# Dynamics of two-layer slushflows

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**ABSTRACT.** A new mathematical model of slushflow dynamics is developed. A slushflow is treated as a two-layer flow. The lower layer consists of pure liquid phase (water) and the upper layer is a floating water saturated snow (slush). The equations of mass and impulse conservation for each layer are written. These equations include an interaction and mass exchange between the layers and between the water or slush layer and snow cover. The model equations were integrated numerically according to a developed for PC program. Series of numerical experiments for uniform slope were carried out. The structure and dynamics of slushflow were investigated. The dependencies of depths, velocities of flow and front coordinates of the upper and the lower layers on the parameters of the model are established. These parameters are the coefficient of snow entrainment, the coefficient of dry friction, the coefficients of turbulent friction, the discharge of water feeding at the rear end of flow, the snow cover thickness and the slope angle. An effect of exhaustion of the water layer is revealed. This effect is due to fast water absorption by entrained masses of snow.

## INTRODUCTION

Slushflows, like debris flows and snow avalanches, belong to destructive gravitational avalanche-type flows (Sapunov, 1985; Onesti and Hestness, 1989, Perov, 1995). The mechanism of their source is basically connected with percolation of melt and/or rain water in snow cover and resultant rising of bottom channel of runoff. Slushflow generation is due to longitudinal pressure gradient of water (Sapunov, 1991; Gude and Scherer, 1995). Usually, slushflow occurence accompanies the motion of one wave-surge in channel. However, P.A.Chernouss observed the pulse motions of slushflows in Khybiny area (personal communication). The similar events were observed in Swedish Lappland (Barsch and others, 1993 ). Structure and dynamics of slushflows are not well studied. There were attempts to use the empirical formula (Sapunov, 1991) or the simplest theoretical estimations of slushflow velocity. For example, the formula for steady motion of snow avalanche down a uniform slope, the formula for velocity of water flow in open channels (Barsch and others, 1993) or the formula based on the estimations according to lateral inclination of free surface of flow in curved sections of path were used (Sapunov, 1991). Recently the mathematical hydraulic-type model describing the motion of slush masses in the trapezoidal cross section channel was developed (Bozhinskiy and others, 1996). The model was in good agreement with the field data in Khybiny area. At the same time slushflows are, in their nature, two-phase flows. Interaction of phases in flow transforms flow structure, changes its dynamics and leads to some effects such as pulse-motion or stratified motion.

In this paper, an attempt is made to investigate structure and dynamics of slushflow treated as a two-layer flow. In other words, the interaction of phases in the flow is approximated with the interaction of two layers. The lower layer consists of pure liquid phase, namely, water and the upper layer is a floating saturated with water snow. Phase transitions are not considered because of short slushflow release (of order several minutes). The interaction of the layers is taken into account by turbulent friction, gradient pressure on the interface boundary and mass exchange. The entrainment of new snow masses is considered. It is assumed the entrained snow instantly saturates with water and floats to the upper layer. Thus the density of the upper layer is variable. The rear end of the flow can be feeded by water accumulated in a snow basin during spring snow melting or intense rainfalls. The flow releases in a rectangular cross section channel. The model is a hydraulic-type one, i.e. all dynamic characteristics of the flow are averaged over the depth of the flow cross section.

## GOVERNING EQUATIONS

Let us write the basic equations of the model.

