# YIELD STRESSES IN SOFT CONTRACTING CLAYS

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### Abstract

The undrained shear strength  $s_u$  along an arbitrary failure plane can be expressed as:

$$s_u = \frac{1}{2}(\sigma'_{ULS} - \sigma'_{LLS}) \tag{1}$$

where  $\sigma'_{ULS}$  and  $\sigma'_{LLS}$  denote the upper (U) and lower (L) effective, normal, compressive failure stresses acting at angles of +45 and -45° with the failure plane. These limiting stresses represent values that cannot be exceeded without yield occurring. In this paper, a study was done to investigate how the friction and attraction are mobilised by the plastic and elastic deformations to which the clay has been subjected. It is then possible to express undrained shear strength as a function of the consolidation stresses and two effective stress-strength parameters, the relative material attraction  $\chi$ , and the effective material friction angle,  $sin\varphi'_M$ . The proposed theory forms the basis for constructing so-called yield envelopes for a clay, based on these strength parameters.

### Introduction

It is well known today that active, undrained shear tests on soft clay undergo extremely small strains up to a certain stress level. This stress level represents a critical state that involves a transition from small elastic to large plastic strains. When exceeding this critical stress, the clay develops high pore pressures, a structural collapse and reduced strength. Such critical stress was found to be far less pronounced in passive undrained shear than in active undrained shear, where maximum shear resistance can appear at a deformation of several percent. The idea of the existence of such limiting or yield stresses has been discussed by several researchers before, for instance, Schofield and Wroth, 1968; Mitchell, 1970; Tavenas and Leroueil, 1977; Wood, 1980; Wroth and Houlsby, 1980; Graham and Houlsby, 1983; and Lau et al., 1988 Many have presented so-called "yield envelopes", which define critical combinations of shear stress and normal effective stress under active and passive shear. These envelopes may be constructed on the basis of stress paths determined in undrained or drained triaxial and plane strain tests. The points on the yield envelope are defined as the start of increased strain rates in drained tests and ultimate shear stress in undrained tests. However, it has been difficult to find an explanation for what these critical limits represent in reality, with respect to factors like friction and attraction or cohesion).

The objective of the present study is to provide a physical explanation on the phenomena, and to express the yield values as a function of effective stresses and a set of (somewhat revised) effective stress strength parameters, the friction and attraction (Aas, 1986).

# **Mobilization of friction**

To understand the principle for mobilizing of friction, one may look at the building up of active shear for an *in situ* clay element which has undergone an uniaxial vertical consolidation for the stresses  $\sigma'_{vo}$  and  $K_o \sigma'_{vo}$ , where  $\sigma'_{vo}$  is the effective vertical stress and  $K_o$  the coefficient of earth pressure at rest. This clay has not experienced any plastic strain in the lateral (horizontal) direction, and the horizontal stress has not contributed to the build-up of any frictional resistance. The vertical stress, however, has resulted in a vertical, plastic compression of the clay, and caused a mobilized friction along a 45° plane equal to  $1/2\sigma'_{vo} \sin\varphi'_M$ . The friction angle  $\varphi'_M$  is denoted *the material friction angle of* the clay. The term "material" indicates that the strength parameter is believed to be a pure material constant and, hence, independent of stress level, stress direction and stress history.

The stress conditions for this clay represent the final stage of a yielding process and are dictated from a simple requirement of static equilibrium. Hence, the mobilized frictional resistance needs to be equal to the applied shear stress:

$$s_{uA} = \frac{l}{2} (\sigma'_{vo} s_{uA} - \sigma'_{ho}) = \frac{l}{2} \sigma'_{vo} \cdot \sin \varphi'_{M}$$
<sup>(2)</sup>

which gives:

$$\sigma'_{ho} / \sigma'_{vo} = K_o = 1 - \sin \varphi'_M \tag{3}$$

Equation (3) is the well-known Jaky formula (Jaky, 1948), expressing the coefficient of earth pressure at rest for a normally consolidated soil.

