Shear Strength of Soft Clay in Terms of Effective Stresses

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Abstract

It is commonly understood and accepted today that soft, contractant soils, when sheared under undrained conditions, develop high pore pressures that result in failure at a critical shear stress, occurring before the soil has been able to fully mobilize its effective stress strength parameters. This has led to the conclusion that in this type of clay, stability analyses, even for natural slopes, should be based on undrained, active and passive triaxial or plane strain strengths and possibly direct simple shear strength (Bjerrum, 1973; Ladd and Foott, 1974). The present study included an analysis of the results from undrained shear strength tests on high quality samples, and aimed at developing an expression for the undrained shear strength and "critical" shear stress as functions of effective stresses. The study demonstrates the importance of considering the effect of shear deformations, the contribution of friction and attraction to undrained strength, and how undrained shear strength is a unique result of the consolidation history of the clay. A procedure is proposed to express the undrained shear strengths measured in active and passive triaxial tests and in direct simple shear tests as functions of the effective stresses and a set of generally valid effective stress strength parameters. The framework also provides a new explanation of the notion of attraction (and cohesion) in soft clays). The study demonstrates how a set of effective stress paths from an active and a passive triaxial test can be used together to establish the consolidation conditions and to determine the effective stress strength parameters, friction angle and attraction. Using tests on clays with plasticity between 5 and 90%, the new framework provides a clear, simple and logical relationship, with decreasing friction angle and increasing attraction with increasing plasticity.

Introduction

In soft, contractant clays, yielding and failure takes place at a critical stress value occurring before the clay has been able to fully mobilize its effective stress strength parameters. The present study puts forward a theory describing how friction and attraction contribute to mobilize the undrained shear strength. In this connection, it was important to focus on the relationship between shear strain and mobilization of strength, and to demonstrate that undrained shear strength is a unique result of the consolidation history of the clay. The new framework was tested with several different clays from around in the world.

The paper first discusses the undrained shear strength for a soft, contractant clay, as determined from triaxial active and passive tests and from direct simple shear tests. A framework to determine the effective stress shear strength parameters from undrained shear tests on soft, contractant clay is then presented. The proposed approach is tested for a number of case records in Norway and abroad. The shear strength parameters are then correlated to plasticity index for 25 clays at 14 locations.

Determination of the undrained shear strength

The undrained shear strength, s_u, along a well-defined, but arbitrary failure plane can be expressed as:

$$s_u = {}^1\!/_2\,(\sigma'_{ULS} - \sigma'_{LLS}) \qquad (1)$$

where σ'_{ULS} and σ'_{LLS} denote the effective, compressive failure stresses acting at angles of ±45° with the failure plane. In the following, these two failure stresses will be denoted the *upper limiting stress* (ULS) and the *lower limiting stress (LLS)*. The meaning of these limiting stresses is that they represent stress-values that cannot be exceeded without yielding occurring. This study wanted to clarify how these two limiting stresses can be determined from the knowledge of the effective stress/deformation history of the soil. In this way, the undrained strength could then be expressed directly in terms of the effective stresses, without having to involve changes in total stresses and pore pressures.

The relationship between undrained shear strength and effective stresses was taken to be governed by two fundamental strength parameters, the *material friction angle*, φ'_M, and the *relative material attraction*, χ. These parameters are believed to be pure material constants and, hence, independent of stress level, stress directions and stress history.

Friction

The build-up of friction in a soil under shear loading constitutes physical work. In physics, a force is said to do work if, when acting, there is a displacement of the point of application in the direction of the force. Work transfers energy from one form to another. In a soil, a soil volume under the influence of an external force mobilizes resistance against being strained and, as strains increase, against failure. Mobilization of friction requires that a plastic (nonreversible) deformation takes place, and that the conditions of work equal to force times distance, with associated deformations, do exist, with coincidence in directions of stresses and strains.

Small elastic strains and larger plastic strains act differently in the process of mobilizing frictional shear strength.

When a clay deposit consolidates without undergoing net lateral strain (K_0-conditions), the ratio between the horizontal and vertical effective stress is equal to the coefficient of earth pressure at rest, K_0 ($K_0 = \sigma'_{h0}/\sigma'_{v0}$). Without lateral expansion, only the vertical consolidation stress contributes to building up frictional resistance capacity. Consequently, the mobilized active frictional strength (along a 45° plane) is limited to the value ${}^1\!/_2\sigma'_{vo}\cdot\sin\varphi'_M$, where M is the mobilized effective friction angle. The stress conditions for this clay represent the final stage of a yielding process and are dictated by a simple requirement of static equilibrium. Hence, the mobilized frictional resistance has to be equal to the applied shear stress:

$$s_{uA} = {}^1\!/_2(\sigma'_{v0} - \sigma'_{h0}) = {}^1\!/_2\,\sigma'_{v0}\cdot\sin\varphi'_M \qquad (2)$$

which gives:

$$K_o = \sigma'_{ho} / \sigma'_{vo} = 1 - \sin\varphi'_M \qquad (3)$$

Equation (3) represents the well-known Jaky's formula (Jaky, 1948) expressing the coefficient of earth pressure at rest for a normally consolidated soil.

In clays, mineral to mineral contacts are supposed to occur only between sand and silt particles. Clay particles are dissociated in the contact points by a thin, firmly bounded layer of pore water. This layer is thought to be of viscous nature, which means that it can transfer, to a certain degree, both compressive stresses and shear stresses. This ability is, however, strongly dependent on the applied rate of strain. In the present study, all shear strengths are, for simplicity, related to a "standard" strain rate corresponding to what is conventionally used in laboratory testing.

The ratio between ultimate friction resistance and effective normal stress at the contact points is substantially higher for the sand and silt fraction than for the clay fraction. Consequently, a plastic clay exhibiting a relatively high content of active clay particles will show a low friction angle compared to a lean (silty) clay.

Attraction

In clays, one assumes that the net attractive forces act at the contact points between clay particles across the water-film. The resulting attractive force in any direction is denoted the *material attraction*, σ_a. This attraction acts like a tension reinforcement in the clay. Compared to an attraction-free soil where the undrained shear strength is governed by friction alone, the attraction force enables the clay to withstand an additional reduction of the lower limiting stress at failure, and thereby increases the undrained shear strength. As the attractive forces already exist prior to any undrained shearing process, practically no strain is necessary to activate the attraction. In a given clay soil, the attractive forces increase with decreasing distance between clay particles. Consequently, the attraction, believed to reflect the effective stresses to which the clay is subjected, can be expressed as follows:

$$\sigma_a = \chi \cdot \sigma'_{na} \qquad (4)$$

where σ'_{na} denotes the effective stress in the direction normal to the direction of the attractive force. For undrained shear under active triaxial loading on a young, normally consolidated clay, the effective vertical consolidation stress σ'_{vo} is σ'_{na}, and the contribution to the active undrained shear strength from the attraction is $^1/_2\, \chi \cdot \sigma'_{vo}$.

As the material attraction is a function of exclusively the clay mineral fraction, its magnitude will increase with increasing content of active clay particles and thereby plasticity.

Failure in soft, contractant soils

An *in situ* clay element consolidated under a vertical stress of σ'_{vo} will undergo a uniaxial plastic deformation and mobilize an active frictional strength equal to $^1/_2 \cdot \sigma'_{vo} \cdot \sin\varphi'_M$. A successive unloading of the clay by reducing the deviator stress to zero results in elastic strains only, which does not influence the clays ability to mobilize friction. A further vertical unloading leads to plastic shear strains and a gradual mobilization of the passive shear strength until σ'_v equals σ'_{ho} $(1\text{-}\sin\varphi'_M)$. The passive shear stress and strength are then both $^1/_2 \cdot \sigma'_{ho} \cdot \sin\varphi'_M$.

These values of active and passive strength assume that no additional plastic deformation has taken place to mobilize friction due to σ'_{ho} in either active or passive shear. Such mobilization implies an important change compared with the previous stage, as the directions of shear deformations here are reversed in relation to the frictional supporting stress component. These reversed conditions in contractant soils are believed to be responsible for a structural reorganization among the clay particles, high pore pressures and decreasing shear strength with increasing strain.