**Water layer (see Fig.1a):** $x \leq x_{wf}$.

The mass conservation equation is

$$(H_w)_t + (H_w U_w)_x = -q_b P - \theta(P - P_w). \quad (1)$$

The momentum equation is

$$(H_w U_w)_t + (H_w U_w^2)_x = -U_w[q_b P + \theta(P - P_w)] + gH_w \sin\psi -$$
$$-(g/2)(H_w^2 \cos\psi)_x - (\rho_s/\rho_w) g H_w (H_s \cos\psi)_x +$$
$$+ k_{sw}(\rho_s/\rho_w)(U_s - U_w)|U_s - U_w| - k_w U_w |U_w|. \quad (2)$$

**Slush layer:** $x \leq x_{sf}$.

The volume conservation equation is

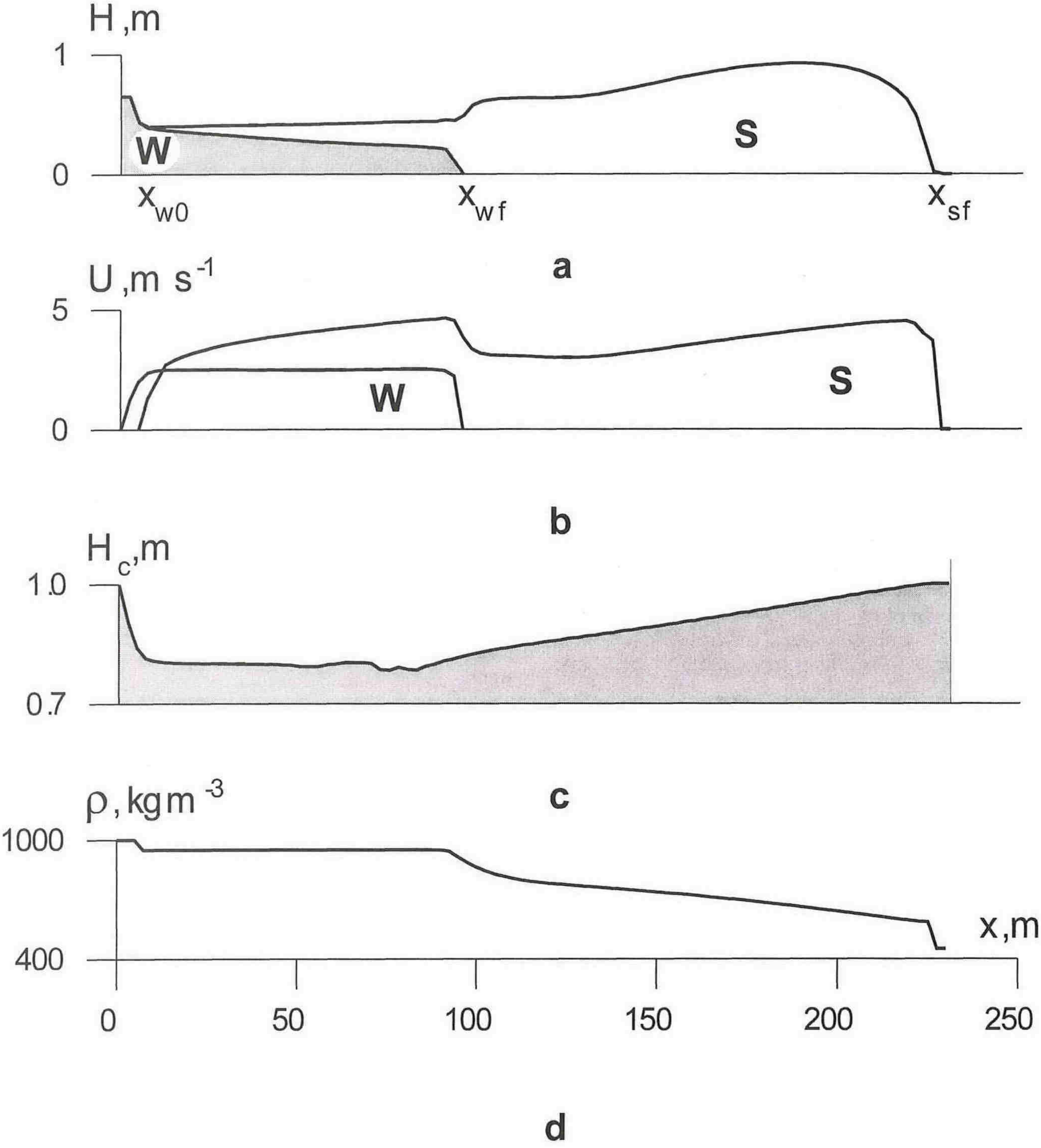


*Fig. 1. Typical distributions, over the length of channel, of structure and dynamic characteristics of two-layer slushflow, obtained by numerical modelling; t = 32 s.*