### **Activation of attraction**

Most clays have in addition a contribution to shear strength due to attraction. This is believed to be resulting of net attractive forces acting between clay particles, like a tension reinforcement in the clay. The attractive forces increase with decreasing distance between the soil particles. Consequently, the attraction,  $\sigma'_a$ , is believed to reflect on the effective stresses to which the clay is subjected:

$$\sigma_a' = \chi \cdot \sigma_{na} \tag{4}$$

where  $\chi$  is *the relative material attraction* and  $\sigma'_{na}$  the value of the effective stress in the direction normal to the attractive force. Activation of the attraction is restricted to conditions of effective stress unloading and does not occur in connection with increasing effective stresses.

### Lower limiting stresses

Figure 1 describes the process leading to the limiting stresses under undrained shear. The clay element described above is first at a state described by the term "Primary consolidation". As a result of the mobilization of friction, the clay has mobilized an active shear resistance equal to  $1/2\sigma'_{vo} \sin\varphi'_{M}$ . The activation of the attraction makes it then possible for this clay to reduce the effective horizontal stress by an amount of  $\chi \cdot \sigma'_h$  in active shear. This operation, which does not involve plastic strain, leads to a lower limiting stress,  $\sigma'_{LLSA}$ :

$$\sigma'_{LLSA} = \sigma'_{vo} \left( 1 - \chi - \sin \varphi'_{M} \right) \tag{5}$$

and an active undrained shear resistance,  $s_{uA}$ :

$$s_{uA} = \frac{1}{2}\sigma'_{vo}(\chi + \sin\varphi'_M) \tag{6}$$

Figure 1 shows that if one adds a shear strain to the clay in order to mobilize friction due to also the horizontal stress, this results in high pore pressures, structural disturbance and failure, as shown in the uppermost and lowermost diagrams.



Figure 1. Mobilization of friction and attraction in active and passive shear.

Figure 1 also shows how one moves from the "Primary consolidation" condition to the lower limiting stress in passive shear. A shear stress relief resulting in  $\sigma'_v = \sigma'_{ho}$  involves only elastic strain and has no influence on the clay's ability to mobilize friction. A further vertical unloading, however, will result in a condition just opposite to that in the active shear above in Figure 1. This leads to a uniaxial vertical plastic expansion. At the point where the effective vertical stress reaches the value of  $\sigma'_v = K_0 \sigma'_{vo} (1 - \sin \varphi'_M)$ , the friction resistance is now balanced by the appearing passive shear stress, so that the friction resistance and the shear stress are both equal to  $1/2\sigma'_{ho} \sin \varphi'_M$ . The activation of the attraction will, in this case, lead to a lower limiting stress,  $\sigma'_{LLSP}$ :

$$\sigma'_{LLSP} = K_0 \sigma'_{vo} \left( l - \chi - \sin \varphi'_M \right) \tag{7}$$

Correspondingly, the passive undrained shear resistance,  $s_{uP}$ , becomes:

$$s_{uP} = \frac{1}{2}\sigma'_{vo}\left(\chi + \sin\varphi'_{M}\right) \tag{8}$$

In this case, a further straining in an attempt to mobilize friction, now due to the vertical stress, results in a structure collapse and reduced strength. The explanation on this behaviour in active and passive shear is believed to be that the transition from uniaxial to three-dimensional shear strain, represents a more brutal reorganization of the clay structure than experienced before.

### **Upper limiting stresses**

There exists also a set of upper limiting stresses which cannot be exceeded, without resulting in yielding and, at the same time, increasing strain. For the young, normally consolidated clay above, the upper limiting stresses in active and passive shear are  $\sigma'_{vo}$  and  $K_{0'}\sigma'_{vo}$ , respectively. However, if the clay has undergone an additional uniaxial vertical deformation due to, for instance, ageing, weathering or mechanical overconsolidation, it will react like a young, normally consolidated clay stressed to a vertical stress  $\sigma'_{vE}$  greater than  $\sigma'_{vo}$ , and experience a strain equal to that of the normally consolidated clay.

