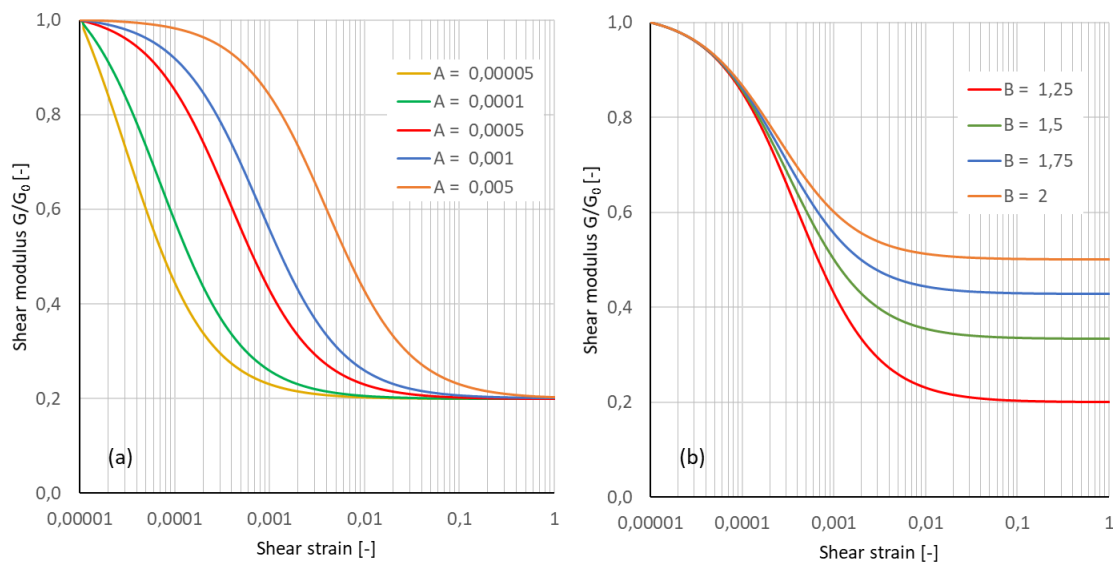

Figure 3. Normalized shear modulus reduction curves function of parameter (a) A for B = 1.25 and (b) B for A = 0.0005

3.3. Calibration of the small strain parameters, A and B

In the absence of data, the model parameter A may be related to the plasticity index (PI) of the clay. In the following, A has been calibrated to the data provided by [25] and a reasonably good fit is obtained as shown in Fig. 5. The second model parameter, B, may be related to the unload reload shear modulus (G_{ur}) as:

$$B = \frac{1}{1 - \frac{G_{ur}}{G_0}}$$ (7)

assuming that the elastoplastic model is good on its own beyond the unload-reload shear stiffness.
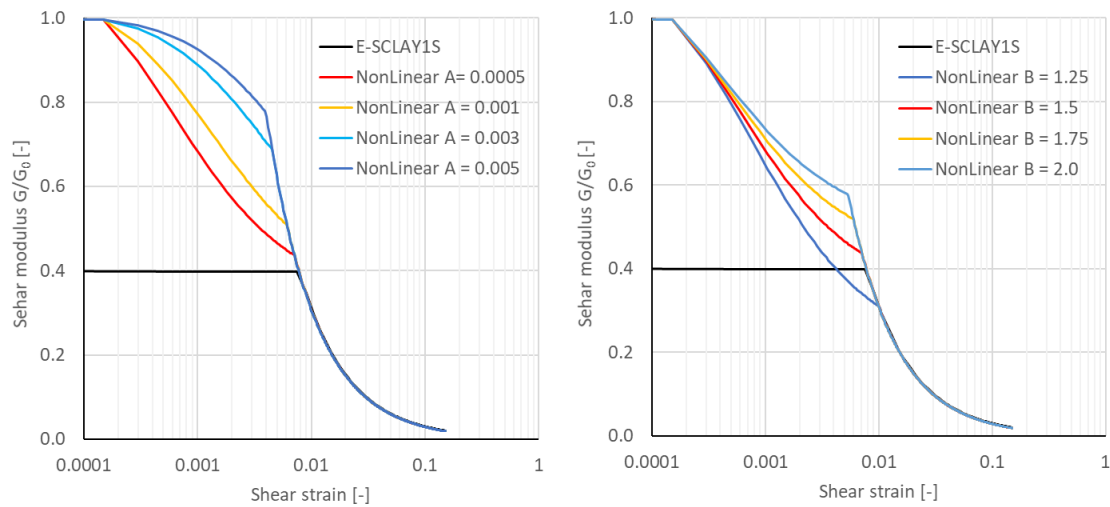
Figure 4. Normalized shear modulus reduction curves during undrained shearing for E-SCLAY1S model as a function of parameter (a) A for B = 1.5 and (b) B for A = 0.0005.
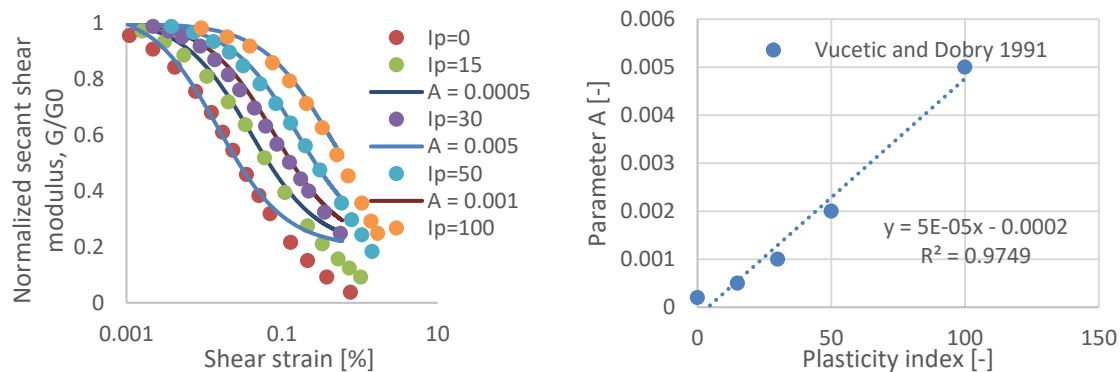