$$(H_s)_t + (H_sU_s)_x = q_b. \tag{3}$$

The mass conservation equation is

$$(\rho_sH_s)_t + (\rho_sH_sU_s)_x = q_b[\alpha\rho_wP + \rho_i(1-P)] + \alpha\rho_w\theta(P-P_w). \tag{4}$$

The momentum equation is

$$(\rho_sH_sU_s)_t + (\rho_sH_sU_s^2)_x = \alpha\rho_wU_w[q_bP + \theta(P-P_w)] + \rho_sgH_s\sin\psi - (g/2)(\rho_sH_s^2\cos\psi)_x - \rho_sgH_s\cos\psi(H_w)_x - \beta_sf\rho_sgH_s\cos\psi - \beta_wk_{sw}\rho_s(U_s-U_w)|U_s-U_w| - \beta_sk_s\rho_sU_s|U_s|. \tag{5}$$

Here $H$, $U$ are the depth and the velocity of the layer, respectively, $\psi$ is the local slope angle, $\rho$ is the mass density of the layer, $P$ is the snow porosity, $q_b$ is the volume rate per unit bottom area, $P_w$ is the water content of slush, $g$ is the gravitational acceleration, $\theta$ is the intensity of water percolation from the lower layer to the upper one, $k_s$, $k_w$, $k_{sw}$ are the drag coefficients of turbulent flow of water and slush over snow cover surface and over slush-water interface, respectively, $f$ is the dry friction coefficient, $\alpha$, $\beta$ are the trace-factors: $\alpha = 1$, when $x \leq x_{wf}$, $\alpha = 0$, if $x_{wf} \leq x \leq x_{sf}$; $\beta_w = 1$, $\beta_s = 0$, when $x \leq x_{wf}$ and $\beta_w = 0$, $\beta_s = 1$ if $x_{wf} \leq x \leq x_{sf}$, $x$ is the down slope coordinate, $t$ is time. The indices $w$, $s$ refer to the water layer and the slush layer, respectively, the index $f$ refers to the front coordinate. The round brackets with attached lower index designate the partial derivative with respect to variable, marked by this index.

The density and the porosity of the slush layer are connected with the relation

$$\rho_s = P_w\rho_w + (1 - P)\rho_i, \quad 0 \leq P_w \leq P, \tag{6}$$

where $\rho_i$ is the ice density.

Intensity of snow mass entrainment is assumed to be proportional to product of the velocity of the eroding layer and its relative density:

$$q_b = m_e U_\varepsilon(\rho_\varepsilon/\rho_w) \tag{7}$$

where $m_e$ is the dimensionless coefficient of entrainment and $\varepsilon$ takes the value $w$, $s$ on dependence of what layer (water or slush) erodes snow cover.

The thickness of snow cover $H_c$ diminishes during the motion. This process is described by the equation

$$(H_c)_t = -q_b. \tag{8}$$

The following boundary and initial conditions are imposed for the problem formulated above, describing the motion of two-layer slushflow. The left (rear) boundary is treated as a dam, therefore,

$$H_s = 0;\ U_w = U_s = 0;\ (H_w)_x = 0, \quad \text{at } x = 0. \tag{9}$$

A possible external water feeding at this boundary is prescribed by the point source with the specific discharge $Q(t)$. The right (frontal) boundary is free. As initial conditions, distributions, over a fixed length of channel, for all field quantities are prescribed.