If reconsolidated in the laboratory under *in situ* stresses, the young clay will have upper limiting stresses equal to σ'_{vo} and $\sigma'_{ho} = \sigma'_{vo}\,(1 - \sin\varphi'_M)$ under active and passive shear. Taking into consideration the contribution from attraction, the lower limiting stresses then become under active and passive shear $\sigma'_{vo}\,(1 - \chi - \sin\varphi'_M)$ and $\sigma'_{ho}\,(1 - \chi - \sin\varphi'_M)$, respectively.'

Active shear strength of loose sand

A loose fine sand may have properties similar to that of a contractant clay, and, in a similar manner, undergo a structural collapse at a critical strain before reaching a classic Mohr-Coulomb failure. As an example of this, Figure 1 reproduces the results from a series of undrained, active triaxial tests on the saturated, loose Ham River Sand (Bishop *et al.*, 1965; Bishop, 1966). All tests gave a friction angle close to 34°, corresponding to the maximum value of σ'_v/σ'_h reached at large strains. However, the peak points on the effective stress paths occur at small strains (about 1 %) and fall approximately on the line describing $\sigma'_h/\sigma'_v = 1 - \sin\varphi'_M$ (line added in diagram by authors). One test was run under drained conditions (initial average stress ½($\sigma'_1 + \sigma'_3$) of 990 psi) until the shear stress by far exceeded the value corresponding to the $\sigma'_h/\sigma'_v = 1 - \sin\varphi'_M$ line, and then continued as an undrained test. No additional shear resistance could be mobilized. A second test was run drained (initial average stress ½($\sigma'_1 + \sigma'_3$) of 500 psi), and the shear stress also here exceeded considerably the value corresponding to the $\sigma'_h/\sigma'_v = 1 - \sin\varphi'_M$ line.

These results support the above results for soft contractant clay, where the upper limiting active stress for a young, normally consolidated clay is σ'_{vo}, and that the active, undrained shear strength of an (attraction-free) contractant soil is limited to the contribution to friction from the vertical stress alone.

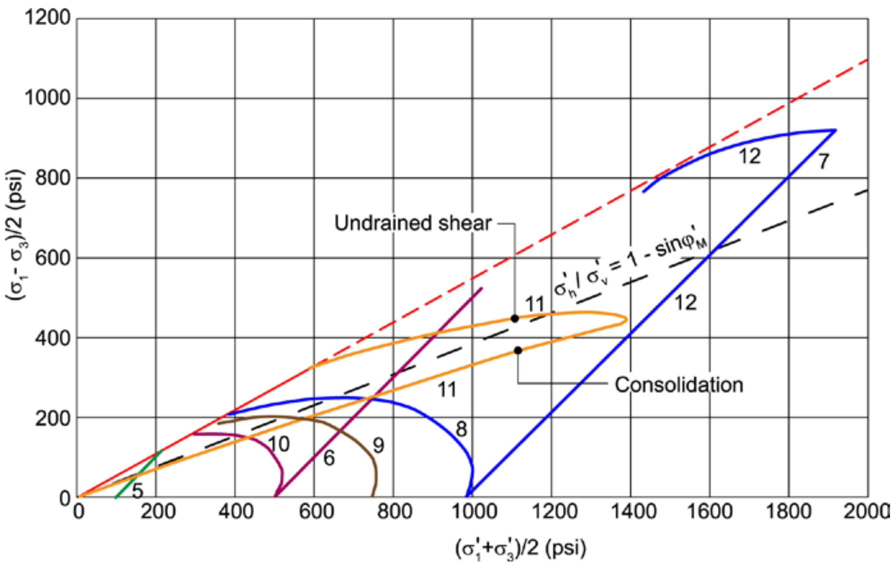

Figure 1. Effective stress paths from undrained active triaxial tests on Ham River Sand (after Bishop et al., 1965; Bishop, 1966)

Undrained shear strength of contractant clays

A perfect, young, normally consolidated clay is rarely encountered in practice. As both frictional resistance and attraction are closely related to consolidation strains, one needs to distinguish among different types of load applications and load history. The following four categories of clays were established for this study:

- Young, normally consolidated clay.
- Aged, normally consolidated clay, which has undergone secondary consolidation.
- Weathered clay, previously subjected to alternating capillary stresses due to, e.g., evaporation or freeze-thaw action.
- Overconsolidated clay, previously subjected to considerably larger loads than at present.

In the following discussion, the clay is assumed, in all cases, to have been consolidated, and if unloaded, under conditions of no lateral yield (K_0-conditions).

As a result of ageing, weathering or overconsolidation, an *in situ* clay deposit has experienced a supplementary vertical strain and behaves correspondingly as if it had been a young clay consolidated under a vertical stress σ'_{vE} higher than σ'_{v0} (Fig. 2). This vertical stress implies an increased horizontal stress of σ'_{vE} (1-sinφ'_M). The earlier strain history enables the clay to activate these "equivalent" stresses without undergoing plastic strains. Hence, the stresses represent an upper stress regime which enables one to determine the upper limiting stresses under active and passive triaxial shear and even under direct simple shear, as will be illustrated in examples later.

Because the existing vertical and horizontal effective stresses are σ'_{v0} and $K_0\sigma'_{v0}$, these stresses form in the same way the basis for the lower limiting stresses under active triaxial shear. The horizontal stress can be lowered to $\sigma'_{v0}(1 - \sin\varphi'_M)$, which means that the shear stress is balanced by the friction due to the vertical stress alone, and further reduced with the activation of the attraction. Hence, the lower limiting stress in active shear is in this case $\sigma'_{v0}(1 - \chi - \sin\varphi'_M)$, where $\chi{\cdot}\sigma'_{v0}$ reflects the contribution from attraction.

In the same way the lower limiting stress under passive shear becomes equal to $K_0\sigma'_{v0}(1 - \chi - \sin\varphi'_M)$. Coefficient K_0 is equal to $1 - \sin\varphi'_M$ for aged and overconsolidated clays. In the case of weathered clays, however, where the clay has been subjected to an alternating isotropic stress change, K_0 may differ from this value, as seen below for one of the case records.

In direct simple shear, where the principal stresses rotate 45°, the same way of reasoning as above leads to values of upper and lower limiting stress equal to $^1/_2\,(\sigma'_{vE} + \sigma'_{vE}\,(1\text{-}\sin\varphi'_M))$ and $^1/_2\,(\sigma'_{vo} + K_0\sigma'_{v0}(1 - \sin\varphi'_M - \chi))$, respectively.

Combining the above limiting stresses with Eqs. 1 and 2, the expressions for the undrained shear strength under active (s_{uA}), passive (s_{uP}) and simple shear (s_{uD}) loadings become:

$$s_{uA} = {}^1/_2\sigma'_{vo}\,[(\chi + \sin\varphi'_M) + \sigma'_{vE}/\sigma'_{vo} - 1] \tag{5}$$

$$s_{uP} = {}^1/_2\sigma'_{vo}\,[K_0\,(\chi + \sin\varphi'_M) + \sigma'_{vE}/\sigma'_{vo}\,(1 - \sin\varphi'_M) - K_0] \tag{6}$$

$$s_{uD} = {}^1/_4\sigma'_{vo}\,[(1 + K_0)\,(\chi + \sin\varphi'_M) + \sigma'_{vE}/\sigma'_{vo}\,(2 - \sin\varphi'_M) - (1 + K_0] \tag{7}$$

The expressions above state the following relationship (for a 45° rotation):

$$s_{uD} = {}^1/_2(s_{uA} + s_{uP}) \tag{8}$$

The expression for the undrained strength on any inclination β (in degrees) of the failure plane ($s_{u\,\beta}$) is then:

$$s_{u\,\beta} = s_{uA}{\cdot}\cos^2(\beta - 45°) + s_{uP}{\cdot}\sin^2(\beta - 45°) \tag{9}$$

In reality, Eqs 5, 6 and 7 are of limited practical importance: it is more convenient to determine the stress-strength parameters directly from the effective stress paths in a Mohr diagram.

