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Estimating Avalanche Triggering Probability using meteorological and local terrain parameters through a fuzzy inference approach

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Review and reference page

1 Introduction and problem definition

The aim of the present study is the quantitative estimation of snow avalanche triggering probability (ATP) for daily local forecasting purposes based on meteorological and local terrain factors.

At present, ATP is indirectly assessed by snow avalanche forecasters (experts) while they conclude upon snowpack stability at the moment and upon its change during the forecasting period based on their personal judgement, knowledge and experience. In the forecasters' decision making, a considerable role is played by the recent meteorological history of the area the forecast is produced for, as well as by forecasted weather development.

In this study, an approach based on fuzzy inference is adopted to estimate ATP through the quantitative parameterization of expert judgement.

2 Fuzzy inference approach: structure and phases

Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic.

Fuzzy logic is an approach to computing based on "degrees of truth" rather than the binary "false or true" (0 or 1) Boolean logic on which the modern computer is based. Fuzzy logic includes 0 and 1 as extreme cases of truth (or "the state of matters" or "fact") but also includes the infinite intermediate states of truth on a closed real number interval bounded inferiorly at 0 and superiorly at 1.

The fuzzy inference process adopted in this study consists of the following sequential steps:

- Definition of the problem
- Identification of relevant inputs
- Design of a fuzzy ensemble
- Fuzzification of input and output variables
- Application of fuzzy operators
- Implication from the antecedent to the consequent
- Aggregation of the consequents across fuzzy rules
- Defuzzification of fuzzy outputs
- Post-processing of fuzzy inference outputs
- Validation and testing of defuzzified outputs

The structure of this report essentially replicates the above steps.

3 Identification of relevant triggering factors

This section addresses the identification of relevant factors contributing to avalanche triggering.

3.1 Meteorological factors

There are many meteorological elements/parameters which affect snowpack stability, thus influencing ATP. Unfortunately, not all of them are observed at ordinary meteorological stations in Norway, if at all. For this study, meteorological elements which are the most essential for ATP and are usually observed by Norwegian meteorological stations are chosen. These data are also used at the Norwegian Local Avalanche Warning service operated by NGI. They often have 1-hour time resolution and are easily available and retrievable online.

The selected parameters are:

- Precipitation (water equivalent)
- Air temperature
- Snow height
- Wind velocity
- Wind direction with respect to terrain orientation

Since snow/snowpack is a continuously evolving medium, ATP is also temporally variable. Thus, in the model, it is important to consider differential parameters such as change in air temperature and change in snow height.

Avalanche hazard estimation (and ATP as a part of it) in daily forecasting service consists essentially of two parts:

1. Assessment of the snowpack stability in present conditions, based on the observed meteorological parameters; and
2. Assessment of the expected variation in stability during a given period (usually in the forthcoming 24 hours) based on forecasted meteorological values.

To enhance the utility and applicability of the model useful for the operative local forecasting, observed meteorological parameters are combined with forecasted values in the present study.

3.2 Non-meteorological factors

Meteorological factors are the most relevant for snowpack stability. However, the same set of meteorological factors (values) will have a different effect on ATP depending on the existence and magnitude of other parameters which should not be neglected when estimating ATP. These are described preliminarily in the following.

3.2.1 Persistent weak layer

A "persistent weak layer" (hereinafter PWL) represents a layer of not bounded snow/ice crystals which may collapse when a critical load is reached. Its presence (or absence), properties and position inside the snowpack is known to be very relevant to avalanche triggering. PWL is not a meteorological element and is not observed by meteorological stations. PWL is accounted for in the study through the probability of its existence, P_{PWL}.

3.2.2 Terrain slope

Terrain inclination/slope represents a particularly important parameter to account for while estimating ATP. Snow avalanches are initiated by gravity, among other processes, and do not occur on flat terrain. At the same time, very steep terrain (over 60-70°) hinders snow accumulation, thus impeding the occurrence of snow avalanches. Terrain steeper than 70° is thus associated to null triggering probability, irrespective of meteorological parameter values. Terrain flatter than 27° is also associated to null triggering probability conditioned on air temperature T being negative. When $T \geq 0°C$, ATP can take positive values even on the terrain flatter than 27°, in this way including wet-snow avalanches and slush flows.

The exposition/aspect of the terrain cell in the question is particularly important when it comes to wind direction (snow drift), specifically in terms of its deviation (in degrees) from the incident angle of the cell. This parameter (exposition/aspect) is indirectly included in the model through the definition of the wind direction (see Section 4.4.1).

4 Expert-based design of the fuzzy ensemble

The expert-based design of the fuzzy inference tool aims at configuring an efficient structure which allows the correct modelling and processing of expert knowledge. In this study, design involved the following sequential steps:

- Expert-based assignment of class boundary values for triggering factors
- Definition of ensemble topology
- Definition of fuzzy inference systems
- Definition of fuzzy sets
- Definition of fuzzy rules
- Definition of fuzzy inference post-processing options

4.1 Expert-based assignment of class boundary values for triggering factors

To be able to reflect expert knowledge into the fuzzy ensemble, all triggering factors were classified. The value ranges of each class of the chosen parameters are overlapping. This reflects the fact that there are often no univocal boundaries in verbal definitions representing expert knowledge. This inherent vagueness lies at the basis of the concept of fuzzy inference. In the context of the fuzzy inference approach, this operation will be reflected in the definition of *fuzzy sets* and *membership functions* (Section 4.4).

The number of classes can differ from parameter to parameter. In the present study, most of the parameters were defined using three or four classes: *(very low / null), low*, *medium* and *high*. However, there also are parameters subdivided in five and even seven classes. For the wind direction parameter, *low, medium* and *high* are termed *windward* (i.e., blowing toward the slope), *parallel* and *leeward* (i.e., blowing away from the slope), respectively. For ATP, the five classes are named *low, moderate, considerable, high* and *very high* to reflect the EAWS Avalanche Danger Scale (http://www.avalanches.org/eaws/en/main_layer.php?layer=basics&id=1) in addition to two extra classes: *null* and *very low.*

The deviation of wind direction from the incident angle of the terrain is accounted for in the classes for the direction of the dominant/relevant wind.

4.2 Definition of the fuzzy ensemble topology

As indicated in Section 3, a total of ten parameters are deemed relevant in the estimation of ATP. Since each of the parameters is divided into three to seven classes (as mentioned in Section 4.1 and detailed in Section 4.3), a single resulting fuzzy system including all parameters would comprise over 40'000 rules, thereby increasing computational expense beyond convenience and actual feasibility.

Therefore, a fuzzy ensemble comprising four fuzzy systems, each having fewer input parameters, was defined. Three systems each contribute a triggering factor, parameterizing the degree of triggering likelihood which the cluster of factors included in that system brings to ATP. The triggering factors from the three systems are then used as inputs to a further system (hereinafter referred to as "terminal system, *TFS*") which yields ATP.

The clustering of the ten input parameters into three systems was based on expert reasoning regarding the physical phenomena leading to avalanche triggering. The topological scheme of the fuzzy ensemble is shown in Figure 1.