Hence, the upper limiting stresses in active and passive shear become:

$$\sigma'_{ULSA} = \sigma'_{vE} \tag{9}$$

$$\sigma'_{ULSP} = \sigma'_{vE} (1 - \sin \varphi'_M) \tag{10}$$

### **Yield envelopes**

Points on a yield envelope may be constructed in the following way:

One starts with the well-known Mohr Coulomb diagram and a clay for which the preconsolidation stress  $p'_c$  ( $= \sigma'_{vE}$ ) and the effective stress strength parameters,  $\chi$  and  $sin \varphi'_M$ , are known. One then chooses a consolidation condition, including the stresses  $\sigma'_{vo}$  and  $\sigma'_{ho}$ , and indicates the tests and effective stress paths for cases where one of the principal stresses is kept constant while the other one is either increased or decreased. As illustrated in Figure 2:

- Unloading cases indicated by the stress paths a-b and a-c end on the  $\chi$ -lines expressing the lower limiting stresses in active and passive shear, corresponding to Eq. (5) and (7).
- The stress path a-d showing the result of an increase in horizontal effective stress, ends on the line expressing  $\sigma'_v = \sigma'_{vE}(1 - \sin\varphi'_M)$  in accordance with Eq. (10).
- The stress path for the loading case a-e ( $\sigma'_v$  increasing) ends on the line B-C in the diagram in Figure 2, which defines that the ratio  $\Delta \sigma'_h / \Delta \sigma'_v$  equals  $(1 \sin \varphi'_M)$ , stating the limit for pure uniaxial and elastic strain under loading (no attraction involved).
- For high values of both  $\sigma'_{vo}$  and  $\sigma'_{ho}$ , the stress path f-g may end on the line expressing  $\sigma'_{vE}$ .

In the above examples, the vertical consolidation stress represents the major principal stress and does not exceed the value of  $\sigma'_{vE}(1 - \sin\varphi'_M)$ . Under this condition, the lines surrounding the area ABCDEFA (in red letters in Fig. 2) constitute the yield envelope for the considered clay. However, the yield envelope is not a definitely unique property of a given clay, as it may vary somewhat depending of the stress path chosen. For instance, for a horizontal unloading case starting with a vertical consolidation stress close to  $p'_c$ , the stress path h-i will end on the extended  $\chi$ - line, due to activation of the attraction (line B-C' in red letters in Fig. 2).



Figyre 2. A theoretical example of a yield envelope

# Conditions that have an influence on the upper limiting stresses

### Strain rate effect

Innumerable examples from the geotechnical literature have demonstrated that the undrained shear strength depends on strain rate, or, alternatively, the duration of failure load application. The above shear strength concept makes it possible to explain the strain rate effect in a rather simple way.

The secondary compression process involves a rate of settlement gradually decreasing with time. Consequently, the process could be related to strain rate, such that to a certain point of elapsed time, there corresponds a certain value of strain rate. This simply means that any aged clay would behave, in undrained shear, as a young, normally consolidated, if exposed to a strain rate exactly equal to the "current" value corresponding to the current age of the clay. If loaded faster than the "current" strain rate, the aged clay will show an apparent preconsolidation in triaxial and oedometer tests. Hence, recorded values of apparent preconsolidation stress,  $p'_c$  or upper limiting stress  $\sigma'_{ULSA}$ , are functions of how high the applied strain rate is in relation to the "age-governed" strain rate for the clay.

