K_0 as a Function of Changes in Stress and Strain Conditions during Consolidation, Unloading and Reloading

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Abstract

There are several examples in the literature that describe empirical correlations for the coefficient of earth pressure at rest, K_0, and undrained shear strength, s_u, in normally consolidated and overconsolidated clays as a function of index properties like plasticity index, overconsolidation ratio, OCR, and friction angle. However, a physical understanding of the reason why the coefficient of earth pressure at rest varies as a function of these parameters is still lacking, which causes an uncertainty in the value of K_0 to use in geotechnical analyses and in design. This paper investigates the relationship between deformation and mobilization of friction. At the same time, it considers that the unloading process has to be looked at as a stability problem where a static, stabile equilibrium needs to be maintained, thus requiring that the mobilized shear strength equals the applied shear stress. The resulting values of coefficient of earth pressure at rest, K_0, interpreted from effective stress paths alone, give realistic results, also in agreement with recent statistical evaluations on the K_0-value in Norwegian clays. The new interpretation contributes to explain the widely used empirical correlations presented in the literature.

Introduction

It is commonly known that when a clay deposit becomes overconsolidated as a result of vertical unloading, the ratio of the horizontal to vertical effective stress increases with the overconsolidation ratio. This paper evaluates how these stress conditions are influenced by isotropic and shear stress changes during consolidation. One particular objective was to look at the difference in the behaviour of elastic and plastic strains and how they contribute to mobilize and reduce the mobilized friction resistance.

These relationships will be visualized by following a clay element in the ground from deposition through consolidation and subsequent stress relief. In addition, an attempt is made to explain what happens with the earth pressure if subsequently reloading the overconsolidated clay vertically. For simplicity, the ground surface is assumed to be horizontal, implying a state of perfect confined compression and expansion.

Model of a clay element under consolidation and drained shear

Theoretical considerations based on the study of the stress paths under uniaxial loading and unloading of a clay were used to determine the value of the coefficient of earth pressure at rest, K_0, for normally consolidated and overconsolidated clay. The analysis studied the changes in effective stresses and strains as a result of a uniaxial unloading and reloading of a clay deposit. The analysis is based on the following two basic assumptions:

Hypothesis 1. In any phase of the loading-unloading cycle, there must exist a static equilibrium, which means that shear stresses and mobilized shear strengths have to be equal.

Hypothesis 2. The mobilized frictional shear resistance is an exclusive result of the plastic deformation that has occurred. Relief of a shear stress then results in a pure elastic deformation that does not influence the mobilized shear strength.

Primary consolidation

When a clay deposit consolidates without undergoing net lateral strain, the ratio between horizontal and vertical effective stress will become equal to the coefficient of earth pressure at rest, K_0. The stress conditions for this clay represent the final stage of a yielding process, and are dictated from a simple requirement of static equilibrium mentioned in Hypothesis 1 above. This means that the shear resistance of the clay has to be fully mobilized under the actual stress-strain conditions, and thus that it is possible to calculate the value of K_0 if a correct shear strength concept is applied.

Hence, it is necessary to determine the magnitude of the shear strength mobilized on a 45° shear plane in a young consolidated soil. It is important to emphasize that the mobilization of friction is physical work which requires that the conditions of force times distance do exist. As no lateral expansion takes place, the horizontal force does not contribute to the mobilization of friction. Thus, the vertical consolidation stress alone will contribute to the build-up of a frictional resistance equal to $½\cdot\sigma'_{vo}\cdot\sin\varphi'_M$. Hence, an applied shear stress has to be equal to the mobilized frictional resistance:

$$½\cdot(\sigma'_{vo} - \sigma'_{ho}) = ½\cdot\sigma'_{vo}\cdot sin\varphi'_M \qquad (1)$$

which gives for a normally consolidated clay:

$$\sigma'_{ho}/\sigma'_{vo} = K_o = 1 - sin\varphi'_M \qquad (2)$$

Equation (2) is the well-known Jaky formula (Jaky, 1948) for the coefficient of earth pressure at rest for a normally consolidated soil. Here φ'_M denotes the *material friction angle* of the clay, indicating a pure material parameter, which is assumed to be independent of stress level, stress direction and stress history.

An indispensable condition for confined compression is that the clay be subjected to plastic stress increments where $\Delta\sigma'_h/\Delta\sigma'_v$ = (1 – sinφ'_M). This requires that the clay undergoes at the same time an increased shear stress equal to $½\cdot\Delta\sigma'_v\cdot\sin\varphi'_M$. The top of the Mohr circle moves along the line from A to B on Figure 1 (the Mohr circle is replaced by an inscribed triangle at peak B). The line AB describes Eq. (2) or Jaky's formula.