Figure 5. a) Model calibration to data from [25] b) Implied dependence of the parameter A on the plasticity index (based data from [25])

4. Model performance in a FE boundary problem

4.1. Problem framework and input parameters

The performance of the E-SCLAY1S model with small strain formulation is evaluated in a FE boundary value problem using Bentley PLAXIS 2D FE code. A typical geotechnical engineering problem where small strain stiffness plays an important role is the deformation of a sheet pile wall induced by an unsupported excavation in soft clay. This problem was studied with PLAXIS 2D 2020 version and the proposed model, which has been implemented as a user-defined soil model, was used to simulate the behaviour of a soft normally consolidated clay.

The excavation is 5 m high and 6 m wide and is modelled as dry. The soil is contained laterally by a sheet pile wall, which is 8 m high and embedded 3 m into the ground, down to the bottom boundary. The geometry is 8 m deep and 15 m wide. The FE mesh (see Figure 6) consists of 15-noded triangular elements of different sizes, with mesh refinement around the sheet pile wall. The problem is modelled as plane strain. Only half of the geometry is modelled because of symmetry. Roller conditions are applied to the lateral boundaries; while the bottom of the model is fully fixed.

The problem is modelled in two stages: firstly, initial stresses are generated through a "K_0 procedure"; secondly, the excavated soil cluster is deactivated in a "plastic" undrained phase. The sheet pile wall is modelled as an elastic plate element with rigid interface.

Table 1 and 2 summarize the input parameters for the soft clay and the sheet pile wall, respectively. Clay parameters are based on [20], except for the new introduced small strain parameters, which are assumed for PI ≈ 60%. Sheet pile wall parameters are based on Tutorial 3 of PLAXIS ([26]).

Table 1. Input parameters for E-SCLAY1S with small strain formulation.

Parameter	Unit	Description	Value
γ	kN/m^3	Unit weight	15
κ	-	Intrinsic slope of swelling line in e-lnp' plot	0.02
λ	-	Intrinsic slope of compression line in e-lnp' plot	0.18
ν	-	Poisson's ratio	0.2
M	-	Slope of critical state line	1.6
	-	Absolute effectiveness of rotation hardening	100
β	-	Relative effectiveness of rotation hardening	1
a	-	Absolute effectiveness of destructuration hardening	8
b	-	Relative effectiveness of destructuration hardening	0.1
POP	kPa	Pre-overburden pressure	1
e_0	-	Initial void ratio	2
α_0	-	Initial yield surface inclination	0.6
x_0	-	Initial bonding effect	10
n_L	-	Contractancy parameter	2
G_0/p'	-	Normalized G_0 with mean effective stress	281*
A	-	Small strain parameter	0.003
B	-	Small strain parameter	1.5

*G_0/p' = 112.5 is used by the basic E-SCLAY1S model following equation (5).

Table 2. Input parameters for sheet pile wall (plate element).

Parameter	Unit	Description	Value
EA	kN/m	Normal stiffness	12E06
EI	kNm^2/m	Flexural rigidity	12E04
w	kN/m/m	Weight	8.3
ν	-	Poisson's ratio	0.15

4.2. Results

Figure 7 shows the wall deflection and the bending moment distribution along the wall. The implemented small strain formulation yields up to ≈30% smaller horizontal displacements and bending moment, compared to the basic E-SCLAY1S model, which generally gives a softer response. This is an expected result and it follows the behaviour observed in Figure 4.

