Mathematical modeling of dense avalanches

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ABSTRACT. The paper presents a brief overview of mathematical models for dense avalanches that were developed by researchers of Moscow State University. Examples of calculations for real avalanches in Caucasus and Khibiny Mountains are given.

MODELS AND ANALYTICAL FORMULAE

Models

Mathematical models for dense avalanches described here are based on the fact that usually the length of an avalanche is much larger than its thickness. So an approach similar to that applied in shallow water theory or in hydraulics can be used. The coupled partial differential equations for an avalanche flow differ from those for an unsteady flow of water due to 1) inclusion of dry component of friction together with hydraulic one (as have been proposed by Voellmy (1955)) and 2) special conditions at the leading edge of an avalanche taking into account that while the avalanche snow behaves like a "liquid", the snow in a snowpack in front of an avalanche does not (this was not taken into account by Voellmy).

The simplest variant of equations for a 1D-motion on a wide slope has the form (Grigorian, Eglit, Yakimov 1967; Eglit 1968)

$$\frac{\partial h}{\partial t} + \frac{\partial hv}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = g\sin\psi - \frac{1}{2h}\frac{\partial}{\partial x}\left(h^2 g\cos\psi\right) - \left(f_1 + f_2\right)$$

Here f_1 and f_2 denote the dry and hydraulic friction, respectively:

$$f_1 = \mu\, g\cos\psi\; sign(v); \quad f_2 = k\frac{v^2}{h} sign(v)$$

In these relations t and x are the time and the coordinate along the slope; $v(x,t)$ and $h(x,t)$ are the velocity and thickness (depth) of the flow; $\psi\,(x)$ is the slope angle; μ is the coefficient of dry friction, k is the coefficient of hydraulic turbulent friction.

Boundary conditions for the equations (1) are conditions at the leading ($x = x_f$) and tailing ($x = x_0$) edges of an avalanche. In the simplest model the snow cover is assumed to be incorporated into an avalanche in its narrow front zone. This zone is modeled by a "hydraulic" jump. Boundary conditions at the front $x = x_f$ follow from the mass and momentum conservation laws:

$$\rho\,\bar{h}\left(w - \bar{v}\right) = \rho_0 h_0 w \qquad (2)$$

$$\rho_0 h_0 w\bar{v} = \frac{1}{2}\rho\bar{h}^2 g\cos\psi - \sigma^* h_0 \equiv P \quad at \quad P \geq 0$$

Here w is the speed of the avalanche front, $\bar{h}$ is its height and $\bar{v}$ is the snow velocity on the front; σ^* is the compressional strength of the snowpack layer involved into motion, h_0 is its thickness and ρ and ρ_0 are densities of snow in the avalanche and in the snow cover. If $P < 0$, i.e., the force acting from the side of the avalanche on the snow cover is insufficient to disrupt it, then instead of the second relation in (2) one of the following conditions can be used

$$w = \bar{v} = 0;\ h_0 = \frac{\rho\,\bar{h}^2 g\cos\psi}{2\sigma^*};\ w = \bar{v},\ \bar{h} = h_0 = 0$$

These conditions mean the stop of the front, motion with entrapment of only part of the snow cover, and motion without snow entrapment, respectively. Calculations show (see Eglit, 1983) that any of these conditions give almost the same run-out distance.

At the tailing edge the conditions $h = 0$, $v = 0$ are assumed.

Later the following modifications and generalization of the model were made.

1. Possible entrainment of snow by all parts of an avalanche (not only by the front part) was included (Ostroumov, 1972; Eglit , 1983).

2. Modified expressions for the friction force were suggested concerning both dry and hydraulic friction components.

2.1. The Coulomb's law for dry friction was modified by introduction of an upper limit for the value of dry friction: the shear stress is equal to $\mu\ p$, where μ is the friction coefficient and p is the pressure, only unless $\mu\ p$ is not larger than the minimal shear strength τ_* of the materials of the avalanche and of the snowpack.

Otherwise it is equal to τ_* (Grigorian, 1979) . So for a bottom friction we have

$$|f_1| = \mu\, g \cos\psi \quad at \;\; h \le h_*, \;\; h_* = \tau_* / \mu \rho g \cos\psi$$

$$|f_1| = \tau_* / h \quad at \;\; h > h_* .$$

Besides it was taken into account that the friction force for stopped parts of an avalanche can be less than the dynamic friction. It is equal to an active force - the sum of gravity force and the pressure gradient (Eglit, 1982, 1983).

2.2. The expression for the hydraulic friction was also changed. It was taken to be proportional to the square of velocity v^2 at $\mathrm{Re} \ge \mathrm{Re}_{cr}$ and to v at $\mathrm{Re} \ge \mathrm{Re}_{cr}$, where Re is the Reynolds number (Eglit, 1983).