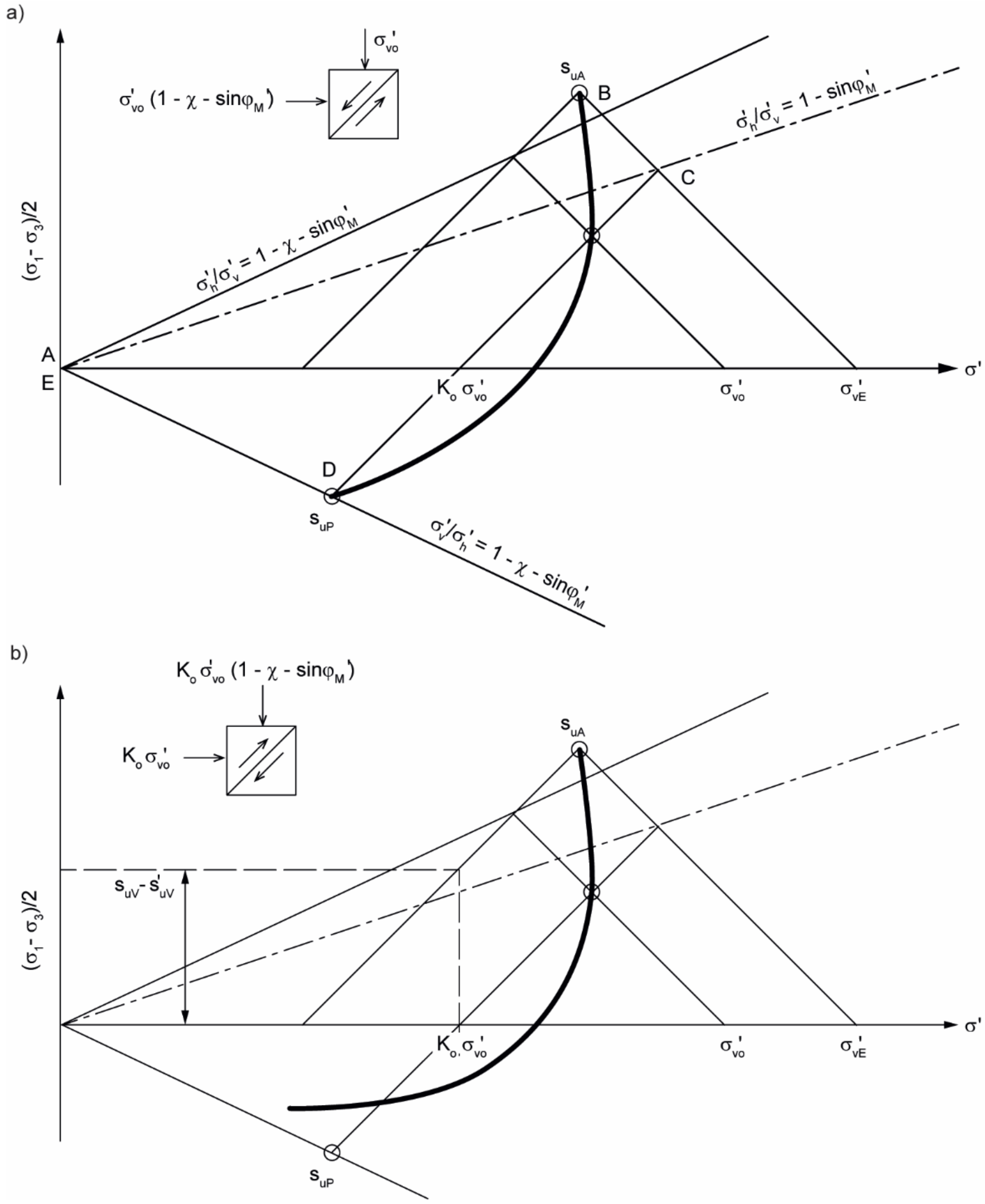

Figure 2. Determination of effective stress and strength parameters on effective stress path from triaxial tests

Framework to determine effective stress strength parameters from undrained shear tests

Figure 2 gives an example of how the effective shear strength parameters φ'_M and χ can be determined from the results of active and passive triaxial tests on an aged, normally consolidated clay. Known are the position of the active and passive shear strengths, s_{uA} and s_{uP} in the Mohr-Coulomb diagram and the effective vertical stress, σ'_{vo}.

The intersection between an upward 45° line from s_{uP} and a downward 45° line from s_{uA} gives a point on the $\sigma'_h / \sigma'_v = (1 - \sin\varphi'_M)$ line. The upward gradient n:1 of this line gives the relation: $\sin\varphi'_M = 2n/(1+n)$.

In the same manner, the intersection between an upward 45° line from σ'_{vo} and a downward 45° line from s_{uA} gives a point on the $\sigma'_h / \sigma'_v = (1 - \chi - \sin\varphi'_M)$ line. The relation between χ and the gradient of this line is $\chi = 2n/(1+n) - \sin\varphi'_M$.

The above derivation requires the knowledge of a sufficiently accurate peak value on the passive effective stress path, which in turn will be used to determines the value of K_0. The determination of the K_0-value may often be problematic in practice, especially in highly sensitive and nearly attraction-free clays. Aas *et al*. (1986) suggested that the data from passive triaxial tests can be replaced by strength data from *in situ* vane shear tests performed at the same depths where the laboratory test specimen was taken. This interpretation is based on the simplified assumption that a clay element in the cylindrical failure zone circumscribing the vane, fails in the same manner as the sample in an active triaxial test. Hence, the vertical vane strength (which in soft clays does not differ much from $s_{uv} - s_{uv}'$ (Fig. 2), is determined by assuming equal lower limiting stress in the two tests:

$$\sigma'_{vo}(1 - \chi - \sin\varphi'_M) = K_o\sigma'_{vo} - (s_{uv} - s_{uv}') \quad (10)$$

which gives:

$$K_o\sigma'_{vo} = (1 - \chi - \sin\varphi'_M)\sigma'_{vo} + (s_{uv} - s_{uv}') \quad (11)$$

Here s_{uv} and s_{uv}' denote undisturbed and remolded vane strength, respectively.

Figure 2a shows a hypothetical example of laboratory tests on a clay where the passive shear strength is sufficiently well defined, and thereby a true value of K_o can be obtained without the need for supporting vane data. There are however numerous examples of research data where vane results has been able to confirm such "constructed" values of K_o. Figure 2b gives an example of such, where a representative vane strength value is necessary to complete the determination of the K_o–value.

In the case of weathered clays. The coefficient of earth pressure at rest, K_o, cannot be determined unless with the help of vane data. This will be examined in more detail in the Calabar case record below.

The lines forming the quadrilateral ABCDE in Figure 2a define the limiting stresses in active and passive shear These have been formerly presented as yield envelopes (e.g. Larsson and Sällfors, 1981). They represent the ultimate value before yield, when one stress is kept constant, while the other one is either reduced or increased. In practice, full mobilization of active and passive shear strength requires in principle that a stress reduction takes place in one direction simultaneously with a stress increase in the other, resulting in points in the diagram located outside the yield envelopes.

The graphic method for the determinations of effective stress strength parameters using these upper and lower limit stresses is illustrated further with a presentation of actual test data on four different clays. It should be noted that the notation of the limiting stresses has been simplified in the forthcoming figures to ULSA, ULSP, ULSD for the upper limiting stresses in active, passive and direct simple shear and to LLSA, LLSP, LLSD for the lower limiting stresses in active, passive and direct simple shear.

Case records

New Jersey Clay (Koutsoftas and Ladd, 1983)

Figure 3 shows the normalized shear stress vs shear strain curve and effective stress paths from plane strain active and passive and direct simple shear tests on undisturbed samples of New Jersey Clay, a plastic clay with a plasticity index I_p of 43 %. The samples were consolidated anisotropically for a vertical stress about twice the *in situ* consolidation stress, and the clay thus consolidated is a "young" clay, not affected by ageing.