The model equations were integrated numerically according to a developed for PC program. This program was preliminary tested against problems for which the analytical solutions are possible. Based on the developed two-layer slushflow model, several series of numerical computations for determining the structure and dynamic characteristics of flow were carried out.

## STRUCTURE AND DYNAMICS OF SLUSHFLOW

The structure of slushflow is shown in Fig.1. Generally, the slush surge, the front coordinate is $x_{sf}$, moves ahead. The rear part (tail) of the flow consists of pure water layer, the front coordinate is $x_{w0}$. Thus the frontal and the rear parts of the flow are essentially one-layer streams. The middle part of the slushflow body ($x_{w0}$, $x_{wf}$) consists of two layers. The upper layer is the slush one and the lower layer is the water one. Both of them wedge in the vicinities of the fronts $x_{w0}$ and $x_{wf}$, respectively, (Fig.1a). One can note that such structure of flow relates to developed slushflow. At the initial period of motion the rear part of flow is also a two-layer stream, i.e. $x_{w0} = 0$. The velocity distributions along the flow are shown in Fig.1b. It is seen the velocity of the slush surge has two maxima. One of them is located near the surge front but the other takes place near the front of the water layer. Here the slush layer rapidly slides down the water layer. The water layer has the more uniform velocity distribution with one smooth maximum in the middle. The distribution of snow cover in the channel is depicted in Fig.1c. One can see the remarkable amount of snow cover that is removed and entrained in the motion along the rear part of the flow. On the other hand, the snow cover thickness near the front $x_{sf}$ approaches the prescribed initial distribution. Figure 1d illustrates the distribution of the upper layer density. Obviously, the flow density is equal to the water density in zone (0, $x_{w0}$) because the depth of the slush layer is equal to zero in this zone. In zone ( $x_{w0}$ , $x_{wf}$ ) the upper layer density for developed slushflow is nearly constant and equal to 950 kg m$^{-3}$. This value corresponds to fully saturated snow because the porosity of the entrained snow is 0.5 and the ice density is 900 kg m$^{-3}$. The slush density diminishes downstream and equals to 450 kg m$^{-3}$ at the front, i.e. the density of snow cover.

## NUMERICAL RESULTS AND DISCUSSION

The special series of numerical computations were carried out to investigate the influence of model parameters on the structure and dynamics of slushflow. All calculations were made for a uniform slope. The following set of model parameters is used as the basis: $\psi = 5°$; $\theta = 0.1$ cm s$^{-1}$; $\rho_c = 450$ kg m$^{-3}$; $P = 0.5$; $f = 0.05$; $k_w = 0.05$; $k_s = 0.02$; $k_{sw} = 0.03$. The uniform initial distributions were prescribed over the length of 50 m: $H_{s0} = 1$ m; $H_{w0} = 0.5$ m; $U_{s0} = U_{w0} = 0.5$ m s$^{-1}$. The initial snow cover thickness was assumed to be constant and equal to 2 m over the length of channel. The specific water feeding at the rear end was assumed to be constant in time and equal to 3 m s$^{-1}$. The computations were carried out using the space step $\Delta x = 2.5$ m; the calculation time of slushflow motion was 1 min.

The influence of model parameters on the structure and dynamics of slushflow was found as follows.

**Coefficient of snow entrainment, $m_e$.**

The range of variation: 0.005 - 0.05. The influence of this coefficient is very complicated. The dependencies of flow characteristics on the coefficient $m_e$ are shown in Fig.2. An effect of exhaustion of the water layer is revealed under relatively low values of the coefficient of snow entrainment. The curves, depicting the front coordinates $x_{sf}$ and $x_{wf}$, are divergent when $m_e$ increases. In other words, the slush front advances far ahead whereas the water front retreats. This effect is due to very rapid water saturation of the new snow