Figure 3 shows effective stress paths from four undrained, active triaxial tests on plastic Drammen clay, all reconsolidated to stresses below the *in situ* stresses (Berre, 1973). The applied rate of strain varied in the test series from 0.0015 to 35 % per hour. As can be seen, the lower limiting stress  $\sigma'_{ULSA}$  is the same in all tests, as it theoretically should be, and that it is an exclusive function of the *in situ* vertical effective stress.



Figure 3. Effects of secondary compression on apparent in situ effective stresses and undrained, active triaxial strength (plastic Drammen Clay)

The slowest test indicates a  $p'_c$ -value approximately equal to  $\sigma'_{vo}$  and, hence, a nearly young, normally consolidated clay. The other tests indicate increasing values of apparent horizontal effective stress and  $p'_c$  (or  $\sigma'_{ULSA}$ ) with increasing strain rate. These tests thus seem to demonstrate clearly that the strength parameters  $\sin\varphi'_M$  and  $\chi$  are fundamental with respect to strain rate.

The effects of secondary compression described above involve that recorded values of *in situ*, horizontal effective stress also are strain dependent.

### Sampling disturbance

Figure 4 illustrates the significant difference in the effective, active stress paths of Ellingsrud clay for a test specimen cut from a block samples and a test specimen from a 95-mm steel tube

sample. The specimens tested were from depths 7.3 and 13.1m. The tests on tube samples gave for this clay 25–30% lower undrained, active shear strength than the tests on block samples. The block samples were taken with the University of Sherbrooke cylindrical block sampler (Lefebvre and Poulin, 1979).

The explanation for the effect of disturbance is that the upper limiting stress in active shear indicated by the top point of the stress path for the block samples reduces significantly in the tube samples. This reduction nullifies the effect of ageing appearing in the block sample tests. Hence, it can be hypothesized that the disturbance and strength reduction in the piston sample, in this extremely quick and nearly attraction-free clay, is due to a plastic elongation of the tube sample in the vertical direction. Such an elongation will have a directly opposite effect to that of secondary compression on the upper limiting stresses in undrained shear.



Figure 4. Effective stress paths from undrained, active triaxial tests on 95-mm and block samples on Ellingsrud clay (Lacasse et al., 1985).

# **Examples of yield envelopes**

## **Backebol Clay**

Figure 5 shows the results from a series of oedometer and drained triaxial and plane strain tests for the determination of the yield envelope for the Backebol clay (Larsson, 1981). In the shear tests, one of the principal stresses was kept constant while the other one was either increased or decreased. Full symbols represent the yield values, and open symbols the failure values. Failure was possible to be determine in unloading tests only. It should be remembered that a test procedure involving effective stress unloading alone provides a necessary condition to activate the attraction of the clay.

At the sampling depth for this test series, the effective vertical stress was about 30 kPa, whereas the preconsolidation stress,  $p'_c$ , recorded in oedometer tests was 48 kPa. Effective horizontal stress measured with Gløtzl-cells and pressuremeter was 25 kPa. The clay was highly plastic ( $I_p = 56\%$ ) and showed signs of weathering at the sampling depths.

With the effective stress strength parameters  $sin\varphi'_M = 0.35$  and  $\chi = 0.40$  for the Backebol clay, all the yielding and failure points in Figure 5 may be fitted to a yield envelope consisting of:

- the upper limiting stress in active shear,  $\sigma'_{vE}$ ;
- the upper limiting stress in passive shear,  $\sigma'_{vE}(1 \sin \varphi'_M)$ ; and
- the two  $\chi$ -lines.

These test results represent an extremely good confirmation of the ideas on which the yield envelope was based upon in Figure 2.



Figure 5. Yield envelope for Backebol clay-

### **Drammen Clay**

Figure 6 shows a yield envelope constructed for a plastic Drammen clay (Berre, 1975). The specimen were first consolidated to the same effective stresses as the clay had been exposed to *in situ* (Point 1 in Fig. 6). The following drained triaxial tests were then performed:

- 1) The vertical stress was kept constant, and the horizontal stress was decreased (path 1-2)
- 2) The vertical stress was increased, and the horizontal stress was kept constant (path 1-3).
- 3) Both vertical and horizontal stresses were increased, such that the specimen area remained constant ( $K_0$ -test, path 1-4).
- 4) The vertical stress was kept constant, and the horizontal stress was increased (path 1-5).
- 5) The vertical stress was decreased, and the horizontal stress was kept constant (path 1-6).