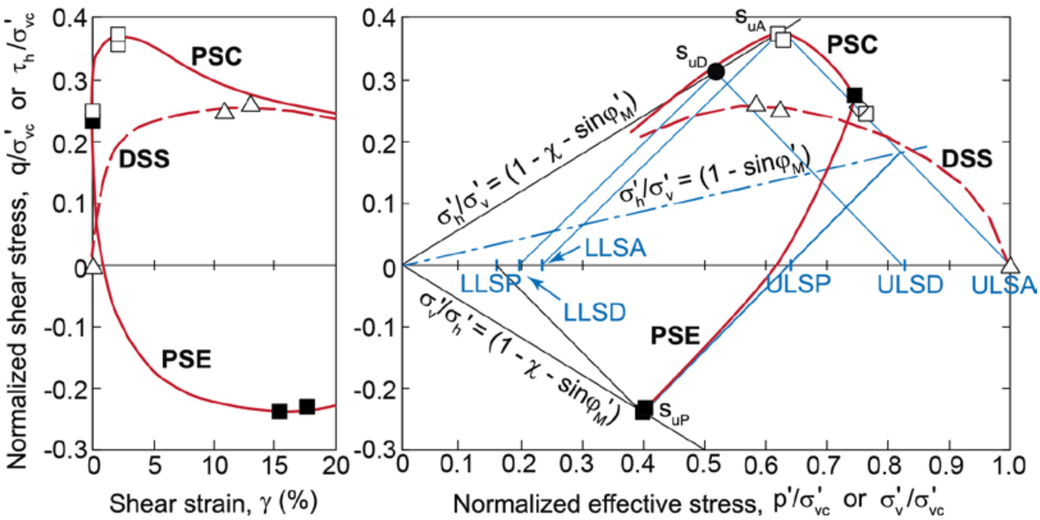

Figure 3. CK_0U plane strain (PS) and direct simple shear (DSS) test data on normally consolidated New Jersey Clay: left: Normalized shear stress vs shear strain; right: Normalized effective stress paths (Koutsoftas and Ladd, 1983) (PSC for active and PSE for passive plane strain tests)

This case, as well as a few other cases in this paper, use plain strain test data instead of triaxial data. Ladd and Edgers (1972) and Ladd *et al*. (1977) published comparisons of the two tests. They suggested higher undrained shear strength in plane strain compression (active) than in triaxial compression (active) by about 10%, and similarly higher by about 5% in extension (passive). In the present paper, the results of the two test types were considered to be equivalent.

The stress stress-shear strain data to the left in Figure 3 show a typical picture for soft, contractant clays. In the active mode of failure, the peak shear strength occurs at a low failure strain, followed by a substantial strength loss and strain-softening in active shear. In contrast, passive shear and direct simple shear, both involving a rotation of the principal stress directions, by 90 and 45° respectively, result in a much lower peak resistance at a significantly higher failure strain, and little strain-softening.

To the right in Figure 3, the effective stress paths are shown. The peak shear stresses on the active and passive stress paths express the active and passive undrained shear strengths. The intersection of the 45° lines starting at the peak shear stress-values with the zero shear stress axis determines upper and lower limiting stresses. The intersections of the 45° lines with the $\sigma'_h / \sigma'_v = 1 - \sin\varphi'_M$ lines are also shown.

Since $\sigma'_{ULSA(=\sigma'vo)}$ equals σ'_{vo}, (see Fig. 2) for this young clay, the peak shear stress on the active stress path represents a point on the "χ-line" ($\sigma'_h/\sigma'_v = 1 - \chi - \sin\varphi'_M$). The data yield the following parameters: $\sin\varphi'_M = 0.37$, $\chi = 0.39$, $K_0 = 0.63$.

The direct simple shear test gives a peak strength significantly lower than the theoretical value s_{uD}, defined as the crossing point between the "χ-line" and a 45° line starting on the abscissa midway between σ'_{ULSP} and σ'_{ULSA}. This divergence was generally observed in the direct simple shear test. Possible explanations include the difficulty in achieving full contact between the clay and the enclosing rubber membrane (so non-K_0 conditions), the non-uniformity of stresses in the simple shear test specimen, or the rotation of the principal stresses not being exactly 45°.

Ellingsrud Clay, Oslo (Lacasse *et al*., 1985)

Ellingsrud Clay is a quick clay exhibiting very low plasticity (I_p = 4 %) and vane shear strength (s_{uv}/σ'_{vo} = 0.07 – 0.10). Figure 4 shows the effective stress paths from triaxial active, triaxial passive and direct simple shear tests on a block sample from 8.1 m depth. In addition, the value of the vane strength difference ($s_{uv} - s_{uv}'$) at the same depth is shown

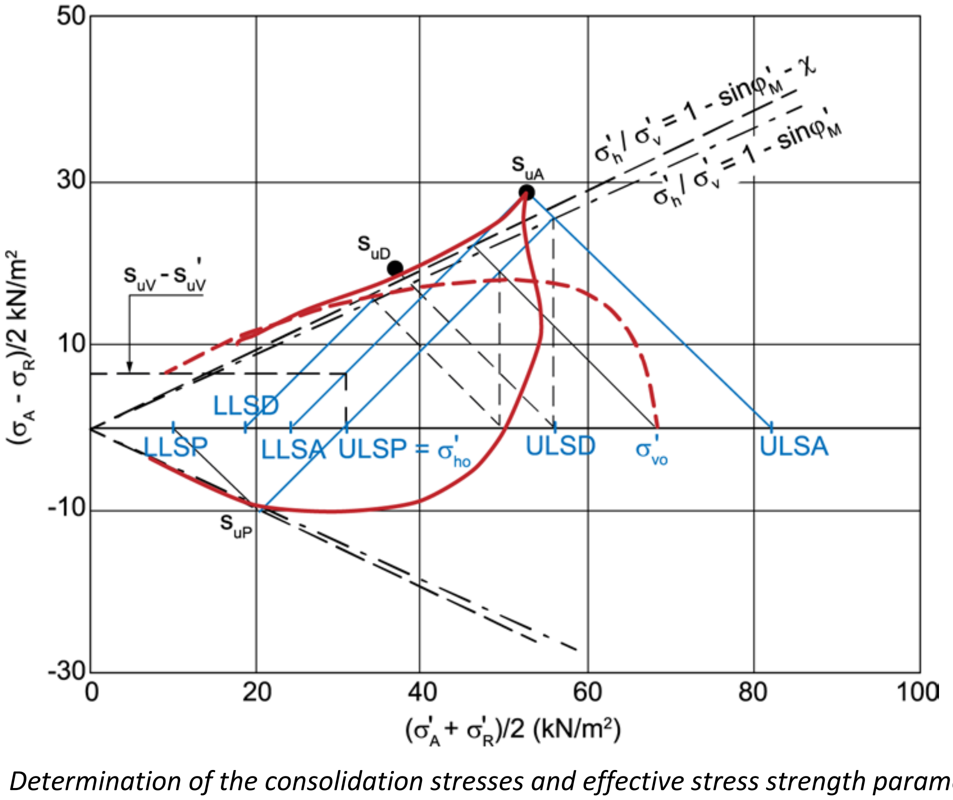

Figure 4. Determination of the consolidation stresses and effective stress strength parameters for the aged Ellingsrud quick clay at depth 8.1 m (Lacasse et al. 1985)

As the stress paths from the passive triaxial and direct simple shear tests show poorly defined maxima for this clay, the value of $\sin\varphi'_M$ had to be determined by the help of the vane test data (Aas and Lacasse, 2021). As illustrated in Figure 4, the intersection of the horizontal dotted line representing $s_{uv} - s'_{uv}$, and the 45° line starting at s_{uA} defines the *in situ* horizontal consolidation stress, σ'_{ho}, and is equal to $(1 - \sin\varphi'_M)\cdot\sigma'_{vE}$. Points on the lines describing the ratio $\sigma'_{ho}/\sigma'_{vo} = 1 - \sin\varphi'_M$ and the "χ-line", respectively, are determined as described in Figure 2.