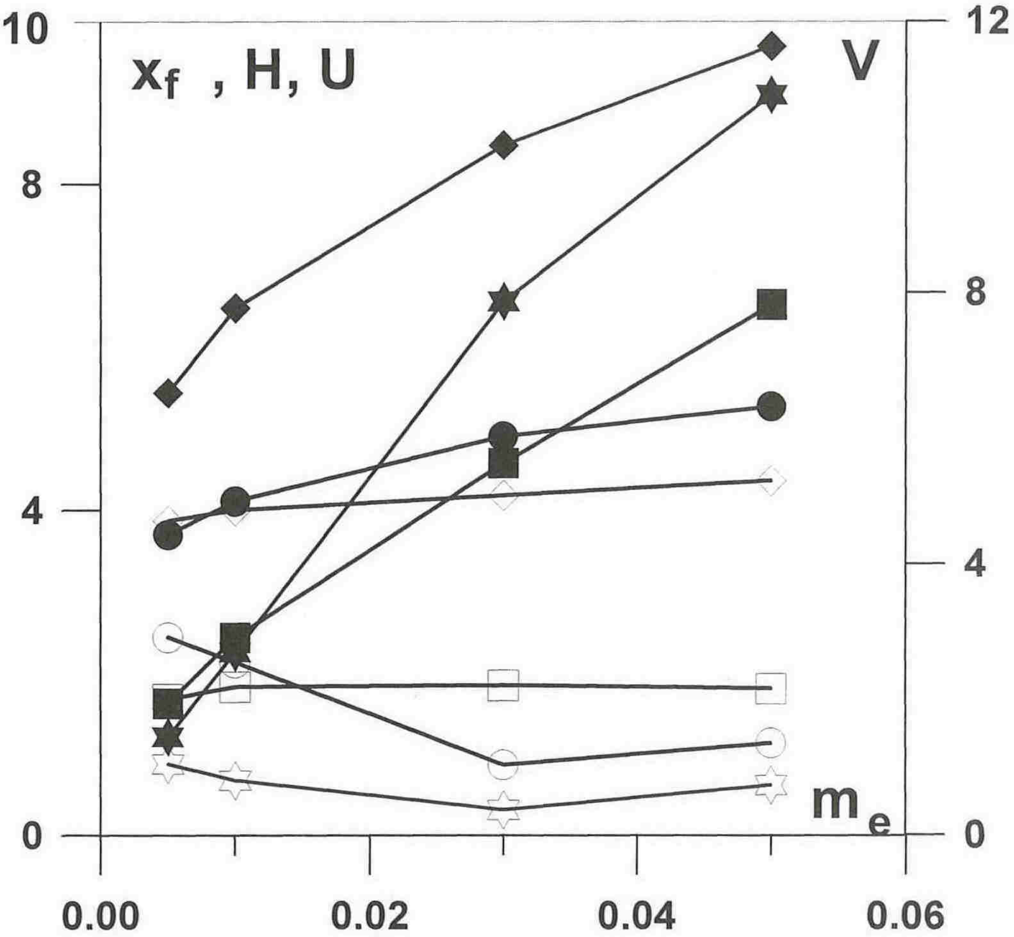


*Fig. 2. Characteristics of two-layer slushflow vs coefficient of snow entrainment, $m_e$ :●,○, ($x_{sf}$, $x_{wf}$) $\cdot 10^{-2}$m;■ , □, $H_{s,\,max}$, $H_{w,max}$, m; ◆, ◊, $U_{s,\,max}$, $U_{w,\,max}$, m $s^{-1}$;✶,✡, specific volumes per unit width of channel ($V_s$, $V_w$) $\cdot$ 0.5 $\cdot$ $10^{-2}m^2$, respectively.*

