Abstract

It has been known for a long time now that in soft and sensitive clays undrained failure occurs apparently before the full friction angle has been mobilized. The main object of this paper is to clarify the importance of deformations in the process of mobilizing undrained shear strength. This has resulted in an improved understanding of the failure process, and made it possible to express undrained shear strength for this type of clays in terms of effective stresses and a set of effective shear strength parameters, friction and attraction. The study made it possible to determine the relationship between the coefficient of earth pressure at rest $K_0$, the undrained shear strength $s_u$ and the overconsolidation ratio OCR during cycles of consolidation, unloading and reloading. The determinations, done both analytically and graphically, were done for a typical low plasticity clay, and the results compared with published data. For comparison, laboratory test results for a somewhat stiffer and dilating offshore clay are also included in this paper.

Introduction

When a soft, sensitive clay mobilizes the full friction under the major principal stress, and thereafter is stressed and mobilizes the contribution to friction from the minor principal stress, this causes a structural collapse, high pore pressures and decreasing strength. This second shearing stage implies an important change compared with the first one, as the direction of axial strain is reversed compared to the friction supporting stress component. These conditions are believed to be responsible for a structural collapse in the soil that is responsible for softening and failure in a loose (contractant) soil or strengthening in a low sensitive (dilatant) clay.

In cases involving rotation of principal stresses, it is the direction of the principal stresses at the end of the first stage that counts. In this first stage, active shear involves a reloading and only small elastic strains. During shear of the soil under other stress conditions, for instance passive shear, the process includes both elastic and plastic deformations.

Assumptions

The results in this paper are based on the following four assumptions:

1) Mobilisation of friction requires a simultaneous development of a plastic deformation in the clay. Elastic deformations do not, in the same way, contribute to build up friction
resistance.

2) Attraction is believed to be due to net attractive forces acting across the water-films constituting the contact points between clay particles. Attraction acts like a tension reinforcement in the clay and requires practically no deformation to be activated.

3) If a clay has been consolidated, thereafter unloaded and then reloaded, at all times without undergoing lateral strain, the active, frictional shear resistance will be a function of the existing vertical stress alone.

4) An attempt to force a brittle (contractant) overconsolidated clay to mobilize a contribution to friction from also the horizontal stress, leads to a structural collapse and decreasing shear strength.

It is important to realize that even though the process involves yielding, the requirement of static equilibrium must always apply.

**Undrained, active shear strength**

The general expression for undrained, active shear strength, $s_{uA}$, of a brittle, pure frictional soil is:

$$s_{uA} = \frac{1}{2} (\sigma'_v - \sigma'_h) = \frac{1}{2} \sigma'_v \sin \phi'_M$$

which gives:

$$\frac{\sigma'_h}{\sigma'_v} = K_0 = 1 - \sin \phi'_M$$

where

- $\sigma'_v$ is the effective vertical stress
- $\sigma'_h$ is the effective horizontal stress
- $\phi'_M$ is the material friction angle
- $K_0$ is the coefficient of earth pressure at rest

Equation (2) is the well-known Jacky formula (Jacky, 1948), expressing the coefficient of earth pressure at rest for a normally consolidated soil.

For a clay exhibiting attraction, this will lead to an increase in strength comparable to the effect of reducing $\sigma'_h$ by a stress equal to the attractive force. Hence, the expression for undrained, active shear strength becomes:

$$s_{uA} = \frac{1}{2} \sigma'_v (\chi + \sin \phi'_M)$$

where $\chi$ is the relative attraction parameter (Aas and Lacasse, 2021).

As long as one is dealing with exclusively clays exposed to a state of perfect confined compression or expansion, $\sigma'_v$ may be exchanged by $K_0 \sigma'_v/(1 - \sin \phi'_M)$, which gives:
\[ s_{ua} = \frac{1}{2} K_0 \sigma'_v (\chi + \sin \phi'_M) / (1 - \sin \phi'_M) \] (4)

Equation (4) defines a proportionality between \( s_{ua} \) and \( K_0 \), and makes it possible to determine how \( s_{ua} \) is related to the overconsolidation ratio, OCR.