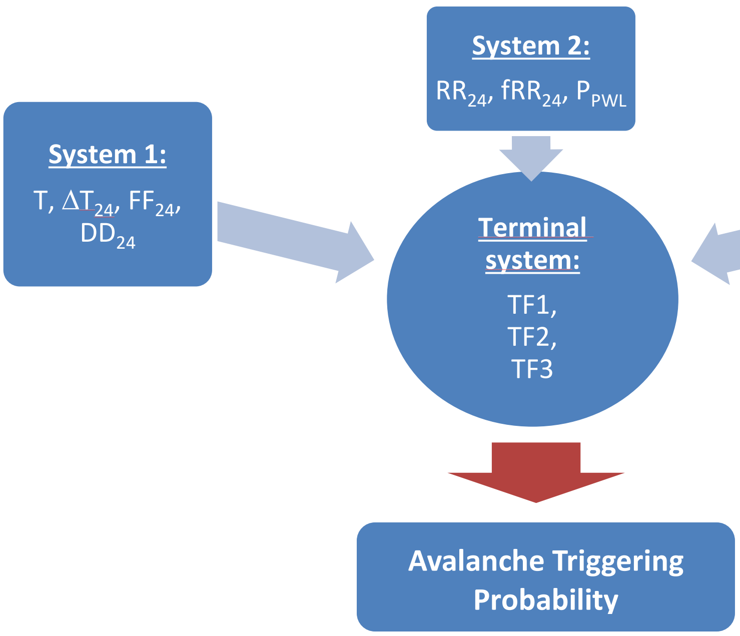

Figure 1 Topology of the fuzzy inference system

The design of the four systems which constitute the fuzzy ensemble is detailed in the following.

4.3 Definition of fuzzy inference systems

4.3.1 Fuzzy system 1

Fuzzy system FS1 investigates the combined effect of wind and air temperature on ATP. Values for all the parameters in the fuzzy system FS1, as shown in Table 1, are forecasted.

Table 1 Input parameters to FS1

Description	Symbol	Units
Air temperature 24 hours ahead	T	°C
Change in air temperature in the next 24 hours	ΔT_{24}	°C
Dominant/relevant wind velocity in the next 24 hours	FF_{24}	m/s
Direction of the dominant/relevant wind in the next 24 hours	DD_{24}	°

The output of FS1 is the dimensionless triggering factor $TF1$, which parameterizes the combined effect of wind and air temperature on the likelihood of avalanche triggering.

4.3.2 Fuzzy system 2

Fuzzy system FS2 investigates the effect of precipitation and PWL on ATP. Inputs to FS2, as shown in Table 2, are a combination of forecasted and observed parameter values.

Table 2 Input parameters to FS2

Description	Symbol	Units
Accumulated precipitation last 24 hours (observed)	RR_{24}	mm
Accumulated precipitation next 24 hours (forecasted)	fRR_{24}	mm
Probability of existence of PWL in the snowpack	P_{PWL}	-

The output of FS2 is the dimensionless triggering factor *TF2*, which parameterizes the combined effect of precipitation and PWL on the likelihood of avalanche triggering.

4.3.3 Fuzzy system 3

Fuzzy system FS3 investigates the combined effect of snow height and terrain inclination on ATP. Input parameters to fuzzy system FS3, as shown in Table 3, are all observed:

Table 3 Input parameters to FS3

Description	Symbol	Units
Snow height at present	SA	m
Change in the snow height last 24 hours	ΔSA_{24}	m
Terrain inclination	θ	°

The output of FS3 is the dimensionless triggering factor $TF3$, which parameterizes the combined effect of snow height and terrain inclination on the likelihood of avalanche triggering.

4.3.4 Terminal system

The terminal system FST uses the triggering factors $TF1$, $TF2$ and $TF3$ as inputs to yield the single output ATP.

4.4 Definition of fuzzy sets

A fuzzy set is a set without a crisp, clearly defined boundary. It can contain elements with only a partial degree of membership. The definition of fuzzy sets thus relies on the assignment of membership functions to each of the classes defined for each of the parameters in all the fuzzy systems.

A membership function is a curve that defines how each point in the input space (also referred to as "universe of discourse") is mapped to a membership value (or "degree of membership") between 0 and 1 in each fuzzy set. The membership value of a number describes how pertinent the definition of a class is to that number.

The only condition a membership function must satisfy is that it must vary between 0 and 1. The function itself can be defined arbitrarily based on objective information and/or subjective judgment.
In the present study, fuzzy membership functions are assigned using a spline-based Pi-shaped membership function. This function is defined piecewise for a generic parameter x by the four parameters p_1, p_2, p_3 and p_4 and yields membership values as follows:

$$\mu(x; p_1, p_2, p_3, p_4) = \left\{ \begin{array}{cc} 0 & x \le p_1 \\ 2\left(\frac{x - p_1}{p_2 - p_1}\right)^2 & p_1 \le x \le \frac{p_1 + p_2}{2} \\ 1 - 2\left(\frac{x - p_2}{p_2 - p_1}\right)^2 & \frac{p_1 + p_2}{2} \le x \le p_2 \\ 1 & p_2 \le x \le p_3 \\ 1 - 2\left(\frac{x - p_3}{p_4 - p_3}\right)^2 & p_3 \le x \le \frac{p_3 + p_4}{2} \\ 2\left(\frac{x - p_4}{p_4 - p_3}\right)^2 & \frac{p_3 + p_4}{2} \le x \le p_4 \\ 0 & x \ge p_4 \end{array} \right\} \tag{1}$$

Sets of values of p_1, p_2, p_3 and p_4 were assigned through expert judgment as detailed in the following.

4.4.1 Definition of fuzzy sets for FS1

Table 4 illustrates the values of the pi-function parameters used in the definition of fuzzy sets for inputs and output parameters of FS1. Numbers in italic indicate auxiliary boundary values used to limit the model calculation and are not connected to any physical assumptions. The deviation of wind direction from the incident angle of the terrain is accounted for in the classes for the direction of the dominant/relevant wind.

Table 4 Parameters of pi-functions for membership value assignment for FS1

		input 1	input 2	input 3	input 4	output 1
		T [°]	ΔT_{24} [°]	FF_{24} [m/sec]	DD_{24} [°]	$TF1$ [-]
VERY LOW	p_1	-45	*	*	*	*
	p_2	-40	*	*	*	*
	p_3	-15	*	*	*	*
	p_4	-10	*	*	*	*
LOW	p_1	-15	-25	-5	-45	-1.000
	p_2	-10	-20	0	0	0.000
	p_3	-5	-5	5	45	0.225
	p_4	-2	2	10	90	0.400
MEDIUM	p_1	-5	-5	5	45	0.250
	p_2	-2	-2	10	89	0.375
	p_3	1	2	15	91	0.625
	p_4	3	5	20	135	0.750
HIGH	p_1	1	2	15	90	0.600
	p_2	3	5	20	135	0.775
	p_3	15	20	30	180	1.000
	p_4	30	30	40	200	2.000

The resulting membership functions for the input parameters and the output parameter of FS1 are shown in Figure 2.