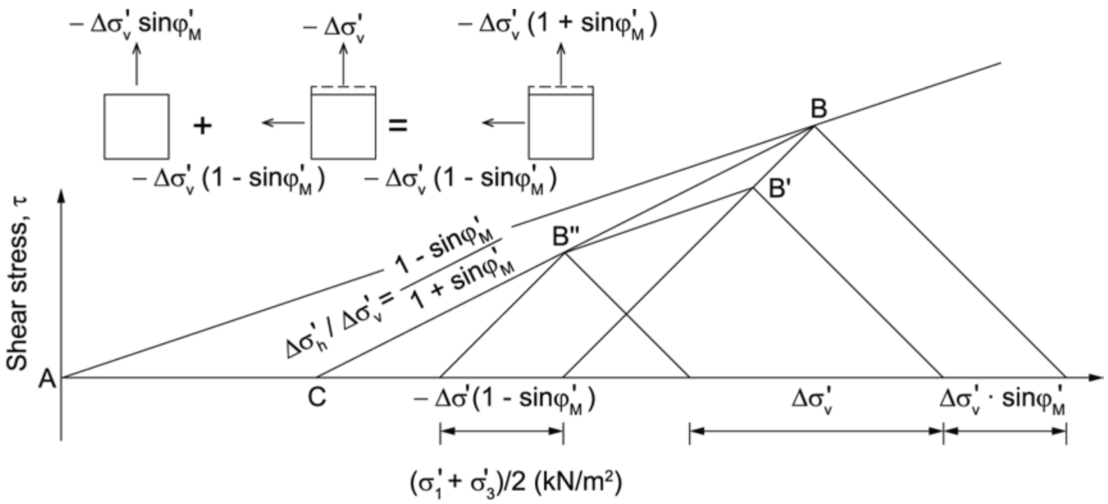

Figure 1. Changes in effective stresses and strains for a clay element during uniaxial loading under K_0-conditions and unloading resulting in an increased in K_0 from $(1 – \sin\varphi'_M)$ to 1

Unloading when $\sigma'_{ho}/\sigma'_{vo} < 1$

A reversed shear stress causes a reversed elastic shear strain, which, as opposed to a plastic strain, does not influence any change in mobilized friction. If the soil element in Figure 1 is unloaded by a reduction of the vertical effective stress equal to $\Delta\sigma'_v$, this implies a subsequent elastic shear stress reduction equal to $½\cdot\Delta\sigma'_v\cdot\sin\varphi'_M$. As illustrated in Figure 1, the unloading process is started by first removing the deviator stress $\Delta\sigma'_v \sin\varphi'_M$ (the top of the Mohr circle moves from point B to B'). For this unloading, the lateral strain remains zero. With further unloading, the stress conditions keep the same ratio between vertical and horizontal stress as for the original loading (from point B' to point B"), leading to pure confined vertical expansion.

The resulting stress change follows by the stress path indicated by B to C in Figure 1, and is described by the function:

$$\Delta\sigma'_h / \Delta\sigma_v = (1 – \sin\varphi'_M)/(1 + \sin\varphi'_M) \quad (3)$$

By introducing the parameters preconsolidation stress, σ'_{vc}, and overconsolidation ratio, OCR expressed as $\sigma'_{vc}/\sigma'_{vo}$, the following equation describes the value of K_0 as a function of OCR:

$$K_0 = (1 + OCR\cdot\sin\varphi'_M)(1 – \sin\varphi'_M)/(1 + \sin\varphi'_M) \quad (4)$$

The value of OCR corresponding to a K_0 of unity (point C in Fig. 1) is:

$$OCR_{K0=1} = 2/(1 – \sin\varphi'_M) \quad (5)$$

As the change in shear stress by unloading along the B to B' line does not influence the amount of mobilized friction, the change in shear strength becomes $½\cdot\Delta\sigma'\cdot\sin\varphi'_M$ and is equal to the change in shear stress.

Unloading when 1< $\sigma'_{ho}/\sigma'_{vo}$ < 1/(1 – sinφ'_M)

With an unloading to stresses corresponding to point C (the isotropic line) in Figure 2, the clay element is subjected to isotropic compressive stress. Under these conditions, further vertical stress relief, equal to $\Delta\sigma'$, occurs. The stress relief does not include any shear stress removal and corresponding elastic strain. Thus, a confined unloading requires that $\Delta\sigma'_h/\Delta\sigma'_v$ = (1 – sinφ'_M). Consequently, for this unloading along the stress path C to D in Figure 2, the effective stress ratio becomes:

$$\Delta\sigma'_{ho} /\Delta\sigma'_{vo} = 1/(1 - \sin\varphi'_M) \qquad (6)$$

And the following relationship between K_0 and OCR can be derived:

$$K_0 = [2 + OCR \cdot (1 - \sin\varphi'_M) \sin\varphi'_M]/2(1 + \sin\varphi'_M) \qquad (7)$$

The value of OCR corresponding to a K_0 equal to 1/(1 - sinφ'_M), i.e. point D in Figure 2, is:

$$OCR_{\ K0=1/(1-\sin\varphi'M)} = 4/(1 - \sin\varphi'_M)^2. \qquad (8)$$

In this case, the change in the applied passive shear stress is ½·$\Delta\sigma'$·sinφ'_M, and is equal to the mobilized passive shear strength.