Figure 1 shows the values of \( \chi \) and \( \sin \phi'_M \) as a function of plasticity index for 25 sets of undrained triaxial tests on a variety of soft clays. As shown on the figure, \( \chi \) increases and \( \sin \phi'_M \) decreases with increasing plasticity. However, the increase and decrease are in such a way that the sum of the two parameters (\( \chi \) and \( \sin \phi'_M \)) is an almost constant value for most clays and equal to a value between 0.7 and 0.8 (Aas and Lacasse, 2021).

In the next two sections, values of \( \chi \) equal to 0.25 and \( \sin \phi'_M \) equal to 0.50 were selected and the approximate values of \( K_0 \) and \( s_{ua} \) for an idealized low plastic, soft clay was calculated for overconsolidated (OC) clays during unloading and reloading.

**Relationship between \( K_0 \), OCR and \( s_{ua} \) for unloaded OC clays**

The following section derives the relationship between \( s_{ua} \) and \( K_0 \) for a clay subjected to uniaxial unloading, and thus having an OCR greater than 1. To demonstrate this, the discussion follows the behavior of a clay from start at the conditions of earth pressures at rest (\( K_0 \)-conditions) at a given preconsolidation stress, \( \sigma'_{vc} \), and up to a state of passive failure.

The conditions for mobilizing friction under confined compression involves a ratio between \( \chi \) and \( \Delta \sigma'_3 \) equal to \( 1/(1 - \sin \phi'_M) \). This means that the clay has been subjected to a combination of an isotropic stress \( \Delta \sigma'_1 \) and a shear stress equal to \( -\frac{1}{2} \Delta \sigma'_1 \sin \phi'_M \), causing plastic deformations.

**Unloading when \( \sigma'_n/\sigma'_v < 1 \)**

The above loading process involves a combination of an isotropic stress, \( \sigma'_v \), and a horizontal deviator stress, \( -\sigma'_v \sin \phi'_M \), causing plastic deformations and contributing to building up frictional resistance. If one applies an unloading involving an isotropic stress \( -\sigma'_v \) and a horizontal deviator stress \( \sigma'_v \sin \phi'_M \), this does not, however, lead to a reversed confined condition. This is due to the fact that the reduction in shear stress causes elastic strains only, which do not influence the mobilized friction. To produce a uniaxial deformation, the clay needs to be exposed to also a horizontal deviator stress \( -\sigma'_v \sin \phi'_M \), so that the combination of stresses causing plastic strains becomes \( -\sigma'_v \) and \( -\sigma'_v \sin \phi'_M \). However, the value of \( K_0 \) reflects also the contribution from elastic stress, and becomes for this case:

\[ K_0 = (1 - \sin \phi'_M) / (1 + \sin \phi'_M) \] (5)

By introducing the preconsolidation stress, \( \sigma'_{vc} \), and the overconsolidation ratio \( OCR = \sigma'_{vc} / \sigma'_v \), the following relationship between \( K_0 \) and OCR can be derived:
\[ K_0 = (1 + OCR \cdot \sin \phi'_M) / (1 + \sin \phi'_M) \]

The particular values of OCR and \( \sigma'_v \) that correspond to a value of \( K_0 \) equal to unity are:

\[ OCR_{K_0=1} = \frac{2}{1 - \sin \phi'_M} \]

\[ \sigma'_{vc\ K_0=1} = \frac{1}{2} \sigma'_{vc\ (1 - \sin \phi'_M)} \]

Figure 1. Relationships between shear strength parameters \( \chi \) and \( \sin \phi'_M \) and plasticity for 25 sets of undrained triaxial active tests.