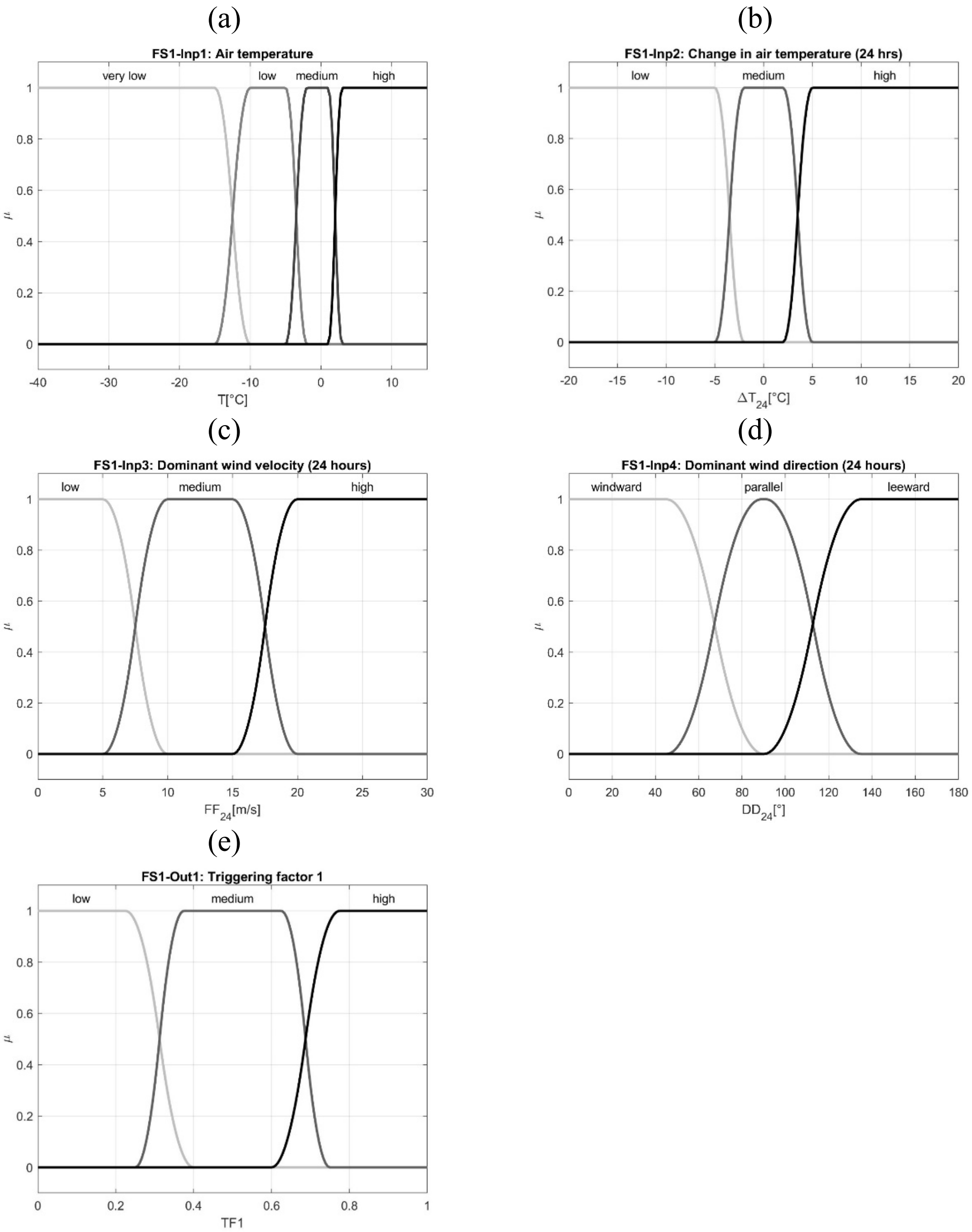

Figure 2. Fuzzy membership functions for FS1: (a) air temperature; (b) change in air temperature; (c) dominant wind velocity; (d) dominant wind direction; (e) triggering factor TF1.

4.4.2 Definition of fuzzy sets for FS2

Table 5 illustrates the values of the pi-function parameters used in the definition of fuzzy sets for inputs and output parameters of FS2. Numbers in italic indicate auxiliary boundary values used to limit the model calculation and are not connected to any physical assumptions.

Table 5 Parameters of pi-functions for membership value assignment for FS2

		inp1 RR_{24} [mm]	inp2 fRR_{24} [mm]	inp3 P_{PWL} [-]	out1 $TF2$ [-]
LOW	p_1	*-10*	*-10*	*-1.000*	*-1.000*
	p_2	0	0	0.000	0.000
	p_3	10	10	0.225	0.225
	p_4	20	20	0.400	0.400
MEDIUM	p_1	10	10	0.250	0.250
	p_2	20	20	0.375	0.375
	p_3	50	50	0.625	0.625
	p_4	60	60	0.750	0.750
HIGH	p_1	50	50	0.600	0.600
	p_2	60	60	0.775	0.775
	p_3	100	100	1.000	1.000
	p_4	*120*	*120*	*2.000*	*2.000*

Membership functions for the input parameters and the output parameter of FS2 are shown in Figure 3.

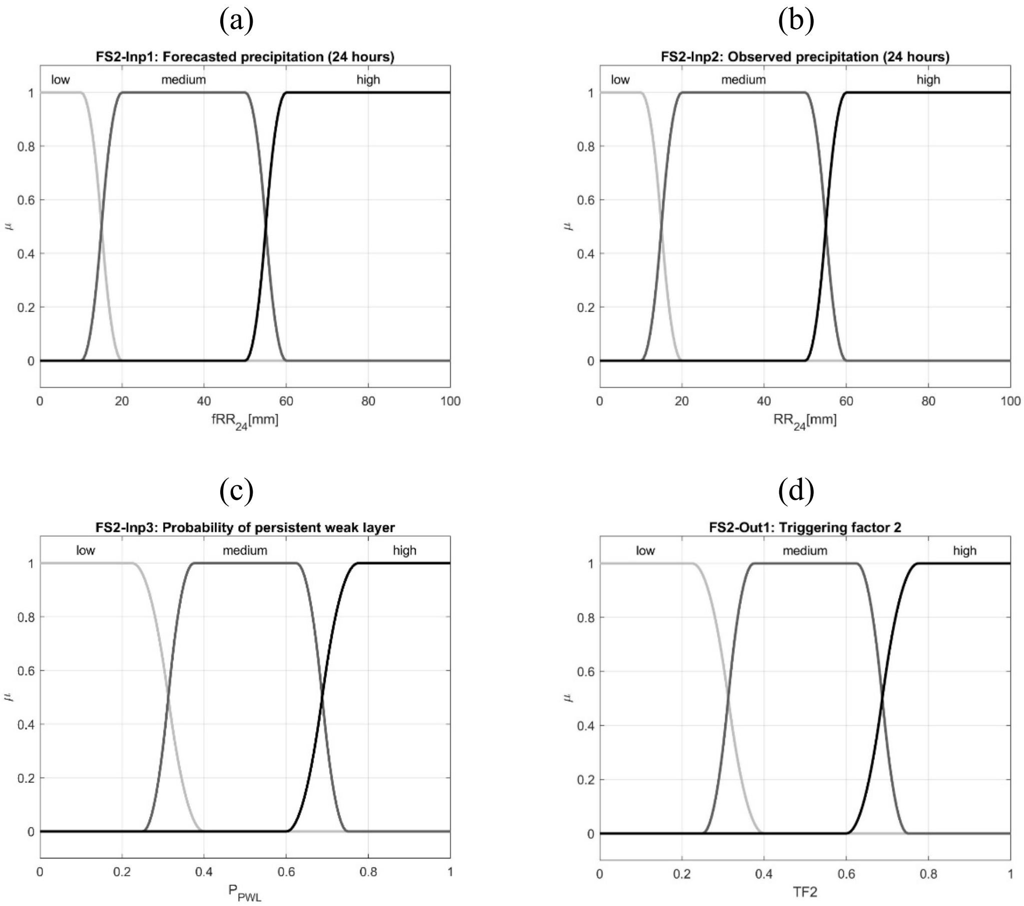

Figure 3. Fuzzy membership functions for FS2: (a) forecasted precipitation; (b) observed precipitation; (c) probability of persistent weak layer; (d) triggering factor TF2.

4.4.3 Definition of fuzzy sets for FS3

Table 6 illustrates the values of the pi-function parameters used in the definition of fuzzy sets for inputs and output parameters of FS3. Numbers in italic indicate auxiliary boundary values used to limit the model calculation and are not connected to any physical assumptions.