Unloading when $\sigma'_{ho} /\sigma'_{vo}$ > 1/(1 – sinφ'_M)

As for the unloading under active shear in Figure 1, an unloading $\Delta\sigma'_v$ leads to a reduction of the passive shear stress equal to ½·$\Delta\sigma'_v$·sinφ'_M, resulting in a pure elastic strain. Again, the conditions of zero lateral strain require an effective stress ratio $\Delta\sigma'_h/\Delta\sigma'_v$ equal to (1 – sinφ'_M). The ratio of the stress change corresponding to the stress path D to E in Figure 3 can be described by:

$$\Delta\sigma'_h/\Delta\sigma'_v = 1 \qquad (9)$$

The following relationship for the coefficient of earth pressure at rest K_0 can be derived:

$$K_0 = 1 + OCR\ (1 - \sin\varphi'_M) \sin\varphi'_M /4 \qquad (10)$$

The maximum value of K_0, corresponding to a passive failure, i.e. point E in Figure 3, is:

$$K_0 = (1 + \sin\varphi'_M)/(1 - \sin\varphi'_M) \qquad (11)$$

The value of OCR corresponding to a value of K_0 equal to (1 + sinφ'_M)/(1 – sinφ'_M), i.e. point E in Figure 3, is:

$$OCR_{\ K0} = (1 + \sin\varphi'_M) /(1 - \sin\varphi'_M) = 8/(1 - \sin\varphi'_M)^2 \qquad (12)$$

In this last phase, the change in shear stress and shear strength are both equal to zero.

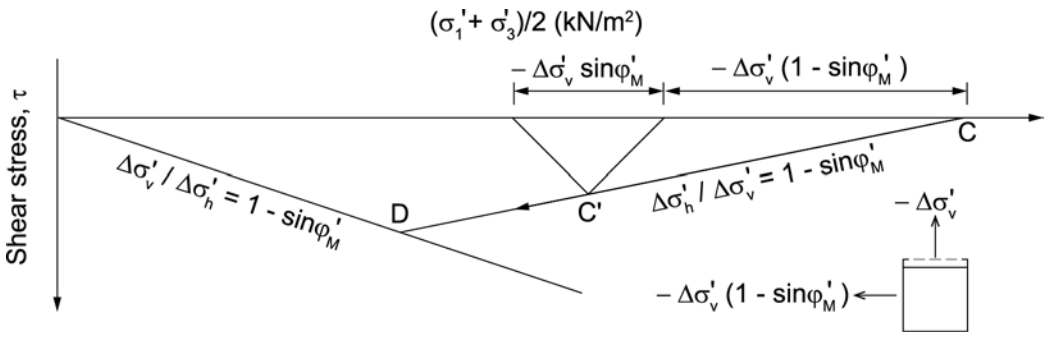

Figure 2. Changes in effective stresses and strains for a clay element during uniaxial unloading resulting in an increase in K_0 from 1 to $1/(1 - \sin\varphi'_M)$

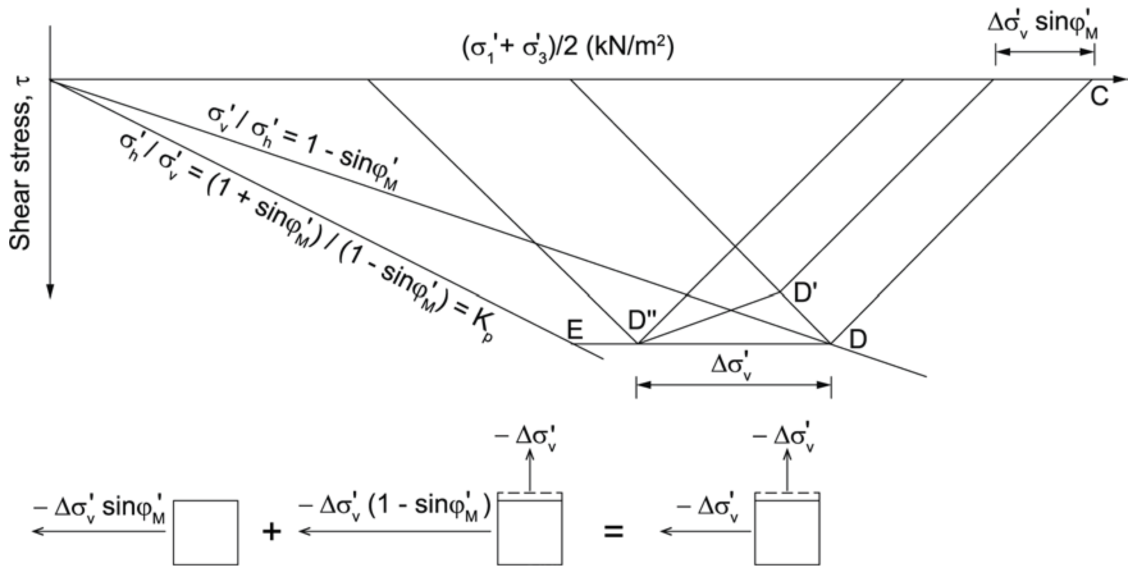

Figure 3. Changes in effective stresses and strains for a clay element during uniaxial unloading for $K_0 > 1/(1 - \sin\varphi'_M)$

Coefficient of earth pressure at rest, K_0

The coefficient of earth pressure K_0 is an important parameter for the design of, e.g., retaining walls, basement walls, pile foundations, pipelines and tunnels. It is also used for generating initial stresses when using advanced numerical methods to solve complex geo-engineering problems. The results of several laboratory tests also strongly depend on the estimate of K_0 (e.g. small strain shear modulus, G_{max}, from resonant column tests, strength and moduli from static and cyclic triaxial tests). Although K_0 can have a significant impact on inputs and calculation results, the reliability in the estimates of K_0 in the laboratory or *in situ* is still uncertain.