**Unloading when \( 1 < \sigma'_h/\sigma'_v < 1/(1 - \sin \phi'_M) \)**

After having been unloaded to stresses corresponding to the OCR value above, the clay is subjected to a condition involving isotropic compressive stresses. Under these conditions, a
further vertical stress relief equal to $\Delta \sigma'$ does not include any shear stress removal nor corresponding elastic strain. Thus, a confined unloading has to follow the simple imposed stresses of $\Delta \sigma_h = \Delta \sigma' (1 - \sin \phi_M)$. Consequently, for this unloading stage, one gets:

$$K_0 = (1 - \sin \phi_M)$$

The following relationship between $K_0$ and OCR can be derived:

$$K_0 = [2 + \text{OCR} (1 - \sin \phi_M) \cdot \sin \phi_M] / 2 (1 + \sin \phi_M)$$

The value of OCR which corresponds to a value of $K_0$ equal to $1/(1 - \sin \phi_M)$ is:

$$\text{OCR}_{K_0} = 4 / (1 - \sin \phi_M)^2$$

In the same way, the OCR corresponding to passive failure, $K_0 = (1 + \sin \phi_M) / (1 - \sin \phi_M)$ is:

$$\text{OCR}_{K_0} = (1 + \sin \phi_M) / (1 - \sin \phi_M) = 8 / (1 - \sin \phi_M)^2$$

Combining Eq. (4) and the above values of $K_0$, one gets a relationship between $su_A$ and OCR:

$$\sigma_h' / \sigma_v' < 1 \quad \text{su}_A / \sigma_v' = \frac{1}{2} [1 + \frac{1}{2} \text{OCR} \sin \phi_M (\chi + \sin \phi_M)] / [2 (1 + \sin \phi_M)]$$

$$1 < \sigma_h' / \sigma_v' < 1 / (1 - \sin \phi_M) \quad \text{su}_A / \sigma_v' = \frac{1}{2} \frac{1}{2} \text{OCR} (1 - \sin \phi_M) \sin \phi_M (\chi + \sin \phi_M) / [2 (1 - \sin \phi_M)]$$

For the case where $\sigma_h' / \sigma_v'$ equals $(1 + \sin \phi_M) / (1 - \sin \phi_M)$, passive failure occurs.

Figure 2 presents a graphical representation of the variation in $K_0$ as a clay is unloaded from an OCR of 1 to 32. The line labeled $\sigma_h' / \sigma_v' = (1 - \sin \phi_M)$ is the original consolidation process resulting in an initially constant $K_0$ equal to $1 - \sin \phi_M$. The red curve below this line shows the value of $K_0$ increasing from $1 - \sin \phi_M$ to $(1 + \sin \phi_M) / (1 - \sin \phi_M)$ when OCR increases from 1 to 32. The latter data point is the passive failure. The dashed lines, normal to each other, demonstrate how one, on the basis of a $K_0$-value, gets to the failure line described by $\sigma_h' / \sigma_v' = (1 - \chi - \sin \phi_M)$. The inset in Figure 2 gives more details for OCRs of 16 and 32 and negative shear stresses.

**Relationship between $K_0$, OCR and $su_A$ for Reloaded OC clays.**

Figure 3 shows how $K_0$ and $su_A / \sigma_h'$ may be determined from a similar graphical construction for the case where the overconsolidated clay is reloaded to its original preconsolidation stress, $\sigma_v'$. The first part of the reloading process (illustrated in the inset) involves the transition of the clay from passive to active stress conditions. This occurs as an elastic recovery to an isotropic stress condition, $\sigma_h'$. The elastic rebound of passive shear stress in this phase does not imply any change in mobilized friction. As the effective vertical stress increases, however, the
ratio \( \frac{s_{uA}}{\sigma'_{v}} \) decreases proportionally with the increasing vertical stress. Thereafter, with further reloading of the clay bringing the clay to a condition of isotropic consolidation, the clay behaves in the same manner as a young clay, implying a stress ratio of:

\[
K_0 = \frac{(\sigma'_h - \sigma'_0)}{(\sigma'_{v} - \sigma'_0)} = (1 - \sin \phi'_M)
\]

(14)

Figure 2. Graphical construction of \( K_0 \) and \( \frac{s_{uA}}{\sigma'_{v}} \) as a function of OCR for soft clays.