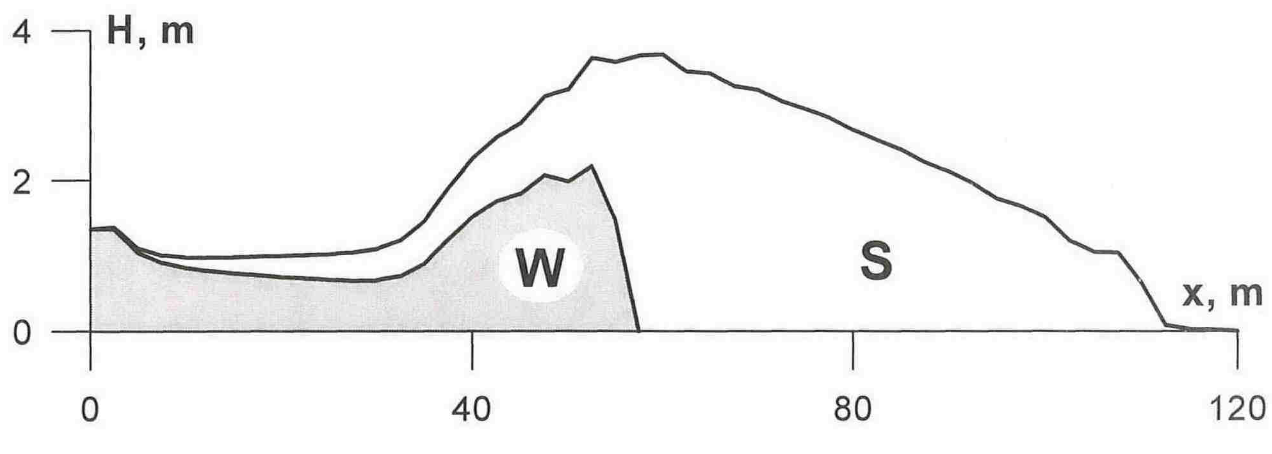


*Fig. 3. Development of slushflow surge; f = 0.15, t = 30 s.*

masses entrained in the motion of slushflow. It is necessary to emphasize that this effect exists only when the snow cover thickness is high enough (2 - 3 m). If the snow cover thickness is smaller than these values, the snow cover is entirely removed and the effect does not exist. The same situation takes place when the coefficient of entrainment is large. Figure 2 depicts that the curves describing the front locations are almost synchronous, when $m_e$ > 0.03. The velocities and the depths of the slush layer increase remarkably when $m_e$ grows. The volumes of slush (per unit width of channel) grow strongly when $m_e$ increases. However, the volume of water layer decreases first (effect of exhaustion) but then begins to grow.

**Coefficient of dry friction, *f*.**

The range of variation: 0.05 - 0.15. The influence of this parameter is very important. Obviously, the change of dry friction coefficient affects the slush layer dynamics. As $f$ increases, the advancement of the front $x_{sf}$, the velocity $U_s$ and the volume $V_s$ fall sharply. At the same time, the maximal depth, $H_{s,\ max}$, reduces slightly first but then it increases considerably. This implies that the height of the slush surge grows due to the stronger snow cover drag. Moreover, the maximum depth shifts to the middle of the slushflow. A different reaction due to an increase of the coefficient $f$ takes place for the water layer. The volume and velocities are almost unchanged and the advancement of the front $x_{wf}$ retards a little. However, the maximal depth grows strongly. Moreover, if the depth distribution of the water layer usually has only one maximum in the rear part of the slushflow (Fig. 1a), then the second maximum of depth arises near the front $x_{wf}$, when $f$ is big. Thus the surge of slush follows the surge of water, Fig. 3.

**Snow cover thickness, $H_c$.**

The range of variation: 1 - 3 m. This parameter exerts a weak influence on the structure and dynamics of the flow. When the snow cover thickness increases, only the volume of slush grows rapidly whereas the volume of the water layer reduces slightly. The front coordinate $x_{sf}$ do not practically depend on $H_c$, but the water layer front retreats that is due to strong water saturation of snow masses entrained in the flow. Maximal velocities and depths of both layers do not depend on $H_c$.

**Water feeding at the rear end, $Q(t)$.**

The range of variation: 1 - 10 m $s^{-1}$. The influence of this parameter is very simple, namely, all characteristics of both layers follows tendencies of change of $Q\ (t)$.