Further, the small strain formulation affects the surface settlement, also in the far field, up to ≈20% difference for the range of parameters considered. This is illustrated in Figure 8.

The small strain parameter A appears to have a larger effect than B on the wall behaviour. This is to be attributed to the shear strain induced by the excavation, which are in the order of 10^{-3}. The parameter B is expected to affect the excavation behaviour at larger strain levels.
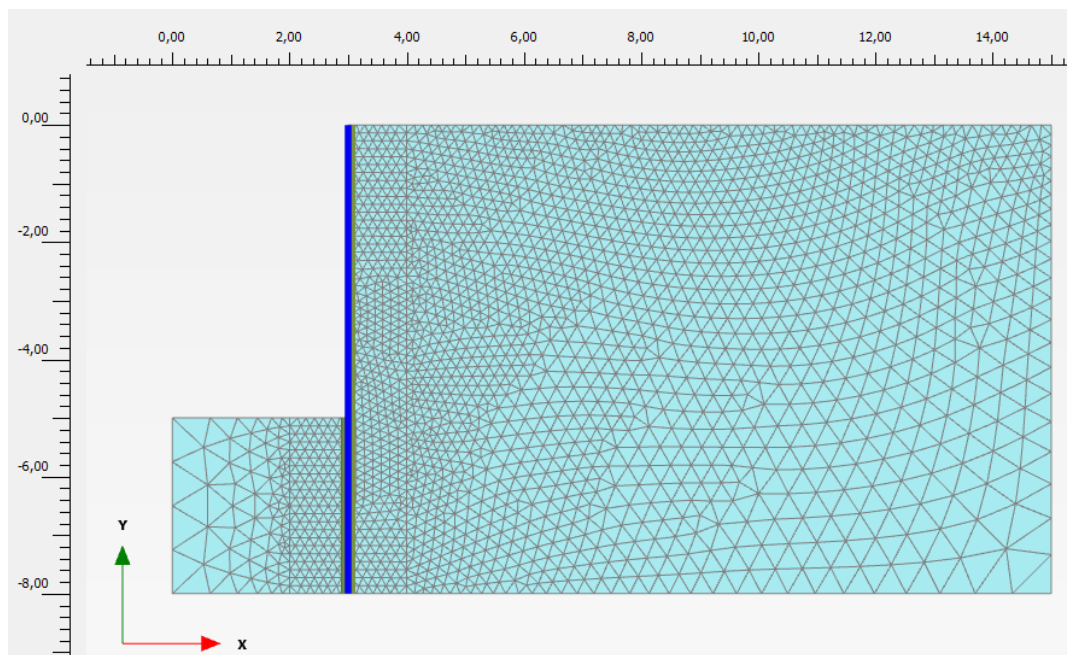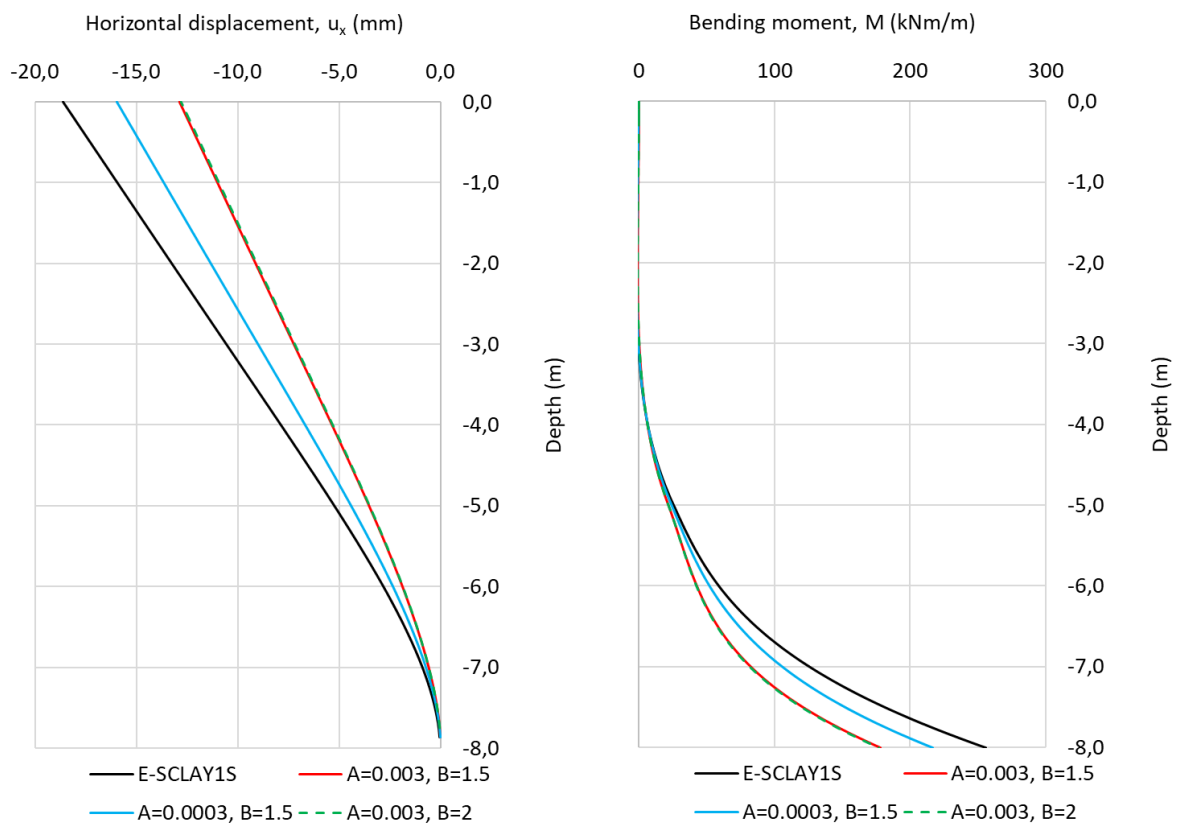
Figure 6. FE discretization.