Figure 3. Theoretical values of \( K_0 \) and \( \frac{s_{uA}}{\sigma'_{v}} \) during reloading of an overconsolidated clay.
The expression for $K_0$ may be determined on the basis of Eqns. (11) and (13):

$$K_0 = 1 + 1/8 \text{OCR} \left(1 - \sin \phi'_M\right) \sin \phi'_M (1 + \sin \phi'_M)$$  \hfill (15)

Figures 4 and 5 show numerical values of $K_0$ and $s_{uM}/\sigma'_v$ based on the processes in Figures 2 and 3 for unloading and reloading of a clay. Values of $\chi$ of 0.25 and $\sin \phi'_M$ of 0.50 were used.

![Figure 4. Change in $K_0$ of a clay as a result of unloading and reloading, based on Figures 2 and 3.](image)
The resulting unloading curves describe very closely the following two well-known expressions introduced by Ladd and Foott (1974) and Ladd et al. (1977):

\[ K_{\theta OC} = K_{\theta NC} \cdot OCR^m \]  \hspace{1cm} (16)

\[ \frac{s_{uA} OC}{\sigma'_{vc}} = \frac{s_{uA} NC}{\sigma'_{vc}} \cdot OCR^m \]  \hspace{1cm} (17)

Figure 5. \( s_{uA}/\sigma'_{vc} \) of a clay as a function of OCR during unloading and reloading, based on Figs 2 and 3.
*Examination of coefficient of earth pressure at rest, $K_0$*

The coefficient of earth pressure at rest $K_0$ can thus be obtained theoretically from Eq. (18), based on the analyses on an idealized clay in Figures 2 and 3. The coefficient in front of the OCR-parameter refers to the $K_0$-value for the normally consolidated clay, as noted in Eq. (16).

$$K_0 = 0.50 \text{OCR}^{0.55} \quad (18)$$

It is important to realize that the $K_0$-value depends on whether a clay has never been reloaded or been exposed to an increased in vertical stress (or virgin consolidation) after unloading and reloading, due to, for instance, the addition of fill or construction of a heavy building.

Figure 6 shows the results of measurements of $K_0$ in triaxial tests during unloading and subsequent reloading of the soft sensitive Haney Clay (Campanella and Vaid, 1972). The $K_0$-values do not deviate much from the values for the theoretical clay in Figure 4, with a value of $K_0$ of about 0.55 at an OCR of 1. The laboratory data support the theoretical model in Figure 4.

![Figure 6](image)

*Figure 6. $K_0$ as a function of OCR as measured in the laboratory on Haney sensitive clay*

The multi-regression statistical analyses by L'Heureux *et al.* (2017) suggested the relationship in Eq. (20), based on laboratory measurements of $K_0$ on eight Norwegian clays with OCR between 1 and 8 and plasticity index between 10 and 40%. The relationship in Eq. (20) is very close to the theoretically derived relationship in Eq. (18).

$$K_0_{OC} = 0.53 \text{OCR}^{0.47} \quad (20)$$
Examination of normalized undrained shear strength ratio $s_{uA}/\sigma_v$

The normalized undrained shear strength ratio $s_{uA}/\sigma_v$ can also be thus obtained theoretically from Eq. (19), based on the analyses on an idealized clay in Figures 2 and 3. The red crosses in Figure 5 represent the theoretical relationship from Eq. (19). The coefficient in front of the OCR-parameter refer to the normalized undrained shear strength for the normally consolidated clay, as noted in Eq. (17).

$$s_{uA}/\sigma_v = 0.36 \text{ OCR } 0.55$$ (19)

Figure 7 presents the theoretically derived relationship in Eq. (19) with the measured data from undrained triaxial active tests on Norwegian soft clays from Karlsrud (1995).