3. The equations for motion in chutes with different cross-sections were written, investigated and used (Ostroumov, 1972; Danilova and Eglit , 1977a; Mironova, 1987).

4. Additional terms that account the curvature of the slope were included into the equations (Eglit 1982, 1983).

5. The two - dimensional motion of an avalanche was considered (Kulikovsky and Eglit, 1973; Mironova, 1987). The lateral spreading of an avalanche is taken into account in a 2D-model and the boundaries of an avalanche run-out zone can be calculated in details.

6. The interaction of dry dense avalanche with an ambient air and formation of a snow powder cloud above it was considered. A two-layer model was constructed to simulate this process (Eglit, 1983; Nazarov, 1992).

Analytical investigation of the models

The models were investigated analytically. In particular, the stability and formation of rolling waves was studied. The approximate analytical formulae were obtained for dense avalanche motion on long steep homogeneous or slowly varying slopes and chutes (Bakhvalov, Eglit, 1970; Bakhvalov et al., 1974, Danilova and Eglit, 1977a,b; Mironova, 1987). These formulae can be used to estimate avalanche parameters without complicated calculations.

It is worth mention that the approximate formula by Voellmy (1955) relating the velocity and thickness for a 1D avalanche at a long constant slope

$$v^2 = \xi\, h\,(\sin\psi - \mu \cos\psi)\,, \qquad \xi = g/k$$

is proved to be valid for tailing parts of an avalanche and not valid for its front part if the snow entrainment takes place.

CALCULATIONS

Numerical investigation of the models

Calculations by the models can be divided into two groups. The first group contains numerical investigation of the models and calculations for different idealized slopes. The second group deals with real avalanches on real mountain slopes. The aim of calculations of the first group was to estimate the effect of different terms and coefficients of the equations as well as the initial conditions and the morphometric parameters of the slope. In particular, the following questions can be answered: 1) what parameters effect the most essentially on an avalanche dynamics; 2) with what accuracy should the parameters of the snow and the slope be measured to obtain the given accuracy of the values of an avalanche parameters? An example of the results is given in the Tables 1, 2 (Blagoveschenskiy, Eglit, 1985).

Table 1. *Errors in the starting zone data leading to the errors in* v *and in* h *less than* Δv *and* Δh.

Errors in v and h	$\frac{\Delta A_1}{A_1}$ %	$\Delta\psi_1$	$\Delta\varphi_1$	$\frac{\Delta h_1}{h_1}$ %
$\Delta v = \pm 1\, m/s$	20	2°	10°	20
$\Delta h = \pm 1\, m$	20	4°	10°	20

Here ΔA_1, $\Delta\psi_1$, $\Delta\varphi_1$, Δh_1 are the errors in values of the area of an avalanche starting zone, the angle of the inclination of this zone, its convergence angle and the thickness of a snow cover in a starting zones, respectively.

Table 2. *Errors in the input data leading to the errors in* v *and in* h *less than* Δv *and* Δh.

Errors in v and h	$\Delta\psi_2$	$\Delta\psi_3$	$\Delta\varphi_3$	$\Delta\mu/\mu$ %	$\Delta k/k$ %
$\Delta v = \pm 1\, m/s$	2°	2°	30°	5	10
$\Delta h = \pm 1\, m$	4°	4°	35°	10	20

In the Table 2 $\Delta\psi_2$, $\Delta\psi_3$, $\Delta\varphi_3$, $\Delta\mu$, Δk are the errors in values of the inclination of the chute, the angle of the inclination in the run-out zone, the angle of lateral spreading in the run-out zone , the dry and hydraulic friction coefficients, respectively.

Numerous calculations with different combinations of values of the parameters and comparison with the data known from observations permitted to obtain the possible ranges of model coefficients. The results of test calculations were used for constructing regional empirical formulae for dynamical parameters and run-out distances of avalanches (Blagoveschenskiy, Mironova and Eglit, 1995).

Calculations with random values of friction coefficients and avalanche volumes made for a given avalanche site were used to obtain the probability of avalanche parameters. This work was done for one avalanche site in Zailiyskiy Alatau Mountains. For initial logarithmic normal distribution of avalanche volumes and normal distribution of friction coefficients the distribution of the front heights was close to normal one and the distribution of velocities was asymmetric with right hand side asymmetry (Blagoveschenskiy, Mironova and Eglit, 1995).

Simulation of real avalanches

The detailed data about dynamical parameters of moving avalanches are still very poor. The values of the flow thickness can be in some cases obtained by measurement of the traces left on the walls of a chute; the velocities can be estimated by measurement of the asymmetry of the traces in the points of turn of the path. Several avalanches were recorded by stereophotogrammetric methods (Brukhanov, 1967; Samoilov, 1977). Some of them were simulated with the use of models described above. The other calculations were based on the data about the boundaries of avalanche deposits and snow distribution before and after the avalanche event only.

