Models for powder snow avalanches : comparison of two approaches

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ABSTRACT. Two models are considered in this paper : one was proposed in Kulikovskiy and Sveshnikova (1977) and is referred here as KS-model, the second was described in Beghin and Brugnot (1983), and is referred here as BB-model. Both models treat the powder avalanche as a cloud of finite length and calculate the front velocity, the dimension and the mean density of the cloud. They are very similar as the geometry of the cloud is assumed to be an elliptic half-cylinder. The BB-model is included in a software presently used in France for design purposes. It contains some simplifying assumptions that are not needed in the KS-model. The aim of this paper is to study the validity of the BB-model simplifications by comparison of the two models. We first simulate the Beghin and others (1981) experiments in water by KS-model and then compare the two models' results in the case of a snow avalanche on a uniform slope. We find that an appropriate choice of the air entrainment coefficient value for the KS-model leads to reasonable similar results for the two models, in both cases for large ranges of slope and cloud density.

INTRODUCTION

Powder snow avalanches constitute a fascinating natural phenomenon. Though in Europe they are not the more common avalanches, they are very often involved in disasters. Field measurements are hardly obtainable and data are scarce. Numerical models (Scheiwiller, 1986; Brandstätter and others, 1992, Hermann and others, 1993 and Naaim, 1995) are useful though too complicated for engineering practice.

In this paper we focus on two analytical models. The main advantage of analytical solutions are their fewer parameters requirements, and the generalisation they offer in relating inputs to system response which are essential for making general inferences, and developing recommendations. Therefore they are interesting tools for engineers.

Both models studied here treat the powder snow avalanche (PSA) as a cloud of prescribed geometrical form, with the purpose to calculate its mean density and dimensions variations during the motion. The first model, proposed by Kulikovskiy and Sveshnikova (1977), is referred here as KS-model. It is based on classic fluid mechanics equations and some hypotheses detailed in the following section. The second model described in Beghin and Brugnot (1983) is referred here as BB-model. In this model thermal theory for a buoyant cloud is considered as well as experimental laboratory results to determine growth rate parameters (Tochon-Danguy, Hopfinger, 1975 ; Beghin and others, 1981). Explanations are also given in the following section. This model is included in a software presently used for engineering purposes (Rapin, 1992). These two models are very similar as the geometry of the cloud is assumed to be an elliptic half cylinder. It is worth to note that the same assumption about the avalanche shape is made in (Fukushima, Parker, 1990). The third section is dedicated to models comparison.

THEORIES

Basic assumptions and equations of the BB-model.

Two basic equations of the BB-model are the equation of the mass conservation and the equation of the momentum conservation along the slope :

$$m = \rho_a (A - A_0) + c_1 \rho_N h_N x_f + \rho_0 A_0 \qquad (1)$$

$$\frac{d}{dt}(K_v \rho_a AU + \rho AU) = (\rho - \rho_a) Ag \sin\theta \qquad (2)$$

Here ρ is the mean density, ρ_a is the ambient fluid (air) density, A is the avalanche volume per width unit, m is the avalanche mass, x_f is the coordinate of the front edge and $c_1 \rho_N h_N x_f$ is equal to the amount of snow involved into the avalanche during motion from a h_N-depth layer of density ρ_N ; U is the mass-centre velocity, K_v is the added mass coefficient. The subscript o refers to initial conditions. Equations (1), (2) contain three unknown functions: m, A, and U (with $\rho = m/A$). So the system is not closed, one more equation is needed. Furthermore, usually, not only the volume A but also the height h of the cloud is important. Therefore two additional equations are required for the calculation of all the parameters.

To obtain the closed system the authors of the BB-model took the following assumption that will be referred in this text as hypothesis S1 : The growth of both height h and length l of the cloud is proportional to the distance x_f travelled by the front of the cloud. The proportionality coefficients $dh/dx_f = \alpha_1$ and $dl/dx_f = \alpha_2$ are known for a given slope angle.

This assumptions is based on thermal theory and experiments in a tank with "clouds" of heavy liquid moving along an incline placed in lighter liquid (Beghin and others, 1981). Experiments to estimate proportionality coefficients were carried out in the laboratory using mainly saline water flowing in fresh water, so with relatively small density difference, on slopes with constant inclination angle.

The other simplifying assumptions included into BB-model are :

1.1. The effect of snow entrainment on avalanche volume variation is neglected.

1.2. The mass of snow incorporated into an avalanche is a known function of the path depending neither on avalanche velocity and dimensions nor on the avalanche density.

1.3. Ground friction is neglected.

1.4. Sedimentation of the snow is neglected.

1.5. The force acting on an avalanche from the surrounding air is connected only to the added mass.

The assumptions 1.1-1.5 together with S1 allow to integrate explicitly the equations and to obtain the algebraic formulas for front velocity and mean density of an avalanche moving on a constant slope.