Examples of relationships and charts for the evaluation of K_0 include those of Jaky (1944), Kenney (1959), Brooker and Ireland (1965), Massarsch (1979) and Mayne and Kulhawy (1990), and more recently L'Heureux et al. (2017). Except for the most recent paper, most of these relationships were developed for clay types and soil history which are not necessarily representative of Norwegian conditions.

Jaky (1944) established a theoretical solution where K_0 was presented as a function of the effective friction angle (φ') of the soil:

$$K_0 = 1 - sin\varphi' \left(\frac{1 + \frac{2}{3} sin\varphi'}{1 + sin\varphi'} \right) \qquad (13)$$

The simplified Eq. [2] was also presented by Jaky (1944):

$$K_0 = 1 - sin\varphi' \qquad (2)$$

The difference between Eq. (13) and (2) is about 8% at low friction angles and 16% at high friction angles. However, considering the difficulty of making an appropriate choice for the friction angle φ' for a given soil, this approximation was suggested as sufficiently accurate for most engineering purposes (Wroth, 1972).

Brooker and Ireland (1965) performed a comprehensive series of laboratory K_0-tests on five clay soils with well-documented properties. In their study, all soils were air-dried and passed through a number 40 sieve before testing on specimens reconstituted at a water content corresponding to a liquidity index of about 0.5. In the oedometer cell, loads were thereafter applied to the sample in increments up to a maximum pressure of 15 MPa. The Brooker and Ireland (1965) plasticity chart for estimating K_0 has been extensively used over the years and is still one of the main reference for the evaluation of K_0 in practice.

$$K_0 = 0.95 - sin\varphi' \qquad (14)$$

The results obtained by Brooker and Ireland (1965) also showed that K_0 in clay soils was strongly dependent on the overconsolidation ratio (OCR) and the plasticity index (I_p).

L'Heureux *et al*. (2017) presented the results of a review of a database for K_0-measurements during first unloading in the oedometer cell, including the results for eight Norwegian clays. A series of multivariate regression analyses were done on the Brooker and Ireland data and the data in the K_0 database to establish guidelines for a reliable estimation of K_0 in Norwegian clays in the absence of site-specific K_0-data. The resulting regression equation for the combined K_0-triaxial tests and K_0-oedometer tests, for OCR between 1 and 8 was:

$$K_0 = 0.48\, I_p^{0.03}\, OCR^{0.47} \qquad (15)$$

where the regression coefficient (R^2) was 0.98. The exponent on the plasticity index, I_p, is very close to zero, indicating little influence of the plasticity index on the K_0-value.

The regression analyses for eight Norwegian clays showed even less influence of plasticity index, I_P, on K_0. For all practical purposes, the exponent in I_p from the regression analyses was zero. Regression analyses were also performed on the Norwegian clay database between K_0 and OCR only. The best fit was given by the following equation:

$$K_0 = 0.53 OCR^{0.47} \qquad (16)$$

The results of this regression analysis showed that all of the K_0-laboratory data on the eight Norwegian clays fell within ±5% of Eq. (16). A similar analysis was performed on the Brooker and Ireland (1965) data and the results were:

$$K_0 = 0.57 OCR^{0.39} \quad (17)$$

In this case the difference in measured K_0 and the K_0 from multivariate regression analysis were larger than for the Norwegian clays and fell within a ±15% interval.

In the following. stress path-based analysis of K_0, both the original relationship by Brooker and Ireland (1965) and the L'Heureux *et al.* (2017) relationship will be used.

Relationship between K_0 and φ'_M for normally consolidated clays

Figure 4 shows the relationship between the coefficient of earth pressure at rest K_0 and plasticity index I_p for 5 different clays and one sand (I_p = 0) after uniaxial consolidation of remoulded material in the laboratory (Brooker and Ireland, 1965). The curve in Figure 4 is for normally consolidated material.

Using the above stress-path considerations on a clay element, the values of K_o determined in this way should come close to the value of $1 - \sin\varphi'_M$. This curve, denoted $1 - \sin\varphi'_M$, is shown on Figure 4. The curve is based on a relationship between friction angle and plasticity index (Aas, 1986). Except for the London Clay, the experimental data by Brooker and Ireland (1965) (and Hendron, 1963) fit exceptionally well to the stress-path based values.