Table 6 Parameters of pi-functions for membership value assignment for FS3

		inp1	inp2	inp3	out1
		SA [cm]	ΔSA_{24} [cm]	θ [°]	$TF3$ [-]
NULL	p_1	*-2.00*	*	*-2*	*-0.400*
	p_2	*-1.00*	*	*-1*	*-0.300*
	p_3	0.00	*	0	0.000
	p_4	1	*	2	0.001
EXTREMELY LOW	p_1	*	*	2	*
	p_2	*	*	4	*
	p_3	*	*	8	*
	p_4	*	*	10	*
VERY LOW	p_1	1	*-30*	8	0.001
	p_2	2.5	*-20*	10	0.025
	p_3	3.5	0	18	0.050
	p_4	5	5	22	0.100
LOW	p_1	0	0	18	0.050
	p_2	5	5	22	0.250
	p_3	70	10	26	0.375
	p_4	140	15	30	0.500
MEDIUM	p_1	70	10	26	0.375
	p_2	140	15	30	0.500
	p_3	200	25	42	0.625
	p_4	280	40	50	0.750
HIGH	p_1	200	25	42	0.600
	p_2	300	40	50	0.775
	p_3	400	60	65	1.000
	p_4	*500*	*70*	70	*2.000*
VERY HIGH	p_1	*	*	65	*
	p_2	*	*	70	*
	p_3	*	*	*80*	*
	p_4	*	*	*90*	*

Membership functions for the input parameters and the output parameter of FS3 are shown in Figure 4.

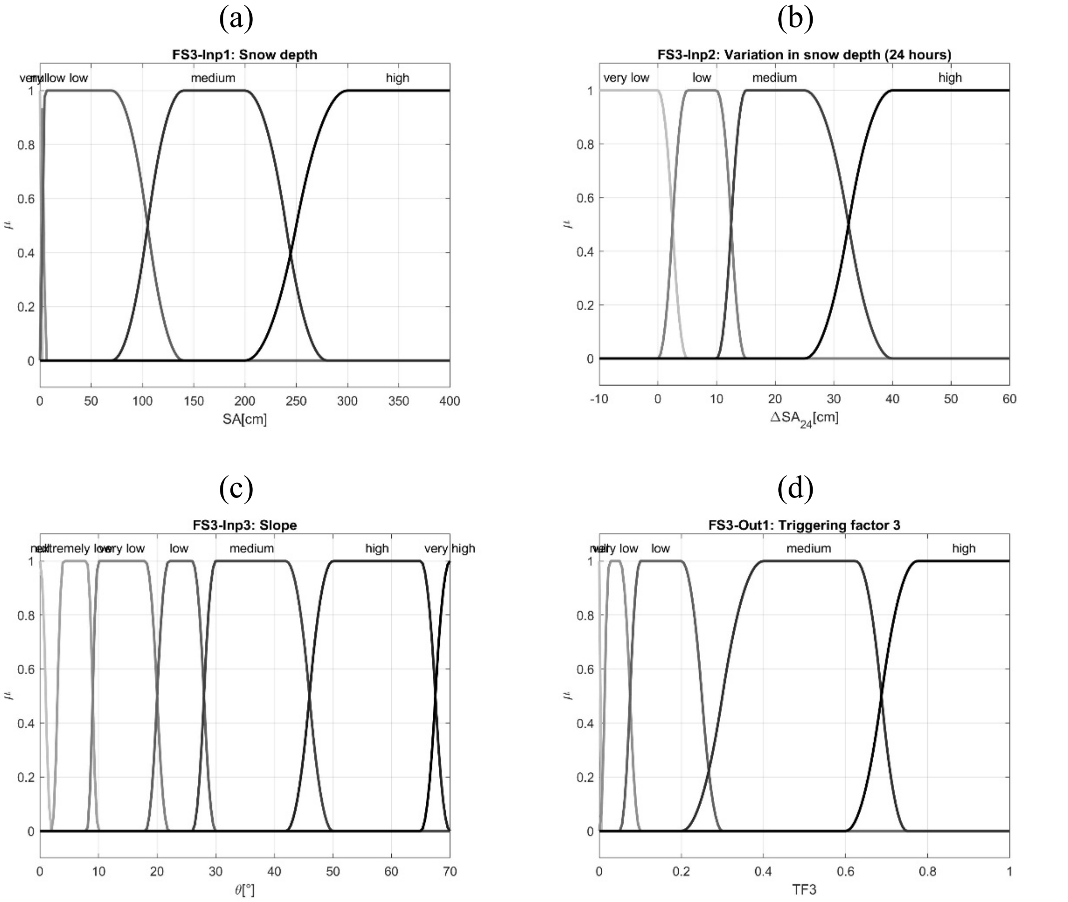

Figure 4. Fuzzy membership functions for FS3: (a) snow depth; (b) variation in snow depth; (c) terrain slope; (d) triggering factor TF3.

4.4.4 Definition of fuzzy sets for FST

Table 7 illustrates the values of the pi-function parameters used in the definition of fuzzy sets for inputs and output parameters of FST. Numbers in italic indicate auxiliary boundary values used to limit the model calculation and are not connected to any physical assumptions.

Table 7 Parameters of pi-functions for membership value assignment for FST

		inp1 $TF1$ [-]	inp2 $TF2$ [-]	inp3 $TF3$ [-]	out1 ATP [-]
NULL	p_1	*	*	-0.400	-0.400
	p_2	*	*	-0.300	-0.300
	p_3	*	*	0.000	0.000
	p_4	*	*	0.001	0.0001
VERY LOW	p_1	*	*	*	0.000
	p_2	*	*	*	0.025
	p_3	*	*	*	0.100
	p_4	*	*	*	0.200
LOW	p_1	-1.000	-1.000	0.001	0.100
	p_2	0.000	0.000	0.025	0.150
	p_3	0.225	0.225	0.375	0.200
	p_4	0.400	0.400	0.500	0.250
MEDIUM	p_1	0.250	0.250	0.375	*
	p_2	0.375	0.375	0.500	*
	p_3	0.625	0.625	0.625	*
	p_4	0.750	0.750	0.750	*
MODERATE	*	*	*	*	0.200
	*	*	*	*	0.300
	*	*	*	*	0.400
	*	*	*	*	0.500
CONSIDERABLE	*	*	*	*	0.400
	*	*	*	*	0.500
	*	*	*	*	0.600
	*	*	*	*	0.700
HIGH	p_1	0.600	0.600	0.600	0.600
	p_2	0.775	0.775	0.775	0.700
	p_3	1.000	1.000	1.000	0.800
	p_4	2.000	2.000	2.000	0.900
VERY HIGH	p_1	*	*	*	0.800
	p_2	*	*	*	0.900
	p_3	*	*	*	1.000
	p_4	*	*	*	2.000

The membership functions of the inputs to FST (i.e., triggering factors *TF1*, *TF2* and *TF3*) have been shown in Figure 2, Figure 3 and Figure 4 respectively. Membership functions for avalanche triggering probability, which is the output of FST, are shown in Figure 5.