Basic assumptions and equations of KS-model.

The basic system of equations in KS-model consists of four equations. Besides the mass and longitudinal momentum conservation equations that are similar to (1), (2), it includes the equation for avalanche volume variation

$$\frac{dA}{dt} = bV_1 \tag{3}$$

and also the Lagrange equation which plays the role of the momentum equation for the direction normal to the slope

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{h}}\right) - \frac{\partial T}{\partial h} = Q_1 + Q_2 \tag{4}$$

Here V_1 is the volume of air involved into the avalanche per time unit and per surface unit, h is the cloud height, b is the length of the top boundary of the cloud, T is the kinetic energy of the internal motion resulting in deformation of the cloud, Q_1 is the gravity force component normal to the slope and Q_2 is the lift force appearing due to the pressure gradient on the top boundary of the cloud (due to air flow over the cloud).

The model takes into account air and snow entrainment, snow sedimentation, ground friction and air drag and does not use the concept of added mass. The laws for the air and snow entrainment rates, for ground friction and air drag are taken to be similar to those used in the theories of turbulent jets and flows in channels. For example the air entrainment rate V_1 is assumed to be proportional to the avalanche velocity U and to the square root of the ratio of the cloud density ρ over the air density ρ_a

$$V_1 = kU\sqrt{\rho / \rho_a} \tag{5}$$

The important points of the model are the expressions for the mean kinetic energy T and generalised forces Q_1, Q_2. To write these expressions the authors of KS-model used several assumptions, the validity of which should be verified by practice. In particular, the lift force Q_2 is taken to be a given fraction of the force calculated for continuous (i.e., without separation) overflow of the avalanche body by air.

COMPARISON AND DISCUSSION

Discussion on the hypotheses

It is known that S1 hypothesis (self-similarity of a cloud shape) is suitable for thermals only if they move under homogeneous external conditions. So, it can be applied for a cloud motion along non-homogeneous slopes only approximately. The system of equations of KS-model contains two extra equations (3) and (4) as compared to the BB-model. That is why the growth rate of an avalanche as well as the ratio of length and height of the cloud need not to be prescribed. They can be found as solution of the basic system of equations. It would be useful to verify the hypothesis S1 with the use of a system of equations based on general physics laws that are true not only for some particular conditions of motion. On this way it would be possible to estimate the validity of the BB-model for the calculations of motion along non-homogeneous slopes and for large density difference between the avalanche material and the ambient air. Let us briefly discuss the other simplifying assumptions 1.1-1.5 included into BB-model :

1.1. This assumption is lawful since the volume snow concentration in powder snow avalanche is small.

1.2. In the other models, i.e. in KS-model and in the model by Fukushima and Parker (1990) the snow entrainment depends on the avalanche velocity and density. If all available snow is entrained into the motion, the BB-model expression for snow entrainment can be valid.

1.3. Calculations by KS-model, which includes ground friction, show that in many cases its effect is small as compared to the snow and air entrainment effect (Eglit and Sveshnikova, 1979).

1.4. Sedimentation of the snow is certainly essential, at least on the final stage of an avalanche motion.

1.5. This could be true if the flow around the avalanche body was continuous, without separation. The flow around an ellipse is always with separation. For such a flow the main part of the force is an aerodynamic drag proportional to the square of the front velocity.

The assumptions 1.1-1.5 are not so crucial as hypothesis S1, furthermore they could be easily changed to take into account the effects which are neglected in the original version of the model.

Comparison by calculations

Simulation of Beghin's and others (1981) experiments

The aim of this part is to simulate Beghin's and others (1981) experiments by KS-model as the BB-model is based on these results. We took their initial conditions : $Uo = 0.0$, $Ao = 80\ cm^2$, the density ratio 1.02, the length of the path 200 cm and put the coefficients of snow entrainment and sedimentation velocity to be zero. The results of calculations are as follows. For the slope angles 10° to 35° the dependence of the avalanche height h and length l are very close to linear after the avalanche passing a short distance from the origin so the hypothesis S1 is well simulated by KS-model. The velocity at first increases till a certain value and then begins to decrease. An example of calculations is shown fig 1. The growth rates of the avalanche height and length, α_1 and α_2, increase with the slope angle. Their values essentially depend on the air entrainment coefficient k. At $k = 0.28$ the measured and simulated values of α_1 practically coincide as can be seen from Table 1.