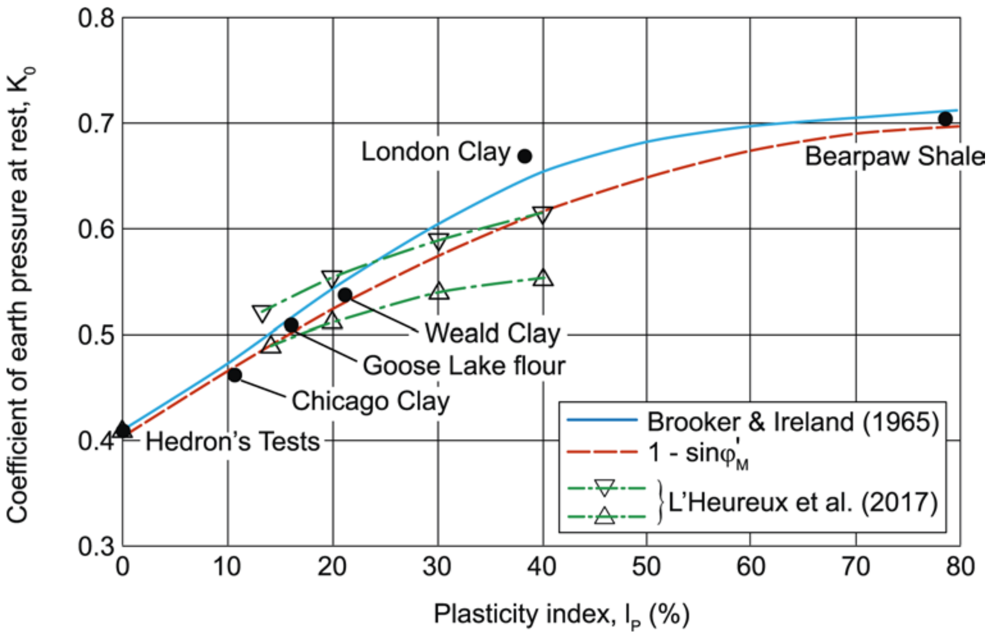

Figure 4. Measured (Brooker and Ireland, 1965; L'Heureux et al, 2017) and stress-path-based values of K_0 as a function of plasticity index I_p for normally consolidated clays

Relationship between K_0 and OCR

These "theoretical" values in Figure 4 are compared in Figure 5 with the Brooker and Ireland (1965) experimental data. The upper diagram in Figure 5 makes the comparison for $1 - \sin\varphi'_M$ equal to 0.5 (ϕ' = 30°). A plasticity index I_p of 16% and a normally consolidated K_0-value of 0.5 was used. The I_p is based on Figure 4, where an I_p of 16% corresponds to a value of ($1 - \sin\varphi'_M$) of 0.5 and $K_0 = 1 - \sin\varphi'_M = 0.5$.