One of the simulated avalanches is a so-called "Home avalanche" that descends from the slope of the Mount Cheghet (Elbrus region, central Caucasus) several times every winter. Moscow State University Research Station is situated quite close to this avalanche path. The data about it can be found in (Zolotariov, 1970). The Home avalanche was simulated by A.V. Ostroumov (1972) with the use of 1D model. The Home avalanche catchment consists of three sections: the starting zone that has a form of a great funnel, the transit zone - a channel, and a run-out zone that has a form of a cone. The author assumed that the snow in a starting zone moves along the lines of the greatest descend, replaced it by a set of channels bounded by these lines , and calculated the velocity and flow thickness at the end of the starting zone using 1D model for a flow in such a representative channel. In the run-out zone the motion was supposed to occur along the elements of the cone being one-dimensional in polar coordinates. A.V. Ostroumov found that the values $\mu = 0.45, k = 0.05$ and $\sigma^* = 20t/ms^2$ lead to a good agreement of calculated and measured path of the avalanche.

Ostroumov simulated also avalanches in a catchment Tubri (Low Svanetia, central Caucasus) with the aim to prove the project of a dam to protect a village. The values of coefficients he used were $\mu = 0.22, k = 0.08$ and $\tau_*/\rho = 16m^2/s^2$ (Grigorian and Ostroumov, 1975).

Simulation of two observed wet avalanches in Apkhiz (west Caucasus) - for two similar avalanche catchments II and III above the Moon glade – was made by E.M. Mironova using the data obtained by Volodicheva and Oleinikov (Volodicheva et al.1986). Calculations for starting zones and channeled parts were made by 1D model. 2D model was applied to calculate the motion in the run-out zones. The data about the avalanche II were used to find the values of model coefficients $\mu = 0.3$, $k = 0.1$, $\tau_*/\rho = 30m^2/s^2$. With these values the motion of the avalanche III was calculated and good agreement with the observed data was obtained concerning the run-out distance, lateral spreading and the deposits area. Calculations for different possible values of avalanches volumes were also made and used to correct the avalanche maps drawn for this region by geographical methods.

The similar method (1D model for chute + 2D model for run-out zone) was applied by Mironova to simulate a dry avalanche recorded by Samoilov (1976) in Khibiny mountains in the avalanche catchment 22. The values of coefficients were found to be $\mu = 0.245$, $k = 0.05$, $\tau_*/\rho = 10m^2/s^2$.

On January 9, 1987 a great avalanche that was called Koghutaiskaya descent from the Mount Koghutai (3819m), Elbrus region. It reached a hotel standing at the foot and destroyed several small buildings. Detailed measurements of avalanche deposits thickness and area were made immediately after the event. The data about snowpack and meteorological conditions were recorded (the mean temperature at 3000m a.s.l. was -11.2°C). The map in scale 1:10000 was also available for the region. Similar large avalanches were observed at the place in 1932 and 1954. Their traces can be seen even now. Simulations of all these avalanches were made by Mironova by 1D and 2D models. To obtain a good agreement with the measured run-out zone boundaries, she modified the equations to include successive layered deposition of the snow in a deposit zone (Volodicheva et al. 1990). The values of coefficients for an avalanche 1987 were taken to be $\mu = 0.2$, $k = 0.02$ and $\tau_*/\rho = 10m^2/s^2$.

Simulations of dry avalanches with formation of snow powder clouds were made by Nazarov (1992) by 1D two-layer model. A Home avalanche (Elbrus region) and an avalanche in Khibiny Mountains were calculated. Both avalanches produced a powder cloud that continued to move after stopping of the dense part. For Home avalanche the dynamical pressure and its variation in time and space were compared with the measured values (Grigorian, Urumbaev, 1975). For Khibiny avalanche the calculated variations of the thickness and velocity of the avalanche front during motion was compared with the measured ones (Samoilov 1976). The agreement was good at $\mu = 0.4$, $k = 0.02$ and $\tau_*/\rho = 6m^2/s^2$ for Home avalanche and $\mu = 0.25$, $k = 0.02$ and $\tau_*/\rho = 10m^2/s^2$ for Khibiny avalanche. One should have in mind that the two-layer model contains a number of extra coefficients besides μ, k and τ_* for a dense layer. These coefficients determine the forces and mass transfer at the dense layer - powder layer and at avalanche - ambient air boundaries.

CONCLUSION

A great work has been done to elaborate models for estimation of dense avalanche dynamic parameters. Still more computations of real avalanche are needed to establish reliable relations between model coefficients values and conditions at a slope.

At present the computer simulations of dense avalanches in Russia are not usual in engineering practice. This is connected mainly with the economical and political reasons. Let us hope that the situation will be much better in future.

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