Points 2 and 6 are failure in the unloading tests as the stress paths reach the upper and lower yield envelopes respectively. The vertical stress at Point 4 equals the upper yield stress  $p_c' (= \sigma'_{vE})$ . The calculated  $K_0$ -value in the current study deviates from the value suggested by Berre (1975), and indicated by the line 1–4 in Figure 6.



Figure 6. Yield envelope for Drammen plastic clay.

Tests by Ramanatha (1975) on the same plastic Drammen clay ( $I_p = 34\%$ ) looked at drained triaxial active and passive tests on undisturbed clay under relatively low effective stresses. These results shown as cross marks in Figure 6. These fall closely to the yield envelope from the undrained tests.

With the effective stress strength parameters  $sin \varphi'_M = 0.37$  and  $\chi = 0.34$  for the plastic Drammen clay ( $I_p = 34\%$ ), the yielding and failure points in Figure 6, in spite of a limited number of test results, can be fitted to a yield envelope consisting of:

- the upper limiting stress in active shear,  $p'_c$ ;
- the upper limiting stress in passive shear,  $p'_c (1 -\sin \varphi'_M)$ ; and
- the two  $\chi$ -lines.

The diagram in Figure 6 deviates from the original version published by Berre (1975) in that the  $\chi$ -lines now replace similar lines which indicated a friction angle equal to 30°.

### **New Jersey Clay**

Figure 7 shows the yield envelope constructed for the marine plastic New Jersey clay ( $I_p = 43\%$ ). The clay had an overconsolidation ratio of 3 to 4. The shear testing program involved, among others, undrained active and passive triaxial and plane strain tests on undisturbed clay samples.



Normalized effective stress, p/op

Figure 7. Yield envelope for New Jersey plastic clay (Koutsoftas and Ladd, 1984) TC and TE: triaxial compression (active) and extension (passive) tests; PSC and PSE: plane strain compression (active ) and extension (passive ) tests; At  $q_f$  = at peak shear stress; At MO = at maximum obliquity;  $q = (\sigma_1 - \sigma_3)/2$ ;  $p = (\sigma'_1 + \sigma'_3)/2$ ;  $\sigma'_p$  = preconsolidation stress; OCR = overconsolidation ratio.

Based on the SHANSEP approach (Ladd and Foott, 1974; Ladd *et al.*, 1977), developed to minimize the effects of sampling disturbance on the normalised shear strength parameters, all specimens were consolidated well beyond the *in situ* preconsolidation stress and then unloading to varying overconsolidation ratios. For compression (active) testing, the vertical stress was increased. For extension (passive) tests, the vertical stress was decreased. Both tests maintained constant cell pressure.

In Figure 7, four sets of active and passive effective stress paths are shown, where the last ones ended on the passive  $\chi$ -line, while the active tests seem to land on the B–C line (see also Fig. 2). The points for two failure criteria are shown: at maximum shear stress ( $q_f$ ) and at maximum obliquity (MO, where the ratio of the major and minor principal streeses  $\sigma'_1/\sigma'_3$ ) is maximum).

The calculated effective stress shear strength parameters are  $sin \varphi'_M = 0.37$  and  $\chi = 0.39$  for the New Jersey plastic clay.

### **Boston Blue Clay**

Figure 8 shows then yield envelope adapted to the test results for Boston Blue clay. The clay had approximately 50% clay size (less than two µm particle size) and a plasticity index of 21%.