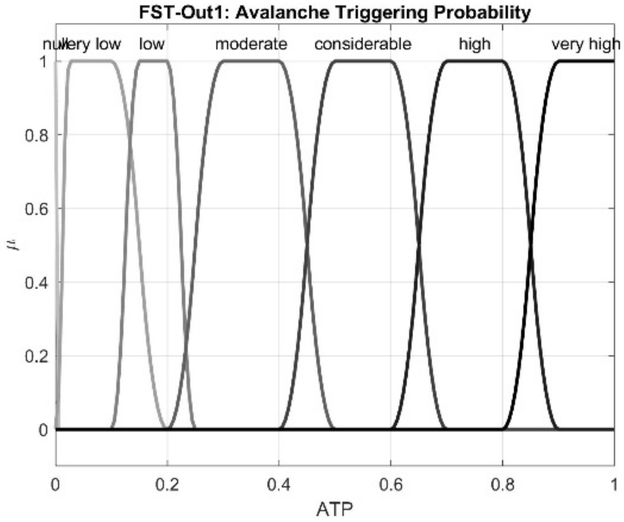

Figure 5. Fuzzy membership functions for avalanche triggering probability

4.5 Definition of fuzzy rules

In fuzzy inference, values in the input vector are interpreted and values are assigned to the output vector on the basis of sets of fuzzy rules.

Fuzzy rules reflect expert knowledge regarding the combined effect of the input parameters of a given fuzzy inference system on the output of the same system. Rules are assigned qualitatively for each fuzzy system, though they acquire a quantitative sense through the information associated with membership functions for each membership class defined for each of the parameters.

Fuzzy rules are compiled according to an *"if-then"* structure, where the *"if"* antecedent leads to a *"then"* consequent. The antecedent can include from one to all input parameters through the logical operator *"and"* (only this operator is used in this study), while the consequent includes one to all outputs (the number of outputs is identically one for all subsets in the present study). Each fuzzy rule can thus be read as an *"if-and-and"*→ *"then"* statement. Fuzzy rule 1-001 (see Appendix A, Table 1), for instance, reads *"If T* is VERY LOW *and* ΔT_{24} is LOW *and* FF_{24} is LOW *and* DD_{24} is LEEWARD, *then TF1* is LOW*"*.

All rules are evaluated in parallel, and the order of the rules is unimportant. The total amount of rules in the four systems is 320. This is a manageable amount both for computation and for revisiting the rules should further expert judgement be elicited (Appendix A).

4.6 Description of the fuzzy inference process

The fuzzy systems constituting the fuzzy ensemble are Mamdani-type systems. Mamdani fuzzy inference (Mamdani & Assilian, 1975) is the most commonly used fuzzy methodology and was among the first control systems built using fuzzy set theory. In Mamdani-type inference, output parameters are modelled as fuzzy sets also requiring membership functions.

The fuzzy inference process employed in the present analysis comprises of five steps:

1. Fuzzification of input variables
2. Application of fuzzy operators
3. Application of the implication method
4. Aggregation of the consequents
5. Defuzzification

These are examined in greater detail in the following.

4.6.1 Fuzzification of input variables

The fuzzification of input variables consists in the resolution of all fuzzy statements in the antecedents of a fuzzy rule to a degree of membership in the range [0,1]. In the present analysis, membership values in each fuzzy set pertaining to a fuzzy input variable are calculated for a specific value of the variable itself as described in Section 4.4.

4.6.2 Application of logical operators

Most of the rules defined in the fuzzy systems for the present analysis have multiple (say, N_a>1) parts. In such case, all N_a parts of the antecedent are calculated simultaneously and are subsequently resolved to a single truth value in the range [0,1], known as the "degree of support" for the fuzzy rule, using logical operators.

Fuzzy logical reasoning is a superset of standard Boolean logic. In the fuzzy rules defined for the ensemble, the "and" operator is equivalent, in logical terms, to implementing intersection (conjunction), and the minimum of the N_a input membership values among those of the variables $x_1, \dots, x_{Na}$ in the antecedent is assumed. The "or" operator is equivalent to union (disjunction), and the maximum of the input membership values among those of the variables in the antecedent is assumed.

4.6.3 Implication

The implication phase consists of two sequential steps for each fuzzy rule. The first step is the multiplication of the rule weight to the rule's degree of support which is calculated in the preceding phase. In the present analysis, all rule weights were assigned identically equal to 1, thus having equal and maximum weight.

The second step consists in the application of the implication method. The consequent of a fuzzy rule assigns an entire fuzzy set to the output. This fuzzy set is represented by a membership function that is chosen to indicate the qualities of the consequent. If the antecedent is only partially true, (i.e., is assigned a value less than 1), then the output fuzzy set is truncated according to the user-defined implication method. The “minimum” implication operator, which truncates the output fuzzy set, was selected among other available methods and used uniformly in all fuzzy systems.

The “minimum” implication method is defined as

$$\phi_{min}[\mu_A(x_1, \dots, x_{Na}), \mu_C(u)] = \mu_A(x_1, \dots, x_{Na}) \wedge \mu_C(u) \tag{2}$$

in which $\mu_A(x_1, \dots, x_{Na})$ is the membership value of the antecedent part as yielded in the preceding phase and $\mu_C(u)$ is the membership function of the consequent part.

4.6.4 Aggregation

Aggregation is the process by which the fuzzy sets that represent the outputs of each rule are combined into a single fuzzy set. Aggregation occurs once for each output variable. The input of the aggregation process is the list of truncated output functions returned by the implication process for each rule. The output of the aggregation process is a single fuzzy set for each output variable. The “maximum” aggregation method, which returns the maximum envelope $\mu_{OUT}(u)$ of the membership functions of the output variable u (either in their original shape for the pertinent fuzzy set or truncated according to the implication operator in the previous phase), was selected among other available methods and used uniformly in all fuzzy systems. Since the adopted aggregation method is commutative, the order in which the rules are executed is unimportant.

4.6.5 Defuzzification

While fuzziness lies at the basis of the rule evaluation process during the intermediate steps described above, the final desired output of fuzzy inference is generally a single number. This is achieved through defuzzification of the fuzzy set output by the aggregation phase.

Several defuzzification algorithms are available. In the present analysis, centroid (Center of Gravity – COG) defuzzification was selected as defuzzification method for all fuzzy systems. In centroid defuzzification, the crisp output value u^* is taken to be the geometrical center of the output fuzzy value $\mu_{OUT}(u)$, where $\mu_{OUT}(u)$ is formed by taking the union of all contribution of rules in the aggregation phase. The center is the point which splits the area under the $\mu_{OUT}(u)$ curve in two equal parts. The COG-defuzzified output is given by

$$u^* = \frac{\int u \cdot \mu_{OUT}(u) du}{\int \mu_{OUT}(u) du} \tag{3}$$

In the present application, for each fuzzy parameter, the universe of discourse is discretized into a variable number of N values. Hence, the expression of the COG-defuzzified output can be rewritten as

$$u^* = \frac{\sum_{i=1}^{N} u_i \mu_{OUT}(u_i)}{\sum_{i=1}^{N} \mu_{OUT}(u_i)} \tag{4}$$

4.7 Inter-system post-processing

One of the disadvantages of splitting a large fuzzy set into several smaller sets, becomes evident when the model requires some rules which include parameters from the different fuzzy sets inside the same system. An additional rule was implemented outside the fuzzy framework to overcome this shortcoming. When the forecasted air temperature, T, (which belongs to *FS1*) is negative and the terrain inclination, θ, (which belongs to *FS3*) is below 27°, then the contribution of $FS3$ to TFS is set equal to 0. To avoid an abrupt drop in ATP values, a smooth transition is implemented for $FS3$ when $\theta \in (27°; 32°)$ and $T < 0$.

5 Fuzzy ensemble outputs

5.1 Implementation of the fuzzy ensemble

The outputs of the fuzzy ensemble inherently reflect expert knowledge and judgment through the definition of fuzzy sets and rules. However, given the complexity of the fuzzy ensemble, it is not straightforward for the expert to predict how each modelling decision influences the end result, i.e., the ATP values output by the model. It should be noted that changing individual rules will result in a change in all three-dimensional projections pertaining to the same fuzzy system.