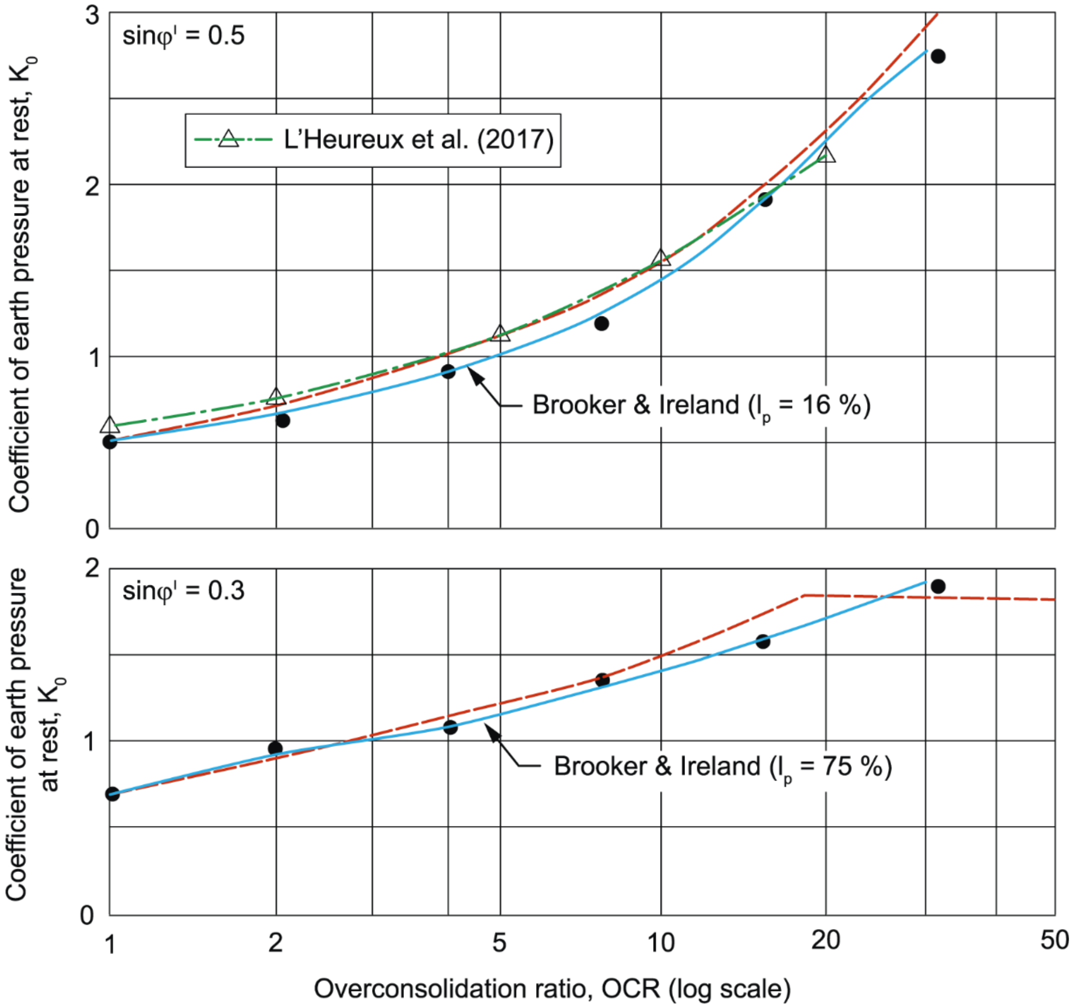

Figure 5. Measured (Brooker and Ireland, 1965) and stress-path-based values of K_0 as a function of overconsolidation ratio for clays 4

Brooker and Ireland (1965) also studied a clay with an I_p of 78% (Bearpaw Shale) and normally consolidated K_0 value of 0.7. Figure 4 indicated that an I_p of 78% corresponds to a value of $1 - \sin\varphi'_M = 0.7$, which in turn confirms the reported value of $K_0 = 1 - \sin\varphi'_M = 0.7$. The lower diagram in Figure 5 compares the Brooker and Ireland data with the stress path-predicted values for a clay with $\sin\varphi'_M = 0.3$. There is a reasonably good agreement between the stress path-calculated values of K_0 and the measured Brooker and Ireland values for both clays.

The three comparisons show that the Brooker and Ireland data support the theoretical explanation given above on effective stress and strain conditions in uniaxial loading of overconsolidated clays.

Schmidt (1966) presented an empirical equation between the overconsolidated and normally consolidated coefficient of earth pressure at rest, $K_{0\,OC}$ and $K_{0\,NC}$, valid for uniaxial unloading of a clay:

$$K_{0\,OC}/K_{0\,NC} = OCR^m \tag{18}$$

and proposed the value of $1.2{\cdot}\sin\varphi'$ for the exponent m.

The theoretical considerations described above make it possible to determine the value of exponent m for different values of $\sin\varphi'$, and gives the following relation:

$$m = 0.34 + 0.73\cdot(\sin\varphi'_M - 0.3) \quad (19)$$

Table 1 compares the m-values for friction angles between 19 and 30°, as proposed by Schmidt and as derived from the stress path-based framework.

Table 1. Comparisons of exponent m for K_0 for overconsolidated clay

Friction angle, φ' (°)	sin φ'	m (Schmidt), Eq. 18	m (Stress paths), Eq. 19
19	0.33	0.39	0.36
20	0.35	0.41	0.37
23	0.40	0.47	0.41
25	0.42	0.51	0.43
28	0.47	0.56	0.45
30	0.50	0.60	0.49

K_0 in an overconsolidated clay under reloading

Figure 6 illustrates the loading-unloading sequence A-B-C-D-E from Figures 1 to 3. In addition, the figure shows what is considered to be the result of a reloading of this overconsolidated clay (Path EFG).

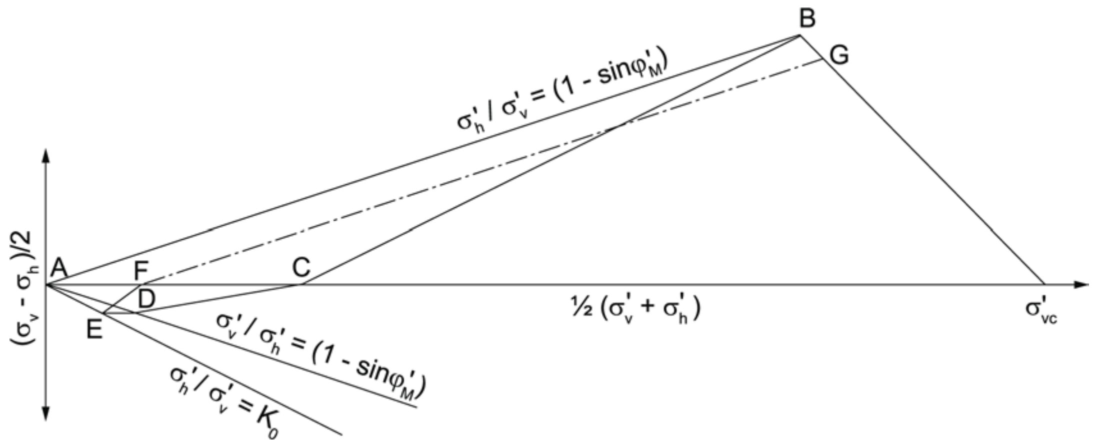

Figure 6. Loading, unloading and reloading of overconsolidated clay

At stage E, the clay is subjected to the largest possible passive shear stress, and is close to yielding. If the deviator stress is removed by increasing the vertical stress, the clay undergoes elastic deformation, and reaches a situation of isotropic consolidation (point F), comparable with that at point A at the start of the first loading.