**Inclination angle of channel, $\psi$.**

The range of variation: 3 - 10°. Naturally, all dynamic characteristics of slushflow grow as the inclination angle of channel increases. Also, $H_{s,max}$ increases, but $H_{w,max}$ decreases slightly.

**Coefficient of turbulent friction, $k_s$.**

The range of variation: 0.01 - 0.08. A basic influence $k_s$ exerts on dynamics of the slush layer. When $k_s$ increases, the characteristics of this layer diminish strongly while the characteristics of the water layer are almost unchanged.

**Coefficient of turbulent friction, $k_w$.**

The range of variation: 0.03 - 0.10. As in the case of $k_s$. the coefficient $k_w$ exerts on dynamics of the water layer whereas the characteristics of the slush layer change a little. When $k_w$ increases the velocities and the volumes of the water layer decrease but the depth grows slightly.

**Coefficient of turbulent friction, $k_{sw}$.**

The range of variation: 0.01 - 0.05. This parameter influences a little on the structure and dynamics of both layers.

## CONCLUSIONS

The results obtained with the mathematical modelling method allow us to extend our understanding of structure and dynamics of slushflows. Field observations of slushflow dynamics are nearly absent. Basically there are measurements of run-out distances and also indirect estimations of velocities using traces on sides of channel (Sapunov, 1991, Barsch and others, 1993 ). It is very difficult and expensive to organize and make field observations on slushflows. Moreover, such observations can be ineffective because of the relatively seldom occurence of this natural phenomena. Therefore, in this case mathematical modelling can serve as a good tool to study this process. The calculated characteristics of two-layer slushflow are within the range of actual observed data. So, the flow depths are 1.5 - 6.5 m, the velocities of slush layer are 5 - 8 m $s^{-1}$, the velocities of water layer are 3 - 5 m $s^{-1}$, the volumes of slush masses are 500 - 2000 $m^3$ (when the width of channel is 10 m) and the volumes of water layer are 100 - 200 $m^3$. The developed model allows the calculation of all dynamic characteristics of slushflow, namely, depth, velocity, density and corresponding pressure on an obstacle. In future, it would be desirable to narrow the range of model parameters using adjusted field data and numerical model results for slushflow motion down a non-uniform slope.

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## REFERENCES

Barsch, D., M.Gude, R.Mausbacher, G.Schucraft,A.Schulte, D.Strauch. 1993. Slush- stream phenomena - process and geomorphic impact. *Zeitschrift fur Geomorphologie N.F. Suppl. - Bd.* 92, 39 - 53.

Bozhinskiy, A.N., A.N.Nazarov, V.N.Sapunov, P.A.Chernouss. 1996. The mathematical model of slushflow dynamics. *Proceedings of the International Conference,* Apatit JSC, Kirovsk, 44 - 49.

Gude, M. and D.Scherer. 1995. Snowmelt and slush torrents preliminary report from a field campaign in Kärkevag - ge, Swedish Lappland. *Geografika Annaler*, **77A** (4), 199 - 206.

Onesti, L.G. and E. Hestness. 1989. Slushflow questionare. *Annals of Glaciology*, **13**, 226 - 230.

Perov, V.F. 1995. Vodosnezhnye potoky [Slushflows] *Mater. Glatsiol. Issled.* 79, 177 - 185.

Sapunov, V.N. 1985. Vodosnezhnye potoky i ikh mesto v ryadu skhodnykh razrushitelnykh yavleniy. [Slushflows and their position in series of similar destructive phenomena]. *Vestn. Mosc. Univ., Ser.5, Geogr.*, 6, 31 - 37.

Sapunov, V.N. 1991. Vodosnezhnye potoky v Khybinakh. [Slushflows in Khybiny]. *Mater. Glyatsiol. Issled.* 71, 94 - 99.