To investigate the sensitivity of ensemble outputs to modelling options, two experts independently prepared distinct sets of rules during the model development process. Three-dimensional projections were obtained for each of these sets (not shown here), and the fuzzy inference outputs from both sets were produced and compared to estimate the influence of possible differences in the experts' judgement. The two experts then analysed the outputs comparatively and critically and agreed on a “hybrid” set of rules which was adopted as reference for future steps.

5.2 Representation of fuzzy ensemble outputs

The four systems defined above are multi-dimensional. While a full visual representation of their outputs as obtained through the fuzzy inference process is not possible, three-dimensional projections of the output surface (showing outputs as three-dimensional surfaces with respect to pairs of input parameters) can be prepared. These allow intuitive appreciation of the outputs themselves. Three-dimensional projections are shown in Figure 6, Figure 7, Figure 8 and Figure 9 for FS1, FS2, FS3 and FST, respectively. It should be noted that the three-dimensional projections shown in the following figures are plotted for intermediate values (with respect to the user-defined ranges previously described) of the parameters which are not explicitly included in the figures themselves. Hence, the shape of the surfaces can be expected to change to varying degrees for different values of the non-represented parameters.

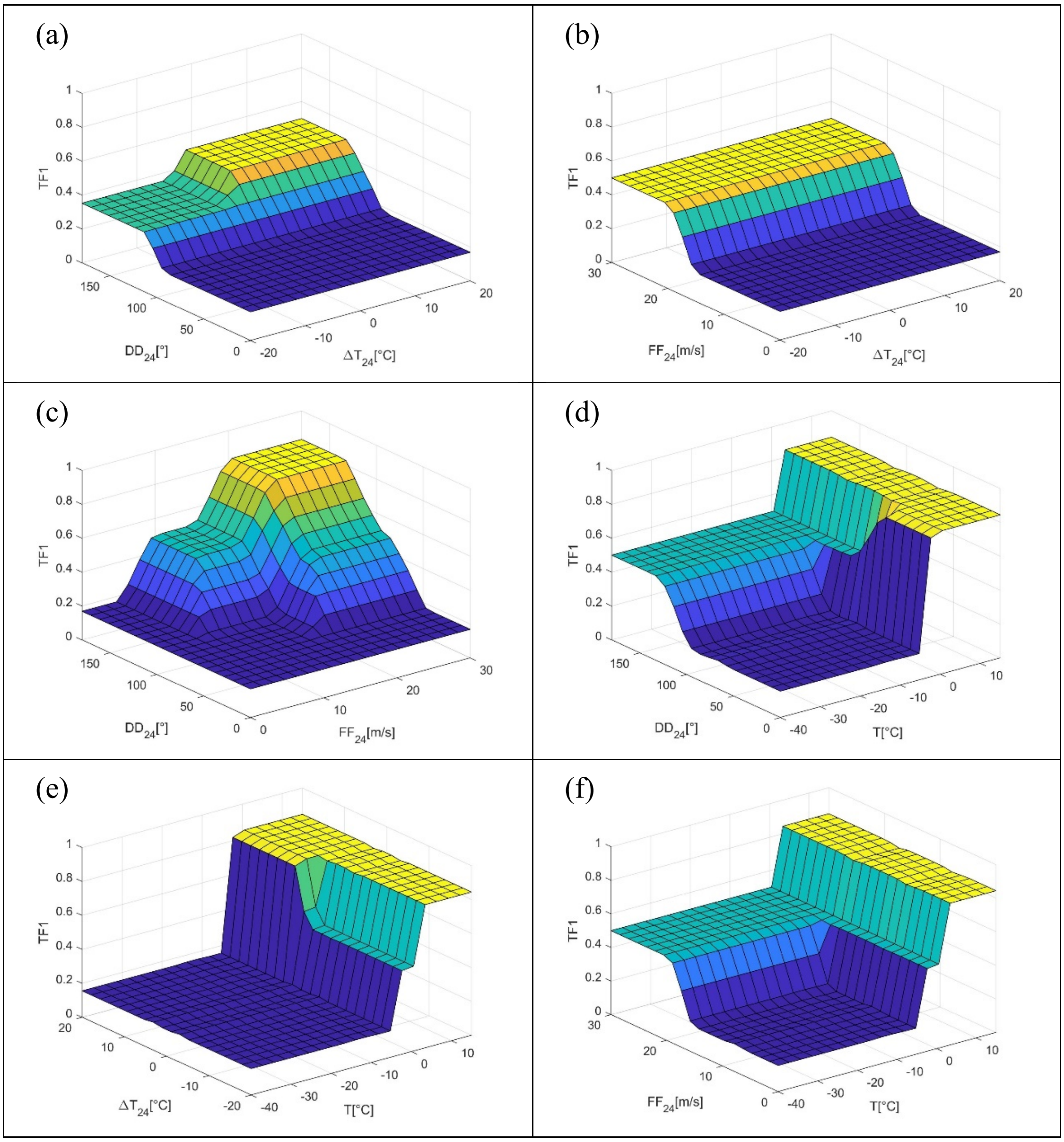

Figure 6 Example three-dimensional projections of the multi-dimensional output surface of FS1 for: (a) ΔT_{24} *vs.* DD_{24}*; (b)* ΔT_{24} *vs.* FF_{24}*; (c)* FF_{24} *vs.* DD_{24}*; (d)* T *vs. .* DD_{24}*; (e)* T *vs.* ΔT_{24}*; (f)* T *vs.* FF_{24}

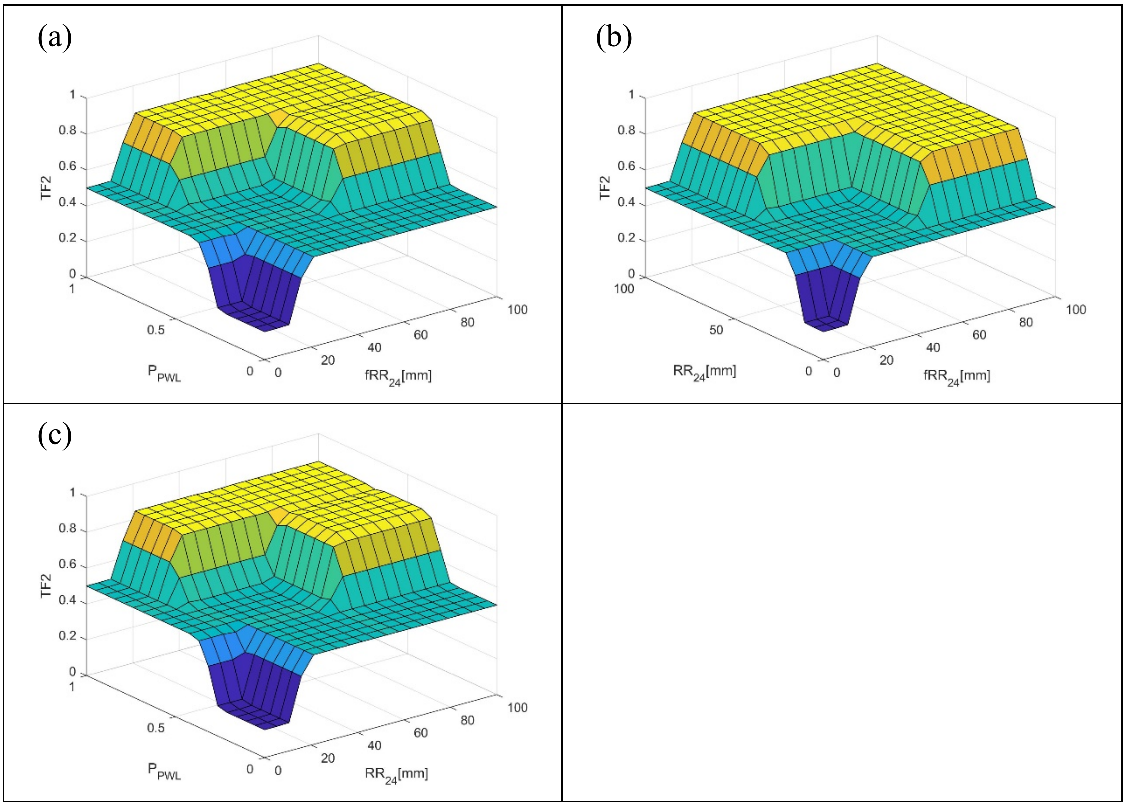

Figure 7 Example three-dimensional projections of the multi-dimensional output surface of FS2 for: (a) fRR_{24} vs. P_{PWL}; (b) fRR_{24} vs. RR_{24}; (c) RR_{24} vs. P_{PWL}