Figure 7. Theoretical shear strength ratio $s_{uA}/\sigma_v$ compared to measurement in the laboratory, with $\chi$ of 0.25 and $\sin \phi'_M$ of 0.50 (data from Karlsrud, 1995). (Note: $\sigma_w$ stands for effective axial consolidation stress in the triaxial test, which in situ corresponds to $\sigma'_{vo}$ effective vertical overburden stress).
Figure 7 also compares the theoretically-derived values with the relationships proposed by Karlsrud et al. (2005) based on Eq. (16):

\[ s_{uA}/\sigma'_{v0} = \alpha \text{OCR}^m \]  \hspace{1cm} (20)

Karlsrud et al. (2005) recommended a range of \( \alpha \)-values between 0.28 and 0.32 and exponent \( m \) between 0.60 and 0.90 (green and blue curves in Fig. 7). The red crosses are again the theoretically-derived values, with for \( s_{uA}/\sigma'_{vc} = 0.36 \text{OCR}^{0.55} \). Figure 7 also shows the relationship \( s_{uA}/\sigma'_{vc} = 0.28 \text{OCR}^{0.85} \), which includes the highest data measured on Eidsvoll clay.

Figure 8 presents the results from triaxial active tests on Norwegian clays. On the upper figure, the data accumulated by Karlsrud and Hernandez-Martinez (2013) are represented by three curves described with the following relationship:

\[ s_{uA}/\sigma'_{v0} = \alpha \text{OCR}^m \]  \hspace{1cm} (21)

where the \( \alpha \)–value represents the ratio \( s_{uA}/\sigma'_{v0} \) for normally consolidated clay noted in Eq. (17). In the upper Figure 8, \( \alpha \) varies from 0.25 to 0.35 and exponent \( m \) from 0.65 to 0.75. The upper bound data are from the Emmerstad and Nybak-Slomarka sites, two of the clays with lowest plasticity index. The lower bound data are mostly from the Kløfta-Nybakk, Klett, Tiller and Stjørdal sites.

Paniagua et al. (2018; 2019) updated the correlations for Norwegian clays, considering only the clays where block samples were taken and thus reduced the effect of sample disturbance to a maximum. Paniagua et al. also did rigorous multi-regression analyses, including water content of the tested specimens as one of the regression variables. The results are illustrated in the lower diagram in Figure 8, and compared with the estimated curves by Karlsrud et al. (2005) earlier, which had suggested \( \alpha \)-values between 0.28 and 0.32 and exponent \( m \) between 0.60 and 0.90.

The updated Paniagua et al. functions for \( s_{uA}/\sigma'_{v0} = \alpha \text{OCR}^m \) (Eq. 21) are also a function of water content. There is an observable trend as a function of the water content. These more recent experimental correlations for \( s_{uA}/\sigma'_{v0} \) have an \( \alpha \)-value of 0.32 for all water contents \( w \). The exponent \( m \) is equal to \((0.20 + 1.17w)\), with \( w \) expressed as a percentage between 0 and 1. The correlations in Figure 8 mean that exponent \( m \) increases from 0.55 to 1 for water contents between 30 and 70%.

D'Ignazio et al. (2017) studied 6 clay sites in Finland Statistical analysis of the normally consolidated shear strength ratio and the exponent \( m \) was done. The ratio \( s_{uA}/\sigma'_{v0} \) varied between 0.30 to 0.48 and the exponent \( m \) could vary between 0.5 and 1.0.
Figure 8. Undrained shear strength ratio $s_{ua}/\sigma'_{w0}$ from triaxial compression tests, with relationships $s_{ua}/\sigma'_{w0} = \alpha \cdot OCR^m$: Top: 21 Norwegian clays (Karlsrud and Hernandez-Martinez, 2013); Bottom: exclusively block sample data, and as a function of water content $w$ (Paniagua et al., 2019).
There is naturally scatter in the laboratory data, and the parameters may depend on other clay characteristics such as, for example, clay mineralogy and/or sensitivity, although Paniagua et al. (2019) concluded that the main significant factor was water content. The laboratory values may also be affected by sample disturbance, even if the samples are taken by block samplers.

Nevertheless, in spite of the scatter in the laboratory results, the theoretically-derived model in this paper (shown as crosses in Fig. 7) provide a good picture of the relationship between normalized active undrained shear strength and overconsolidation ratio for low plasticity soft clays. The experimental exponent $m$ may be somewhat higher than the theoretical value, but this may be a reflection of the water content, as exponent $m$ seems to increase with increasing water contents.