Figure 8. Yield envelope for Boston Blue clay  $\sigma'_p = preconsolidation stress; OCR = overconsolidation ratio; K_{ONC}=K_0$  for normally consolidated clay (data after Jamiolkowski et al., 1985; O'Neill, 1985)

Batches of resedimented Boston Blue clay were prepared by consolidating a clay slurry under conditions that resulted in strength and consolidation properties similar to those of natural Boston Blue Clay. The consolidated cake was extruded and stored in transformer oil prior to testing. Specimen were then cut from the cake and subjected to consolidated undrained plane strain tests under a maximum effective consolidation stress of about 40 kPa. The test program included tests run on samples with nominal overconsolidation ratios (ratios) of 1, 2 and 4.

The calculated effective shear strength parameters were  $sin\varphi'_M = 0.46$  and  $\chi = 0.29$ .

## James Bay, B-6 Marine Clay

At 12 meter depth, the James Bay, B-6 marine clay had a natural water content of about 40%, a plasticity index of 10%, a sensitivity of 280 and a clay fraction (less than 2  $\mu$ m particle size) of 71%. The clay was overconsolidated, probably partly due to weathering.

Figure 9 shows the results from active and passive triaxial tests on normally consolidated specimens of this brittle clay.



Figure 9. Yield envelopes for the James Bay B-6 Marine Clay.



The following consolidation conditions, based on a stipulated value of  $K_0 = 0.55$  were used:

- 1) Consolidation to  $\sigma'_{vc} = \sigma'_{vo}$  on specimens at five depths, to measure the behaviour of "intact clay" at the *in situ* overconsolidation ratio (OCR), which varied from 1.8 to 3.3.
- 2) Consolidation to  $\sigma'_{vc}/\sigma'_p$  between 0.6 and 1.0 at two depths to measure the effect of recompression on intact clay.
- 3) Consolidation to  $\sigma'_{vc}$  ranging from 1.3 to 3 times the *in situ* preconsolidation stress  $\sigma'_p$  to measure the behaviour of "de-structured" clay (Leroueil *et al.*, 1979).

The yield envelopes on Figure 9 were constructed on the basis of the active triaxial peak strengths and effective stress paths. The following additions were done: the revised yield envelopes were drawn for both the undisturbed and the de-structured clay. With the help of passive triaxial effective stress paths from a sample at depth 12 m (Koutsoftas and Ladd, 1984), it was possible to add also the passive parts to the envelopes.

The calculated effective stress shear strength parameters were  $\sin \phi'_M = 0.60$  and  $\chi = 0.26$  for the undisturbed clay, and  $\sin \phi'_M = 0.60$  and  $\chi = 0.12$  for the disturbed clay. The large reduction in attraction can be due to a certain destruction of the clay fabric, leading to a reduction of effective clay particle contacts.

## James Bay, Olga Clay

The clays from the James Bay district are often described as overconsolidated due to some type of brittle cohesive bonds, due to, e.g., cementation. Earlier experience indicates that such bonds might be destroyed as a result of de-structuration or the application of stresses well above the preconsolidation stress. Test results for a second James Bay clay deposit (NGI, 1977a; 1977b) might throw further light on this phenomenon. The Olga Clay had a plasticity index of 10%.

Figure 10 presents the results of laboratory tests conducted on block samples of Olga clay from about 7 m depth. Leftmost in the diagram, the effective stress paths from active and passive plane strain and triaxial tests on specimens reconsolidated to the *in situ* stresses are shown. The result from a vane test at a corresponding depth is also shown.

The plane strain tests, supported by the 'net vane strength' data  $(s_{uV} - s'_{uV})$ , give the following effective strength parameters:  $sin\varphi'_M = 0.45$  and  $\chi = 0.31$ . The parameters  $s_{uV}$  and  $s'_{uV}$  denote the average vane shear strength and the average remoulded vane shear strength. The difference in the Olga strength values compared with the B-6 Marine Clay above is a result of different plasticity, 25% versus 10%. The two types of active tests (triaxial and plane strain tests) gave nearly identic result, whereas the triaxial test in passive shear gave a very low value, probably due to testing shortcomings.