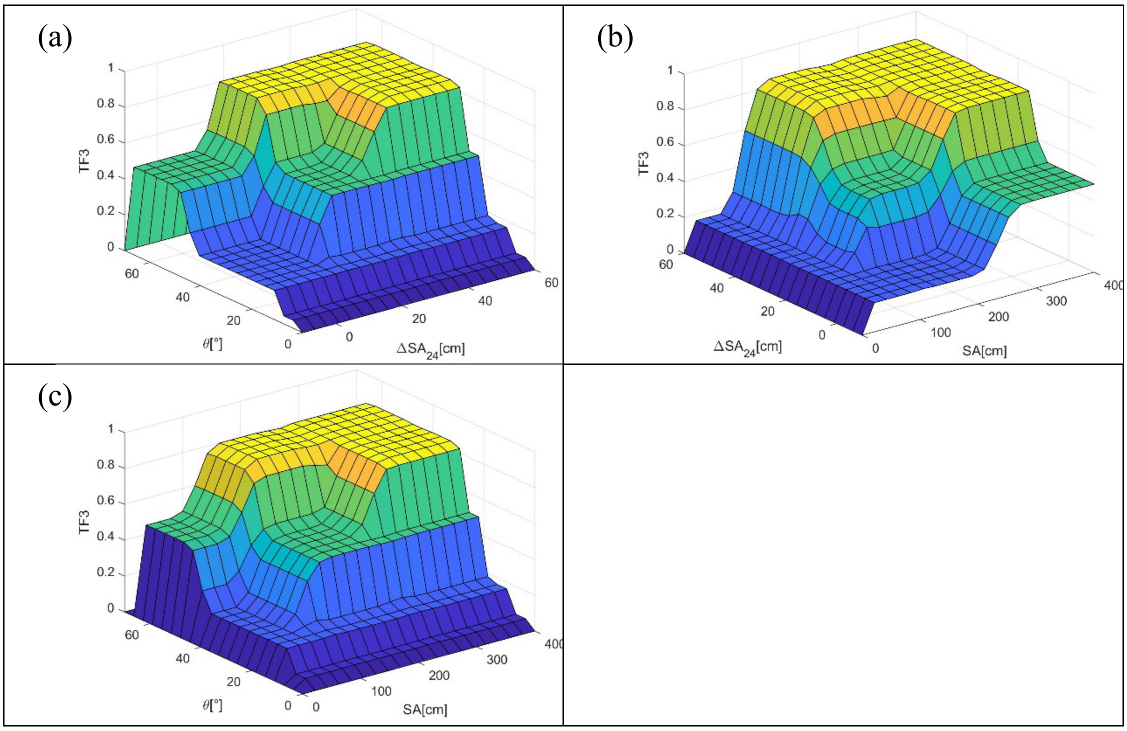

Figure 8 Example three-dimensional projections of the multi-dimensional output surface of FS3 for: (a) ΔSA_{24} vs. θ; (b) SA_{24} vs. ΔSA_{24}; (c) SA_{24} vs. θ

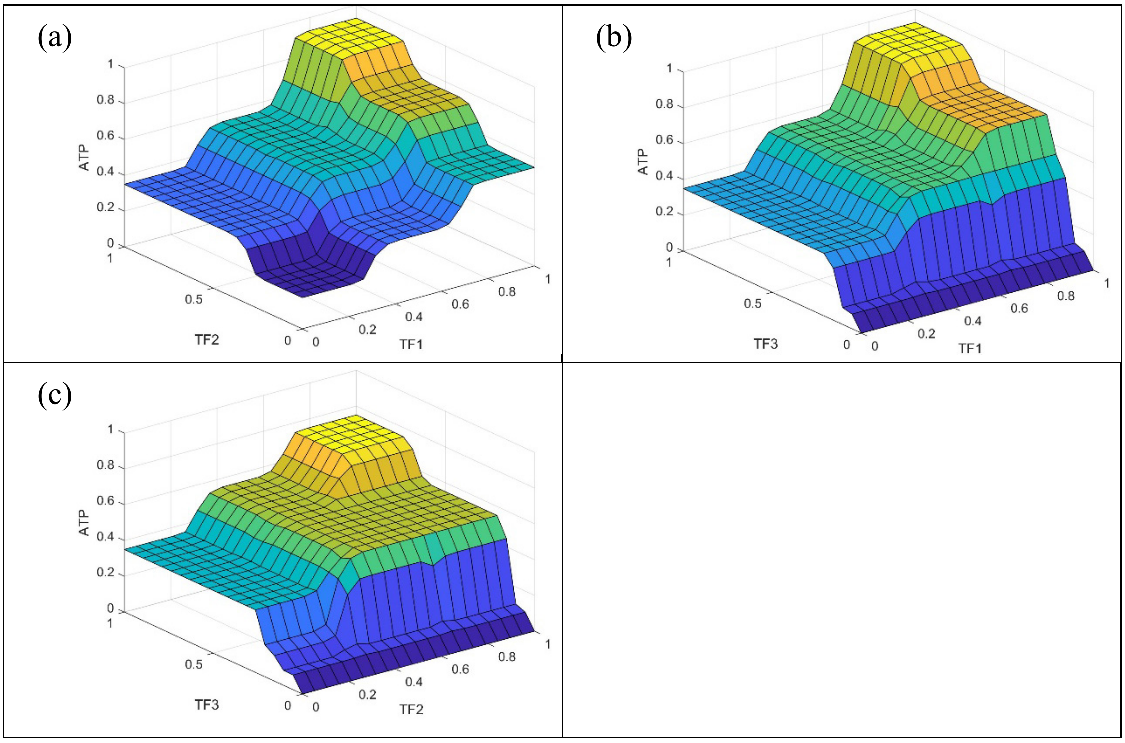

Figure 9 Example three-dimensional projections of the multi-dimensional output surface of FST for: (a) TF1 vs. TF2; (b) TF1 vs. TF3; (c) TF2 vs. TF3

Visual inspection of the three-dimensional projections of the "hybrid" fuzzy systems, presented in Figure 6 – Figure 9, did not find any obvious inconsistencies.

6 Sensitivity testing

The model's behaviour can be well understood by looking at the results of parametric sensitivity tests. However, performing sensitivity tests on the entire model is neither possible nor meaningful because the model comprises a potentially unlimited number of scenarios. Varying each parameter one at a time given that all the other parameters are fixed, would yield different ranges of triggering probabilities for each scenario. Thus, to acquire a general idea of how each parameter may influence ATP, sensitivity tests were performed for the following five notable scenarios:

- Scenario 1 – "Very unlikely avalanche release";
- Scenario 2 – "Very probable avalanche release";
- Scenario 3 – "Moderately possible avalanche release";
- Scenario 4 – "Spring avalanche"; and
- Scenario 5 – "Wind-slab avalanche".