The tests indicate that for this aged clay the following strength parameters are the most representative: $\chi = 0.02$ and $\sin\varphi'_M = 0.62$. In addition, the tests give a value of 0.46 for the coefficient of earth pressure at rest, K_0. For the sample from a depth of 13.1 m, similar parameters were determined: $\chi = 0$, $\sin\varphi'_M = 0.62$ and $K_0 = 0.49$. It should be mentioned that these K_0-values correspond well with values measured by the self- boring pressuremeter tests at this site (Lacasse and Lunne, 1982) and with the newest statistically based correlations developed by L'Heureux *et al*., 2017.

For this very soft clay, there is a fairly good agreement between the laboratory-inferred values of LLSD values from the laboratory-measured s_{uD} and the theoretical value halfway between LLSA and LLSP on the zero shear stress line.

Calabar Clay, Kenya (NGI, 1976)

Figure 5 presents the results from undrained triaxial active and passive tests from 9.2 m depth on a soft highly plastic clay (I_p = 88%). In addition, the measured field vane shear strength at the same depth is shown. Calabar clay is a weathered clay, where two distinct sets of consolidation stresses were used: the higher ones, σ'_{vE} and $\sigma'_{vE}(1 - \sin\varphi'_M)$, represent the upper limiting stresses, while the *in situ* stresses σ'_{vo} and $K_0\sigma'_{vo}$ represent the lower limiting stresses.

For this weathered clay, $K_0\sigma'_{vo}$ does not coincide with $\sigma'_{vE}\cdot(1 - \sin\varphi'_M)$, which means that the passive undrained shear strength s_{uP} has to be constructed with the help of a lower value of effective horizontal stress and using the shear vane strength, as described earlier. This highly plastic clay shows, as expected, a very high value for attraction ($\chi = 0.50$) and a correspondingly low friction angle ($\sin\varphi'_M = 0.28$). For this highly plastic clay, the field vane undrained shear strength is close to the undrained shear strength from triaxial active tests.

Haga Clay, Akershus (Christoffersen and Lacasse, 1985)

The Haga clay is a leached marine clay with salt content of 1 g/l, a plasticity index of 11 to 14% and a sensitivity of 4 to 5. The clay is highly overconsolidated at shallow depths, and the derived overconsolidation ratio OCR is about 10 at a 2 m depth.

An overconsolidated clay has undergone in its past history a loading and unloading under conditions of no lateral yield. This has led to a state of static equilibrium implying a ratio between the vertical and horizontal effective stress equal to that for the corresponding young clay, if the deformations are the same. Hence, the overconsolidated clay will behave like a young, normally consolidated clay, however taking into consideration the higher level on the upper limiting stresses.

Figure 6 presents the effective stress paths from consolidated undrained triaxial active and passive tests on a 95 mm tube sample from 2.2 m depth. Using the procedure described earlier, the Haga tests indicate the following soil parameters: $\sin\varphi'_M = 0.50$, $\chi = 0.21$, $K_0 = 2.00$.

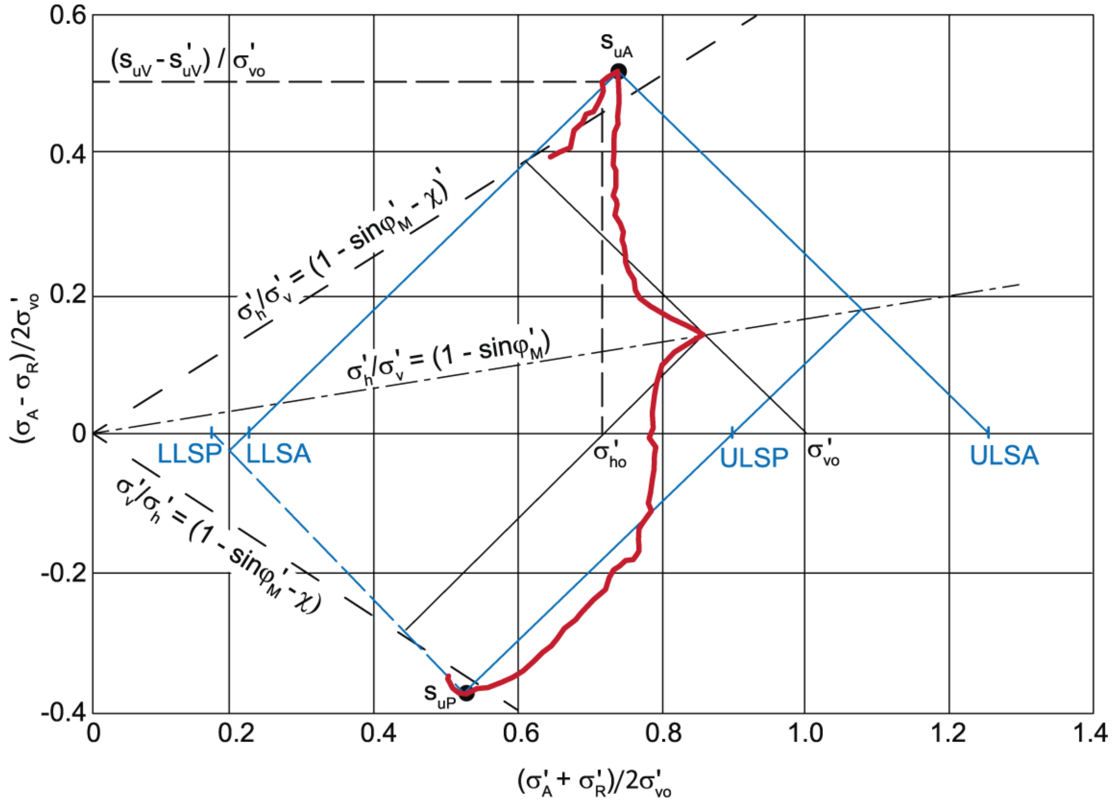

Figure 5. Determination of the consolidation stresses and effective stress parameters for soft, highly plastic Calabar clay at 9.2 m depth

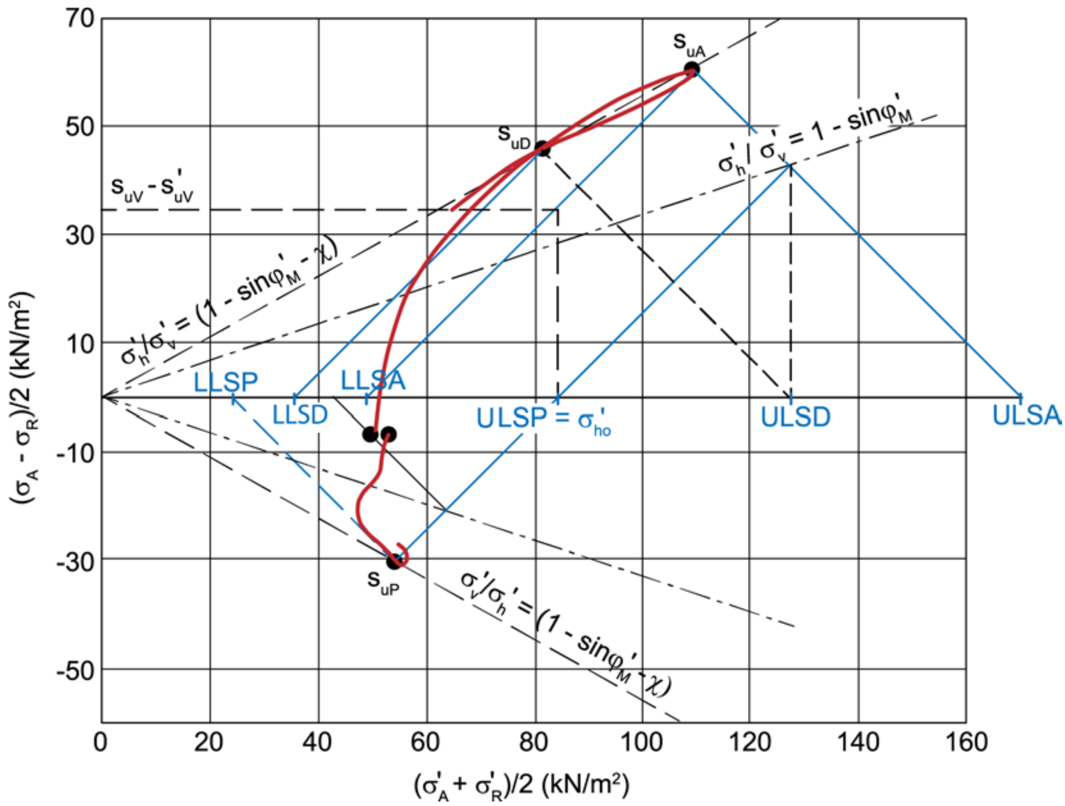

Figure 6. Determination of the consolidation stresses and effective stress strength parameters for the highly overconsolidated Haga clay at 2.2 m depth

The same value of K_0 is obtained if one combines the results of vane and triaxial data. It should also be mentioned that this K_0 value corresponds well with values recorded in self-boring pressuremeter tests at this location (Lacasse and Lunne, 1982).