Figure 10 also contains active, effective stress paths from an active/plane strain and a triaxial test, where both specimens were consolidated under a vertical stress equal to  $3.5\sigma'_{vo}$ . This consolidation stress exceeded the apparent preconsolidation stress measured in an oedometer tests. It is interesting to note that these tests suggest a normally consolidated clay exhibiting the same effective stress shear strength parameters  $sin\phi'_{M}$  and  $\chi$  as the weathered clay. At least for this clay, the test results indicate that the shear strength can be expressed on the basis of a set of two "true" strength parameters and the existing effective stress conditions.



Figure 10. Determination of effective stress strength parameters for Olga clay at depth 7 m.

# Effective shear strength parameters

The effective stress shear strength parameters for the six clays reported above were compared with an earlier published relationship (Aas *et al*, 1986; Aas and Lacasse, 2021) between the effective stress shear strength parameters and the clay's plasticity index. Figure 11 shows that the results from these new clay sites agree well with the earlier publications. Table 1 summarizes the values of the effective stress shear strength parameters obtained for the six clays in this paper. Except for the undisturbed cemented James Bay B-6 clay, the sum of the effective stress parameters  $\chi + \sin\varphi'_{M}$  is between 0.71 and 0.76, with an average of 0.74, in agreement with the data presented earlier in Aas and Lacasse (2021a; b).

Clay	Attraction parameter,	Friction parameter,	Sum
	χ	$\sin arphi'_M$	$\chi$ + sin $\varphi'_M$
Backebol	0.40	0.35	0.75
Plastic Drammen	0.34	0.37	0.71
Plastic New Jersey	0.39	0.37	0.76
Boston Blue	0.29	0.46	0.75
James Bay B-6 ("undisturbed")	0.26	0.60	0.86
James Bay B-6 ("disturbed")	0.12	0.60	0.72
Olga	0.31	0.45	0.76

Table 1. Effective stress shear strength parameters for the six clays investigated in this paper



Figure 11. Friction and attraction effective stress parameters as functions of plasticity index, Ip.

# Conclusions

Yield stress has often been described as a shear stress limit for a clay, representing failure or a sudden transition from small elastic to large plastic strains. It is important, however, to recognise that the general definition of a so-called yield envelope is not about strain magnitude. The yield envelope should be rather linked to relations between deformations and mobilization of the effective shear strength parameters of the clay, attraction  $\chi$  and friction angle  $sin\varphi'_M$ . The activation of the attraction is restricted to unloading cases, and the mobilization of friction follows strict rules with respect to appearing simultaneously with plastic deformations. Actually, yielding occurs before friction has been mobilized due to the horizontal stress under active shear and the vertical stress under passive shear.

The upper boundaries of a yield envelope depend on the rate of strain and the degree of sample disturbance. The test results in this paper indicate, however, that the rate of strain and disturbance only affect the upper limiting stresses, and have no influence on the lower limiting stresses.

A yield envelope, as suggested in this paper, can be used to determine the preconsolidation stress and the effective stress shear strength parameters of the clay; or vice versa, the yield envelope can be constructed on the basis of the above mentioned soil parameters. Five examples on yield envelopes constructed from the literature, were analysed. It was possible to demonstrate a common yield envelope pattern for the five different clays.

The effective stress shear strength parameters determined for these clays fit well with the model presented by Aas and Lacasse (2021), expressing the attraction  $\chi$  and friction angle  $sin\varphi'_M$  parameters as a function of the plasticity index,  $I_p$ . Although the values of  $\chi + sin\varphi'_M$  vary somewhat for the six clays, the sum of the two parameters  $\chi$  and  $sin\varphi'_M$  was invariably close to 0.8 as observed earlier.

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