Table 1. Values of height growth rate coefficient α_1 for different slope angles θ, exp. : as measured by Beghin and others (1981), sim. : simulated by KS-model ($k = 0.28$)

θ (°)	10	15	20	25	35	40
α_1 *exp.*	0.07	0.08	0.11	0.12	0.17	0.19*
α_1 *sim.*	0.07	0.08	0.10	0.11	0.19	0.26

*obtained from the top 50cm only

In Kulikovskiy and Svèshnikova (1977) the value $k = 0.055$ was used based on experiments about turbulent jets. In our calculations this value gave too low values of α_1. The possible explanation is that due to internal rotation of the fluid in an avalanche body (see great vortexes shown at the fig. 3 in Beghin and others (1981)) the velocity at the boundary of an avalanche is larger than the centre-mass velocity that enters in the formula (5). The length growth rate α_2 obtained is more than recommended in Beghin and others (1981). This parameter has not as clear physical meaning as h, because it is equal to the length of the half-ellipse having the same area as the avalanche and it can be quite different from the real length of the avalanche (which is a very conventional parameter itself). At the slope angles higher than 40° the height and the length of a cloud show the approximately linear dependence on x_f in our calculations only at the first part of the path (about 100-50 cm) and then begins the fast growth of h and decrease of l. We will say that there appears a "splash", the air-snow mixture rises very fast. The authors of KS model noticed this phenomenon. It can be explained by the action of the lift force Q2 that increase with the increase of h. After the splash began the cloud became very high and short and it is inappropriate to consider it as a cloud of elliptic form. We agree with the authors of KS-model that in this case an avalanche should be considered as a great vortex ring. It is well known that thermals can transform into such rings during the motion.

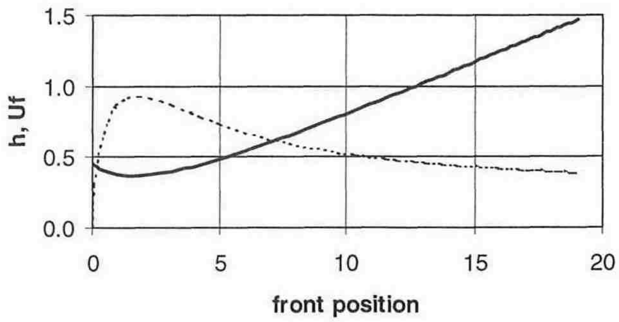

Fig.1 Example of KS-model simulation : cloud height h (bold line) and front velocity (dashed line). Data are adimensionalised as suggested in Hopfinger and others (1981), initial conditions as in the text with $\theta = 10°$

Simulation of an avalanche on a constant slope

The complete comparison of the two models, with complex path and snow entrainment and deposition, extends beyond the scope of this paper. In this part we focus on a uniform slope without snow entrainment and deposition. The following case have been arbitrarily chosen : a 200-m. slope of 30° angle, an initial 10 x 20 m cloud of density ratio 1.5 and an initial front velocity of 10 m/s (see Voellmy, 1955). The KS-model k coefficient was fixed to 0.3. Sensibility of the results to some of these parameters, the slope angle and the density, will be discussed later.

Results are presented in Fig. 2, 3, 4 and 5. We can see that there is a reasonable concordance between the two models, especially if we consider that no attempt of calibration was made. As for simulations in water the growth of h simulated by KS-model is not linear at the beginning and is lower than those of BB-model. This explains the difference in final values of h, 30 m for KS-model and 38 m for BB-model. Front velocities also show a slight difference in their evolution at the beginning, besides the initial value for KS-model is not exactly 10 m/s as the initial parameter to be fixed is the centre of mass velocity. These two points explain the difference in final velocities : respectively 5.6 and 4.1 m/s for KS and BB-model. Though the two simulations give results of the same order, as the densities are the same, in term of pressure the BB-model underestimation is of 46% compared with KS-model. As for simulations in water, the avalanche length is overestimated by the KS-model, here by a factor 2.

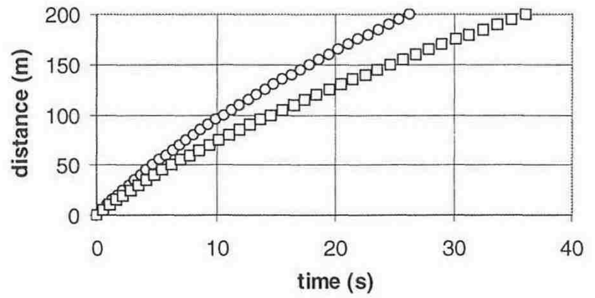

Figure 2 Time evolution of the avalanche front as simulated by KS (o) and BB-model (□).