Under these conditions, a further isotropic stress increase $\Delta\sigma'$, combined with a decrease in horizontal stress equal to $\Delta\sigma'\cdot\sin\varphi'_M$, contributes to maintain zero net lateral strain. As illustrated in Figure 6, the resulting effective stress path follows the F to G line, parallel to the path of the first loading A to B:

$$\Delta\sigma'_h / \Delta\sigma'_v = 1 - \sin\varphi'_M \quad (20)$$

Figure 7 shows the relationship between K_0 and OCR for a clay with $\sin\varphi'_M = 0.5$, as calculated with the stress path model. Curve A describes unloading from OCR = 1 to 30 (the same curve as shown in the upper diagram in Fig. 5), Curve B reloading from OCR = 10 to 1, and Curve C reloading from OCR = 30 to 1.

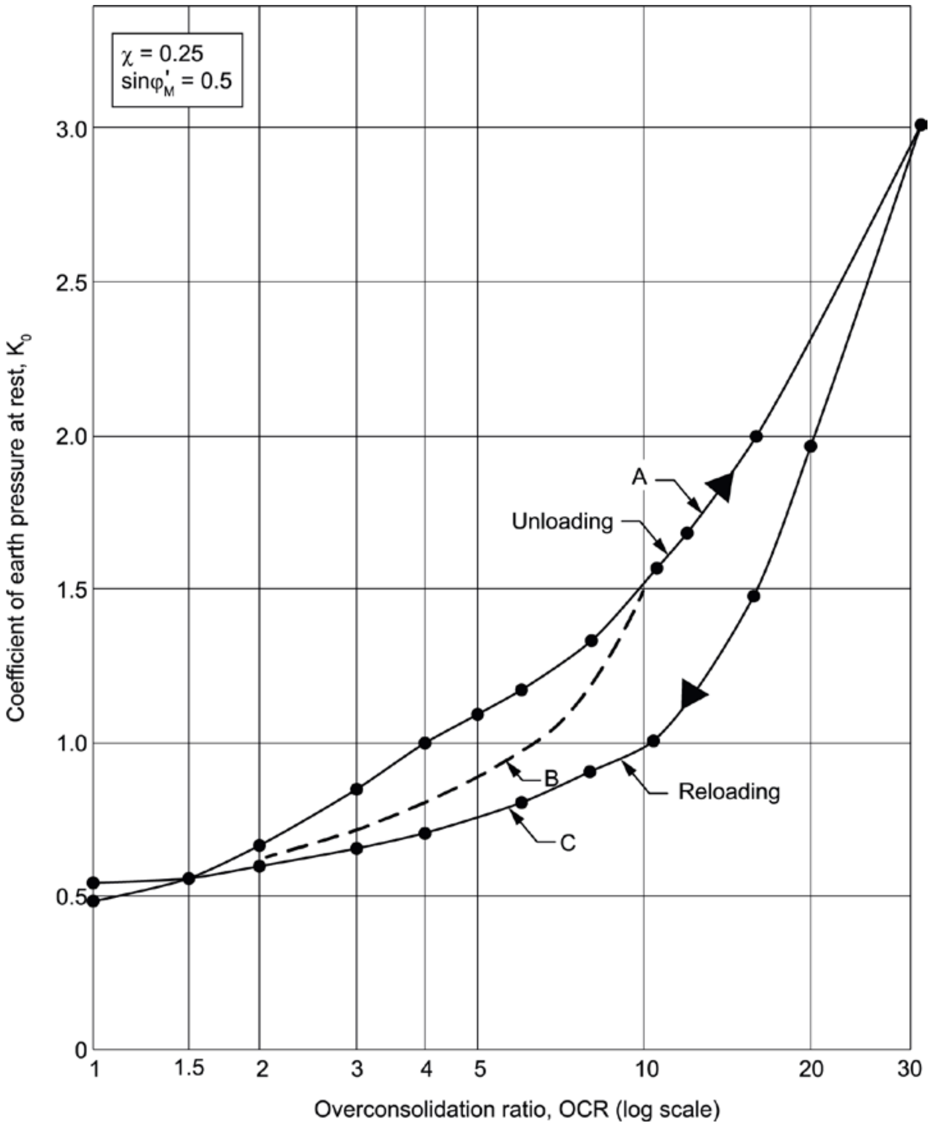

Fig. 7: Relationship between K_0 and OCR during first loading, unloading and reloading

Without implying the general validity of expression

$$K_{0\,OC}/K_{0\,NC} = \mathrm{OCR}^m \tag{21}$$

the value of m that gives the best fit to the curve A is about 0.5. Introducing a value of K_{oNC} of 0.5 in Eq. 21 gives m = 0.486, while Schmidt's equation yields 0.6. The multi-regression statistical analysis by L'Heureux *et al.* (2017) suggested an m-value of 0.47 for a K_{oNC} of 0.53, based on laboratory measurements on eight Norwegian clays with OCR between 1 and 8 and plasticity index between 10 and 40%.