Figure 7. Horizontal displacement and bending moment in the sheet pile wall as a function of small strain model parameters.
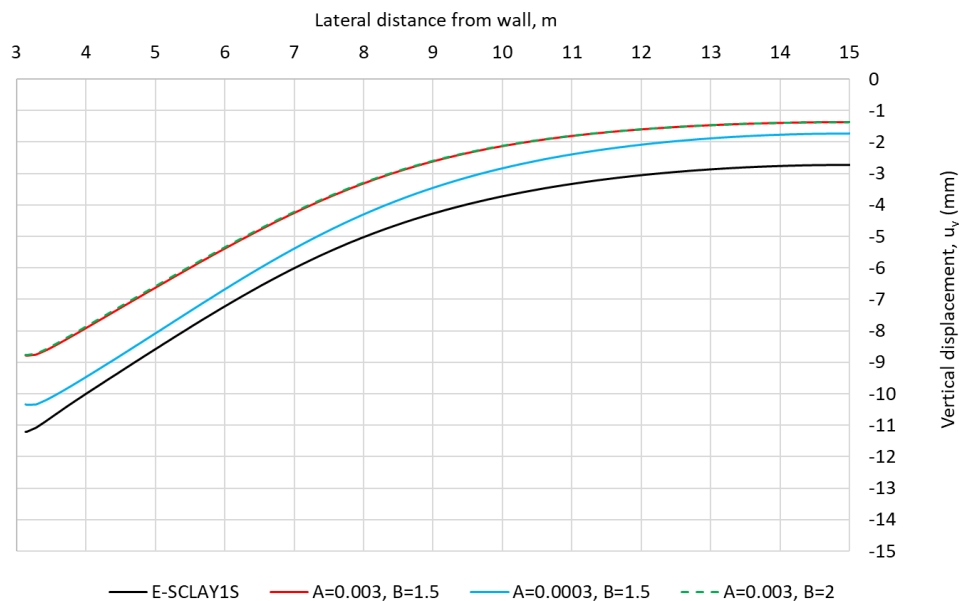
Figure 8. Ground settlement as a function of small strain model parameters.

5. Summary and conclusions

This paper discusses the implementation of a small strain stiffness formulation within the anisotropic constitutive model for clays E-SCLAY1S which can predicts initial structure and its degradation. The model is rate-independent and characterized by an anisotropic yield surface controlled by a logaritmic contractancy parameter n_L. In particular, n_L defines the shape of the yield surface and can be tuned to fit both the earth pressure coefficient at rest K_0 and the undrained shear strength. Additionally, the model version presented in this paper is capable of modelling small strain behaviour, which is described by a function that considers a limiting shear strain value and two curve fitting parameters A and B. The parameter A appears to correlate with the plasticity index.

The performance of the model was evaluated in single element test simulations and in a FE simulation of an undrained excavation in clay. Results demonstrated how the small strain formulation affects the deformation behaviour of the excavation and the bending forces in the sheet pile wall. Therefore, this highlights the importance of including an appropriate small strain formulation in design as it not only affects serviceability but also the design of the structural elements involved.

As a future research work, the model requires validation against actual measurements (i.e., benchmark tests) covering a wide range of geotechnical applications. Additionally, the relationship between input parameters A and B and basic soil parameters shall be investigated for different soil types and calibrated from high-quality regional as well as global datasets.

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