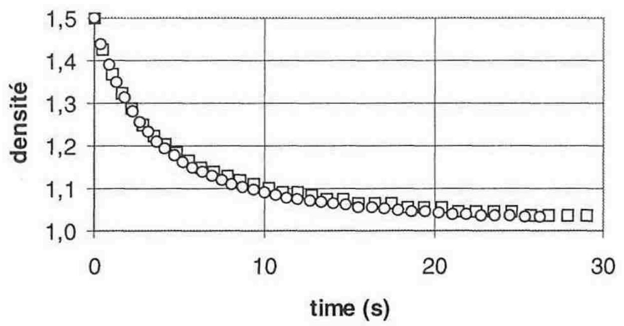

Figure 3 Time evolution of the avalanche density ratio as simulated by KS (o) and BB-model (□).

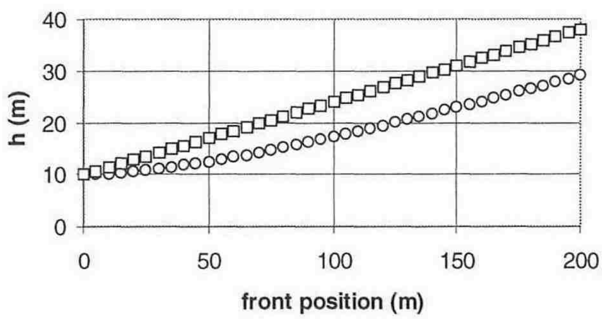

Figure 4 Avalanche height, h, evolution along the path, as simulated by KS (o) and BB-model (□).

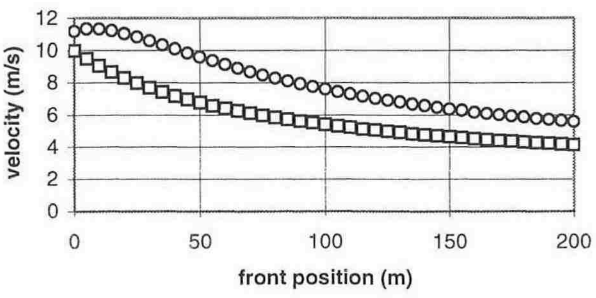

Figure 5 Avalanche front velocity evolution along the path, as simulated by KS (o) and BB-model (□).

In order to evaluate the sensibility of these findings to the slope angle and the cloud density, additional calculations have been made with the same other initial values. They are presented in Table 2. A linear behaviour appears after an avalanche front motion of about 100 m for ρ/ρ_a= 1.5 and 0 to 100 m for $\rho/\rho_a = 50$.

The results are satisfying for α_1 shows little dependence on density, so the above analysis can be probably extended to larger density. However one must keep in mind that all these simulations were made with an initial front velocity value of about 10 m/s. Consequently, the complex slope case, treated as a piecewise of constant slopes, remains to be studied.

Table 2. Values of height growth rate coefficient α_1 for different slope angles θ , exp. : as measured by Beghin and others (1981), sim. : simulated by KS-model (k = 0.30) for two densities (see the text for the other parameters).

θ (°)		5	10	20	30	40	60
α_1 *exp.*	ρ/ρ_a = 1.02	0.06	0.07	0.11	0.15	0.19	0.24
α_1 *sim.*	ρ/ρ_a = 1.5	0.03	0.05	0.08	0.12	0.20	*
α_1 *sim.*	ρ/ρ_a = 50	0.05	0.07	0.10	0.12	0.14	0.22

* no linear behaviour

CONCLUSION

We have compared two simplified models for powder-snow avalanche, namely KS-model (Kulikovskiy and Sveshnikova, 1977) and BB-model (Beghin and Brugnot 1983). The most important difference between this two models is the hypothesis S1 in BB-model : both height and length of the cloud are proportional to the front position, and the proportionality coefficients are known for a given slope angle. Our comparison is based first on simulation of Beghin and others (1981) experiments in water and second on the simulation of an avalanche on a constant slope.

We find that KS-model well simulates the experiments in water, provided the use of an ambient fluid entrainment coefficient fixed to k = 0.28 which is far higher than suggested by Kulikovskiy and Sveshnikova (1977). The hypothesis S1 is well simulated by KS-model, a while after the start of motion.

In the constant slope case the two models agree reasonably well, except for the avalanche length. KS-model shows little dependency of the height growth rate coefficients on the cloud density in a large range of slope angles.

These results are reassuring as only one parameter (k) allows the two models to give very similar simulations, however, this cannot be considered as a validation. The complex slope case has not been studied here, nor the snow entrainment and deposition which are very important in the avalanche dynamics. The general conclusion is that further investigations are useful to determine the domain of applicability of the KS and BB-models. Engineers still must

use these models with caution. At last only complete and reliable field data on actual powder-snow avalanches would allow models validation.
For engineering purpose it would be interesting to extend the KS-model to the three-dimensional case as it was done for the BB-model by Beghin and Olagne (1991).

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