**Behaviour of North Sea dilatant clays**

Triaxial tests on clay from the offshore Sleipner field in the North Sea were included in the present study. The reason for including a dilatant clay was to demonstrate that a contractant and a dilatant clay behave similarly up to a critical point. Once this critical point is reached, the contractant clay starts building up pore pressures and losing strength, whereas the dilatant clay generates negative pore pressures and increases in strength.

The three series of active and passive triaxial tests examined herein were run on samples taken from depths of 3.35, 5.6 and 8.0 m below seabed. Each sample was reconsolidated to the in situ effective stresses and sheared under undrained conditions. As illustrated in Figures 9, 10 and 11, the three series of tests showed remarkably consistent behavior.

The behavior of the dilatant clay can be described as follows: the clay from the three depths showed a "material" friction ($\sin\phi'$) varying from 0.55 to 0.59, based on the highest value of $\sigma'_h/\sigma'_v$ in active shear and $\sigma'_v/\sigma'_h$ in passive shear. Both active and passive tests showed a small (elastic) strain of about 0.5%. At this point, large deformations started to develop suddenly.

By drawing straight lines from the origin and through these critical points, the lines should, based on the experience with the soft clays, correspond to:

\[
\frac{\sigma'_v}{\sigma'_h} = 1 - \sin\phi'_M \quad \text{in active shear} \tag{22}
\]

\[
\frac{\sigma'_v}{\sigma'_h} = 1 - \sin\phi'_M \quad \text{in passive shear} \tag{23}
\]

The values of $\sin\phi'_M$, determined from these lines lie between 0.55 and 0.57, and support the theoretical model developed for the idealized soft clay in the previous sections. The data show that dilatant and contractant clays behave similarly up to a stress level where only $\sigma'_v$ contributes to the mobilized active strength and only $\sigma'_h$ contributes to the passive strength.
Figure 9. Effective stress path for active and passive triaxial tests on dilatant Sleipner clay from 3.35 m depth.
Figure 10. Effective stress path for active and passive triaxial tests on dilatant Sleipner clay from 5.6 m depth.
CONCLUSIONS

When consolidating under confined compression, a soft clay is subjected to a combination of an isotropic stress and a shear stress, causing plastic deformations and responsible for mobilizing friction resistance. Since no lateral strain takes place, the change in vertical stress only is responsible for mobilizing friction. The requirement for static equilibrium then defines a generally valid ratio between the principal stresses equal to $1/(1 - \sin \phi_M')$, where $\sin \phi_M'$ is the ("material ") friction angle of the clay.

If unloading occurs, this clay will now be subjected to a reduction of both an isotropic stress and a shear stress. In this case, however; the reversion of shear stress results in an elastic strain
only, which has no influence on the mobilized friction. The contribution to friction from plastic strains which fulfils the condition of no lateral strain, then has to be based on the "general" principal stress ratio for virgin consolidation.

These principles have made it possible to calculate, both analytically and graphically, how $K_0$ varies as a function of overconsolidation ratio (OCR) as a low plastic clay is unloaded from a condition of earth pressure at rest ($K_0$) to passive failure, and then reloaded back to the initial stresses. In confined compression, there exist a constant ratio between $K_0$ and normalized shear strength ratio $s_{u}/\sigma_\text{vc}$, and where the effective stress strength parameters, $x$ and $\sin\phi'$ are two defining parameters. The paper compares theoretical-derived values of $K_0$ and normalized shear strength ratio $s_{u}/\sigma_\text{vc}$ with the values measured in the laboratory for a large number of Norwegian clays. The agreement is good and the theoretical approach can be used when there is a lack of experimental data.

Laboratory test results from a dilatant offshore clay show that the dilatant clay behaves exactly as a contractant clay up to a certain critical stress level, where a soft and sensitive clay will continue to failure, whereas a dilatant clay increases in strength, and until failure following a conventional Mohr Coulomb failure.

REFERENCES