Results from a direct simple shear test, K_0-consolidated in the conventional manner to the *in situ* effective vertical stress, showed an undrained shear strength reaching only about 50% of the theoretical value of the simple shear strength, s_{uD}.

A series of three additional direct simple shear tests were run on specimens from depth 2.2 m. Each specimen was first consolidated under a vertical stress substantially higher than the *in situ* value σ'_{vo}, and then unloaded to a value equal to σ'_{vo} (similar to the original SHANSEP approach (Ladd *et al.*, 1977).

The results from the three additional tests showed quite remarkably that the effect of a preloading above the preconsolidation stress and unloading in the laboratory is considerable. In the test where the OCR was 11, both the magnitude of the measured shear strength and the position of the peak shear stress on the stress path corresponded well with the theoretical values shown in Figure 6.

Such preloading procedure is believed to be necessary in a direct simple shear test to ensure a full contact between the clay specimen and the enclosing rubber membrane to ensure zero lateral strain and to reestablish the correct *in situ* stress conditions in weathered and overconsolidated clays.

Relationship between strength parameters and plasticity

Is it reasonable to assume that in a clay the material friction decreases and the attraction increases with increasing content of active clay minerals, i.e., with increasing plasticity index. However, some reservations need to be made because certain variations in mineral composition and chemistry in the clay may not have exactly the same effect on attraction and plasticity.

Mobilized friction angle and attraction

Figure 7 presents the values of $\sin\varphi'_M$ and the attraction ratio χ determined with the above procedures versus the plasticity index, I_p, for 25 sets of triaxial active and passive effective stress paths. Table 1 lists each of the clays the data in Figure 7 are based on. There is some scatter. However, even with scatter, there is a definite relationship correlation with plasticity index: $\sin\varphi'_M$ decreases from a value a little above 0.6 ($\varphi' = 37°$) in a very low plastic clay to about 0.3 ($\varphi' = 17.5°$) in a highly plastic clay. Correspondingly, the attraction χ varies from zero to about 0.5 as I_p increases from zero to about 90%. The reason for the very low attraction values for a few clays in Figure 7 is not understood at the present time.

There exist very little published data on the relationship between $\sin\varphi'_M$ and I_p. Kenney (1959) showed a decreasing friction angle with increasing plasticity. The data from Table 1 agree with Kenney's trend for low plasticity index ($I_p < 20\%$). For higher plasticity indices, the Kenney $\sin\varphi'$-values gradually diverge with the friction angle values deduced from the effective stress paths from triaxial tests. The difference is believed to be due to the following: even though the triaxial clay sample has lost some of its undrained shear strength when maximum obliquity is reached (when σ'_v/σ'_h is maximum, which forms the basis for determining the friction angle), there still exists attractive forces between the clay particles and there is attraction in the clay. In a standard undrained triaxial test, a residual attraction will act to give an apparently friction angle that is too high, with the discrepancy increasing with higher plasticity and attraction.

Undrained shear strength

A closer look at the two diagrams in Figure 7 reveals that the sum of χ + $\sin\varphi'_M$ varies little with increasing plasticity. The sum $\sin\phi'_M + \chi$ for the clays in Table 1 has a mean value of 0.76. This means that a normally consolidated clay ($\sigma'_{vE}/\sigma'_{vo} = 1$) according to Eq. 4 would have an active, undrained shear strength equal to about $0.38\sigma'_{vo}$, independently of I_p.

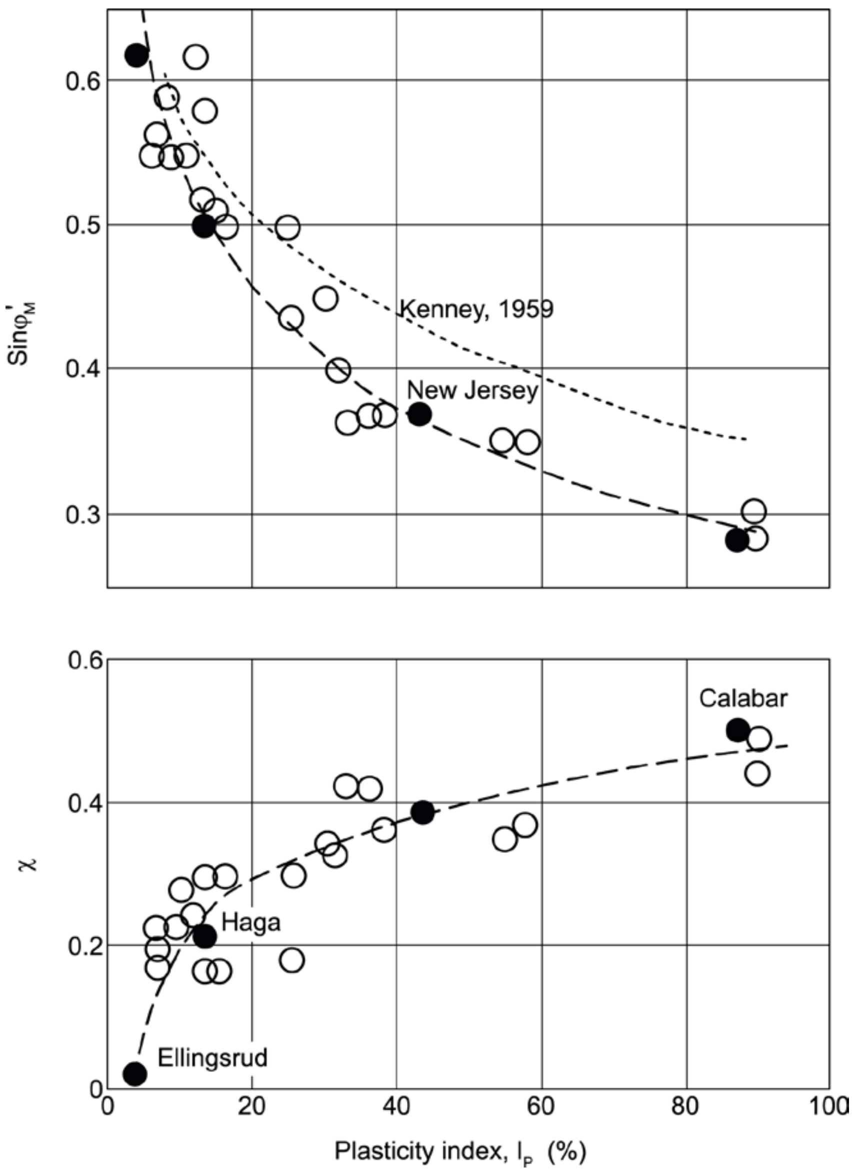

Figure 7. Relationship between friction angel and attraction parameters and plasticity

The same clay would theoretically (Eqs 5 and 6) show the following values in passive and direct simple shear: $(1 - \sin\varphi'_M)\ s_{uA}/\sigma'_{vo}$ and $(1 - {}^1/_2{\cdot}\sin\varphi'_M)\ s_{uA}/\sigma'_{vo}$, respectively. Combining the two expressions with the deduced ratio between I_p and $\sin\varphi'_M$ in Figure 7, one obtains the results in Table 2 for normally consolidated clay.