It is important to realize that the value of K_0 for a given overconsolidation ratio OCR depends significantly on whether the clay has been subjected to a first loading, unloading or reloading.

Limited evidence so far (NGI database and in L'Heureux *et al.*, 2017) indicates the K_0-value under unloading and second unloading are very similar. In practice, it is therefore important to consider whether a soil is subjected to an increased vertical stress due to subsequent construction activities, such as embankments or buildings.

Figure 8 gives the result of an actual measurement of the relationship between K_o and OCR during unloading and subsequent reloading for a soft sensitive clay in an instrumented oedometer in the laboratory (Campanella and Vaid, 1972). The K_0-values for this clay agree well with the stress path derived K_0-values: it shows a value of K_0 of about 0.55 at OCR = 1, and a reloading curve very similar to that shown in Figure 7.

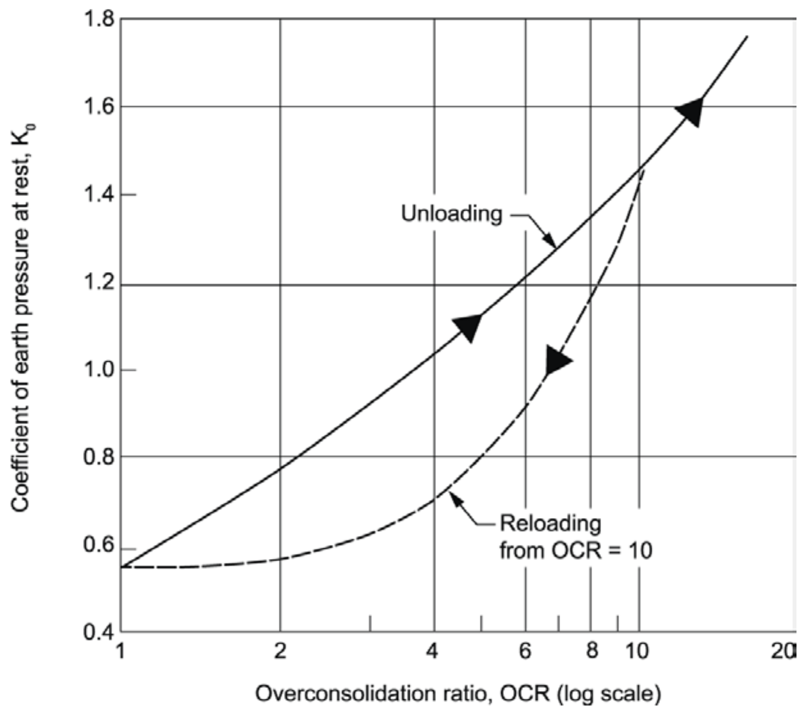

Figure 8. K_0 as a function of OCR for Haney sensitive clay (Campanella and Vaid, 1972).

Concluding remarks

Theoretical considerations based on the study of the stress paths under uniaxial loading and unloading of a clay were used to determine the value of the coefficient of earth pressure at rest, K_0, for normally consolidated and overconsolidated clay. The analysis studied the changes in effective stresses and strains as a result of a uniaxial unloading and reloading of a clay deposit. The analysis is based on the following two basic assumptions:

- In any phase of the loading-unloading cycle, there must exist a static equilibrium, which means that shear stresses and mobilized shear strengths have to be equal.
- The mobilized frictional shear resistance is an exclusive result of the plastic deformation that have occurred. Relief of a shear stress results in a pure elastic deformation that does not influence the mobilized shear strength.

The paper provides an explanation for the values of K_0 through an interpretation of the effective stress paths during uniaxial loading. One particular consideration was looking at the difference in the behaviour of elastic and plastic strains and how they contribute to mobilize and reduce the friction resistance.

The analysis led to relationships between the coefficient of earth pressure at rest K_0 and $\sin\varphi'_M$ (the mobilized friction angle) and between K_0 and overconsolidation ratio OCR for unloading. The stress path-derived values of K_0 agreed closely with the well-known results presented by Brooker and Ireland (1965) and with recent statistical analyses of laboratory-measured K_0 data on eight Norwegian clays presented by L'Heureux *et al*. (2017).

The reloading phase with the stress path model was found to give lower values of K_0 for the same OCR than the unloading phase. This was reported by Ladd *et al*. (1977), where they presented a loading-reloading curve (OCR = 1−10) for a sensitive clay with a normally consolidated value of K_0 equal to 0.55. The stress-path derived values of K_0 during reloading also agreed well with the measured data presented by Campanella and Vaid (1972).

In general, the calculated curves with the stress path model, across a range of OCR-values, harmonized well with the measured values presented in the literature.

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