Table 1. Values of sinφ'$_M$ and χ vs plasticity index from triaxial active and passive tests for 25 clays

Site	Depth (m)	I_p (%)	$\sin\phi'_M$	χ	$\sin\phi'_M + \chi$	Clay category
Ellingsrud, Norway	8.1	4	0.62	0.02	0.64	Aged
Emmerstad, Norway	5.9	7	0.55	0.23	0.78	OC
Emmerstad, Norway	7.8	7	0.55	0.20	0.75	OC
Olga, Canada	3.9	36	0.37	0.42	0.79	Weathered
Olga, Canada	5.0	33	0.36	0.42	0.78	Weathered
Olga, Canada	6.9	25	0.44	0.30	0.77	Weathered
Olga, Canada	7.1	30	0.45	0.35	0.80	Weathered
Rupert, Canada	5.3	14	0.58	0.30	0.88	OC
Rupert, Canada	9.3	7	0.56	0.19	0.77	OC
Haga, Norway	2.2	13	0.50	0.21	0.71	OC
Haga, Norway	5.3	38	0.37	0.36	0.73	OC
Arendal, Norway	7.5	10	0.55	0.28	0.83	OC
Arendal, Norway	15.3	16	0.50	0.30	0.80	OC
Åndalsnes, Norway	7.2	25	0.50	0.19	0.69	OC
Åndalsnes, Norway	21.2	13	0.52	0.16	0.68	OC
Bangkok, Thailand	5.3	90	0.30	0.49	0.79	Weathered
Bangkok. Thailand	8.5	90	0.28	0.43	0.71	Weathered
Drammen, silty, Norway	10.2	15	0.50	0.16	0.66	Aged
Lille Melløsa, Sweden	9.0	58	0.35	0.37	0.72	OC
Singapore	18.5	55	0.35	0.35	0.70	NC, young
Gøteborg, Sweden	14.0	31	0.40	0.33	0.73	NC, young
Calabar, Kenya	9.2	88	0.28	0.50	0.78	Weathered
New Jersey, USA	?	43	0.37	0.39	0.76	NC, young
Québec, B-6, Canada	12.0	9	0.59	0.22	0.81	OC
Québec, B-2, Canada	7.2	12	0.62	0.24	0.86	OC

Table 2. Effective strength parameters for I_p of 10 and 90%

I_p (%)	$\sin\varphi'_M$	s_{uA}/σ'_{vo}	s_{uP}/σ'_{vo}	s_{uD}/σ'_{vo}
10	0.55	0.38	0.17	0.27
90	0.28	0.38	0.27	0.33

Consequently, the strength anisotropy of a clay decreases with increasing plasticity as also demonstrated by for instance Ladd *et al*. (1977) (Fig. 8) and Jamiolkowski *et al*. (1985). The explanation for this is apparently that strength anisotropy is associated exclusively with the friction component of the shear strength, and of minor importance where attraction is responsible for the major part of the shear strength.

Generally, one has to run both active and passive triaxial tests as basis for a stability analysis to model the parts of the failure surface under active and passive conditions. However, a compilation of 46 different cases, including the 25 cases in Table 1, indicated the following approximate relationship (Fig. 9):

$$\tfrac{1}{2}\,(s_{uA} + s_{uP}) = \tfrac{3}{4}\,s_{uA} \tag{12}$$

and therefore

$$s_{uP}/s_{uA} = \tfrac{1}{2} \tag{13}$$

Figure 9 indicates that for the softest clays, this ratio might be lower:

$$^{1}/_{2}\,(s_{uA} + s_{uP}) = {}^{2}/_{3}\,s_{uA} \qquad (14)$$

and therefore

$$s_{uP}/s_{uA} = {}^{1}/_{3} \qquad (15)$$

One can question, however, whether sample disturbance could have affected the lower values, especially in silty and sensitive clays with low plasticity.

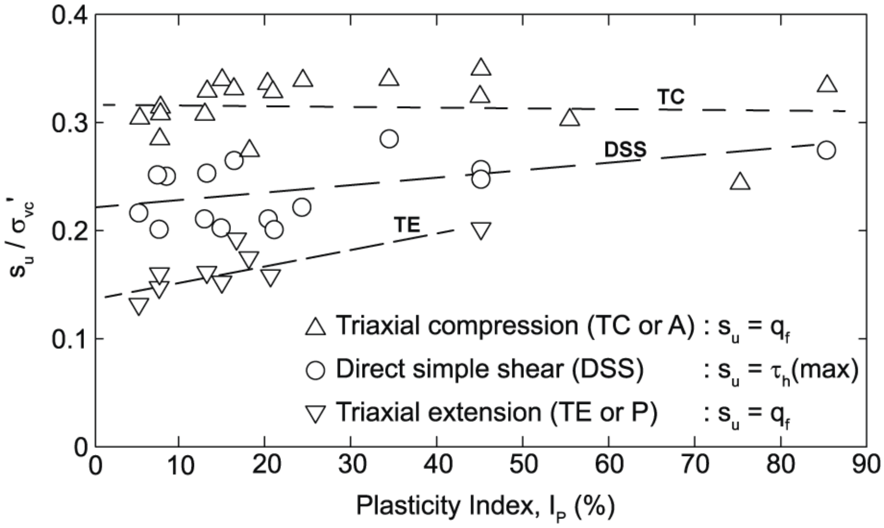

Figure 8. Anisotropy of undrained shear strength as function of plasticity (Ladd et al., 1977; Jamiolkowski et al., 1985)

Since the direct simple shear strength theoretically lies midways between the active and passive strength (Equation 7), s_{uD} can be calculated approximately on the bases of active strength and the expressions above.

Figure 9 presents the relationship between the active undrained shear strength, s_{uA}, and the s_{uLAB} for overconsolidation ratios between 1 and 3, where s_{uLAB} is the average of s_{uA} and s_{uP} (Eq. 12).

Conclusions

The study showed that the undrained shear strength of soft, contractant clays can be expressed as a function of two effective stress strength parameters, the *material friction* and the *relative material attraction*, χ. The two effective stress strength parameters are pure material constants and thus independent of stress level, stress directions and stress history. The material friction decreases and the relative attraction increases with increasing clay plasticity. These basic strength parameters can be determined from sets of effective stress path tests from active and passive triaxial tests and direct simple shear tests, and possibly, in addition, vane strength.

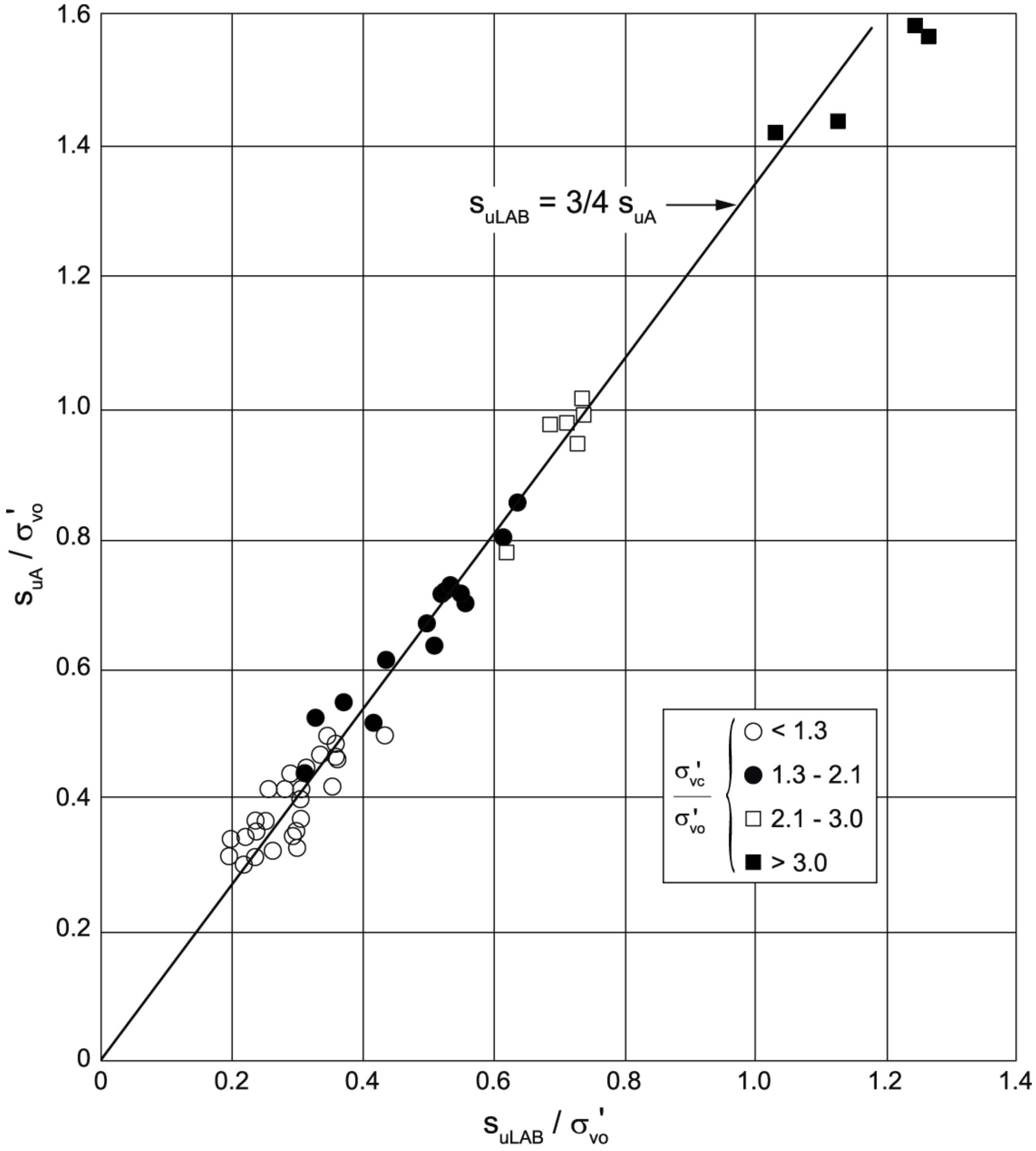

Figure 9. Relationship between normalized shear strength $s_{uLAB} = {}^{1}/_{2}\,(s_{uA}+s_{uP})$ and s_{uA} for overconsolidation ratios between 1.0 and 3.0

The results show that the mobilization of the material friction and relative attraction under undrained shear is closely related to the consolidation stresses and strains, and to the nature of the deformations imposed on the clay before failure. The mobilization of the material friction constitutes physical work, resulting in a special relationship for the interaction between charging force and resulting plastic deformation (where work is force times distance). On the other hand, only a small elastic strain is necessary to activate attraction, which is also a function of consolidation stresses. The attraction acts like a tension reinforcement in the clay.

In a soft, contractant clay, the yielding conditions imply that only friction due to the vertical stress contributes to the undrained, active shear, and only friction due to the horizontal stress contributes to the undrained, passive strength. Any further loading to mobilize friction from either the horizontal stress in active shear or the vertical stress in passive shear, leads to structural collapse and failure.

The study included several different clay types from around in the world. The data, embracing from silty Norwegian clay to highly plastic Nigerian clay, support that the proposed framework applies in general to soft, contractant clays and can provide a realistic measure of stress-strain-strength behavior of soft clays under undrained shear.

The consequence of these findings is that a conventional Mohr-Coulomb failure criterion is, in reality, not valid for contractant soils such as soft, sensitive clay and loose, narrow graded sand/silt. In these soils, failure takes place as a result of a structural collapse before the full mobilization of friction, in accordance with the presuppositions for this failure criterion. Therefore, in lieu of a conventional effective stress analysis, one needs to use, for stability calculations of natural slopes, an "ADP-type" of analysis based on representative values of the undrained strengths from active (A) and passive (P) triaxial (or plane strain) tests and direct simple shear (D) tests.

The results of the above study show that if one takes into consideration the relationships between deformations and mobilization of friction and attraction, each of the different undrained shear strengths can be explained with the effective stress strength concept, based on the actual values of the effective stress strength parameters.

References

Aas, G. and Lacasse, S. (2021). Vane Shear Strength in Terms of Effective Stress. 3rd paper in this NGI publication.

Aas, G., Lacasse, S., Lunne, T. and Høeg, K. (1986). Use of in Situ Tests for Foundation Design on Clay. ASCE. Conf. in Situ ’86 Blacksburg, Virginia, USA, 1986. 1−30.

Bishop, A.W., Webb, D.L. and Skinner, A.E. (1965). Triaxial tests on soil at elevated cell pressures. International Conference on Soil Mechanics and Foundation Engineering, 6. Montréal 1965. Proceedings. **1**: 170−174.

Bishop, A.W. (1966). The strength of soils as engineering materials. 6th Rankine Lecture. *Geotechnique*. **16**(2): 91−128.

Bjerrum, L. (1973). Problems of soil mechanics and construction on soft clays. State-of-the-art report to session IV. International Conference on Soil Mechanics and Foundation Engineering, 8. Moscow 1973. Proceedings. **3**: 111−159.

Christoffersen, H.P. and Lacasse, S. (1985). Offshore site investigation techniques. Laboratory tests on Haga clay. Norwegian Geotechnical Institute, Oslo. Internal Report 40019-8.

Jaky, J. (1948). On the bearing capacity of piles. International Conference on Soil Mechanics and Foundation Engineering, 2. Rotterdam 1948. Proceedings. **1**: 100−103.

Jamiolkowski, M., Ladd, C.C, Germaine, J.T. and Lancellotta, R. (1985). New developments in field and laboratory testing of soils. State-of-the-art Paper. International Conference on Soil Mechanics and Foundation Engineering, 11. San Francisco 1985. Proceedings. **1**: 57−153.

Kenney, T.C. (1959). Discussion of "Geotechnical properties of glacial lake clays" by T.H. Wu. ASCE. *Soil Mech. and Foundations Div.*, Paper 1732. **85** (SM3): 67−79.

Koutsoftas, D.C. and Ladd, C.C. (1985). Design strengths of an offshore clay. J. *Geotech. Eng.*, ASCE, **111**(3): 337−355.

L'Heureux, J.S., Ozkul, Z., Lacasse, S., D'Ignazio, M. and Lunne, T. (2017). A revised look at the coefficient of earth pressure at rest for Norwegian Clays. Fjellsprengningsteknikk - bergmekanikk - geoteknikk. Oslo 2017. Chapter 35.

Lacasse, S. and Lunne, T. (1982). *In situ* horizontal stress from pressuremeter tests. Symp.on the Pressuremeter and its Marine Applications. Paris, Editions Technip. 187–208.

Lacasse, S., Berre, T. and Lefebvre, G. (1985). Block sampling of sensitive clays. International Conference on Soil Mechanics and Foundation Engineering, 11. San Francisco 1985. Proceedings. **2**: 887−892.

Ladd, C.C. and Edgers, L. (1972). Consolidated-Undrained Direct-Simple Shear Tests on Saturated Clays. Research Report R72–82, Dept. of Civil Engineering, MIT, Cambridge, MA. USA.

Ladd, C.C. and Foott. R. (1974). New design procedure for stability of soft clays. *J. Geotech. Eng.* ASCE. **100**(GT7): 763−786.

Ladd, C.C, Foott, R. Ishihara, K., Schlosser, F. and Poulos, H.G. (1977). Stress-deformation and strength characteristics. State-of-the-art Paper. International Conference on Soil Mechanics and Foundation Engineering, 9. Tokyo 1977. Proceedings. **2**: 421−494.

Larsson R. and Sällfors, G. (1981). Hypothetical Yield Envelope at Stress Rotation. International Conference on Soil Mechanics and Foundation Engineering, 12. Stockholm. Proceedings. Proc. **4**: 693−696.

NGI (1976). Laboratory investigations on a series of clay samples from the swampy areas. Norwegian Geotechnical Institute, Internal Report 76010-01. /G. Aas) 1976-0622. 35pp.