The sets of values of input parameters to the fuzzy ensemble are given in Table 8.

Table 8 Input parameters for sensitivity analysis

Scenario	T	ΔT_{24}	FF_{24}	DD_{24}	RR_{24}	fRR_{24}	P_{PWL}	SA	ΔSA_{24}	θ
1	-15	-5	1	W	1	0	0.1	30	-1	30
2	+7	+12	25	W	50	30	0.9	200	+30	57
3	0	+3	6	W	10	10	0.5	110	+10	37
4	0	0	3	S	0	0	0.9	150	-2	45
5	-8	-1	16	W	30	30	0.5	150	+50	37

Since the wind direction would have different effect on the ATP depending on the cell's exposition, the results of the sensitivity analysis are shown in Figure 10 – Figure 19 for eight chosen expositions; namely: *N, NE, E, SE, S, SW, W* and *NW*, separately for each of the scenarios. To ease the reading of the figures, the lines have been slightly offset: from –1.2% for the north exposition through +1.6% for the northwest exposition. The small inconsistences in the courses of these parameters observed in the figures are inherited from the fuzzy approach, explained by the transition from one parameter class to the next one.

In Scenario 1, in which all the parameters take their "least dangerous" values for an avalanche release, the model seems to be scarcely sensitive to the changes in one individual parameter: eight of the ten input parameters (everyone except for snow height, SA, and terrain inclination, θ) do not have any effect on the ATP (Figure 10 and Figure 11). The ATP in this scenario may attain a maximum value of approximately 20% when SA exceeds 100cm. For the terrain cells flatter than about 30° and/or steeper than about 65°, ATP falls to 0.

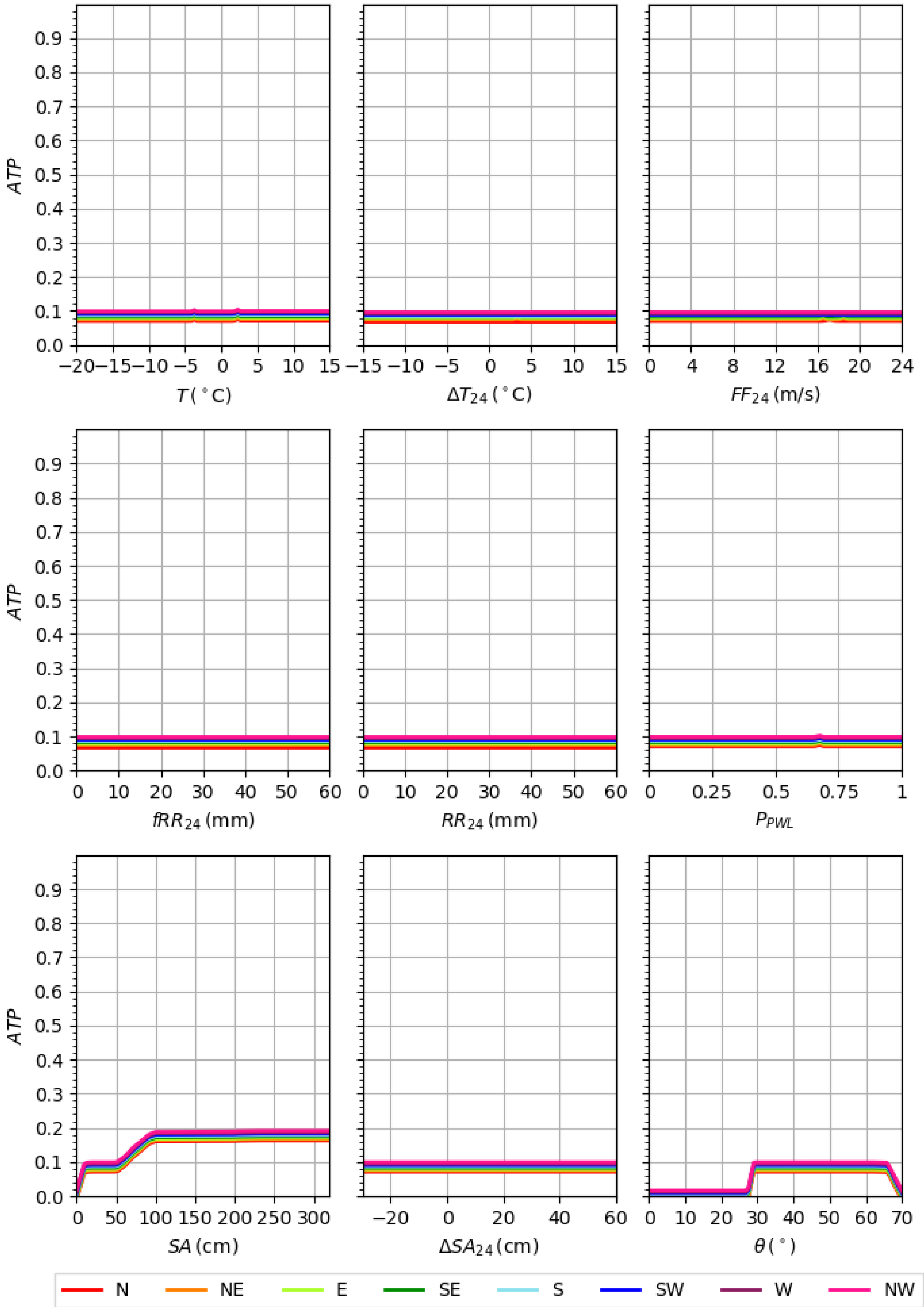

Figure 10 Sensitivity analysis for Scenario 1 ("Very unlikely avalanche release")

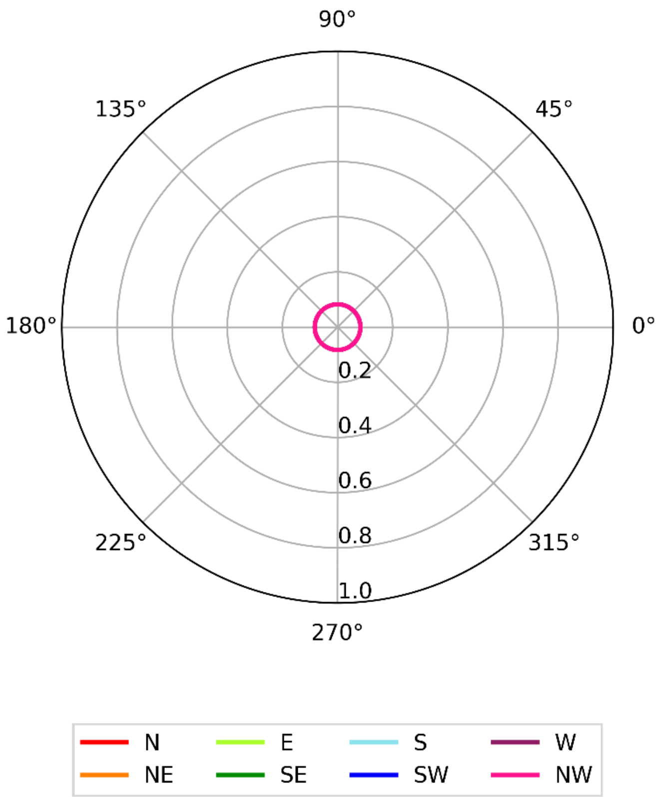

Figure 11 Sensitivity towards the forecasted wind direction during next 24h, $\boldsymbol{DD_{24}}$*, in the Scenario 1*

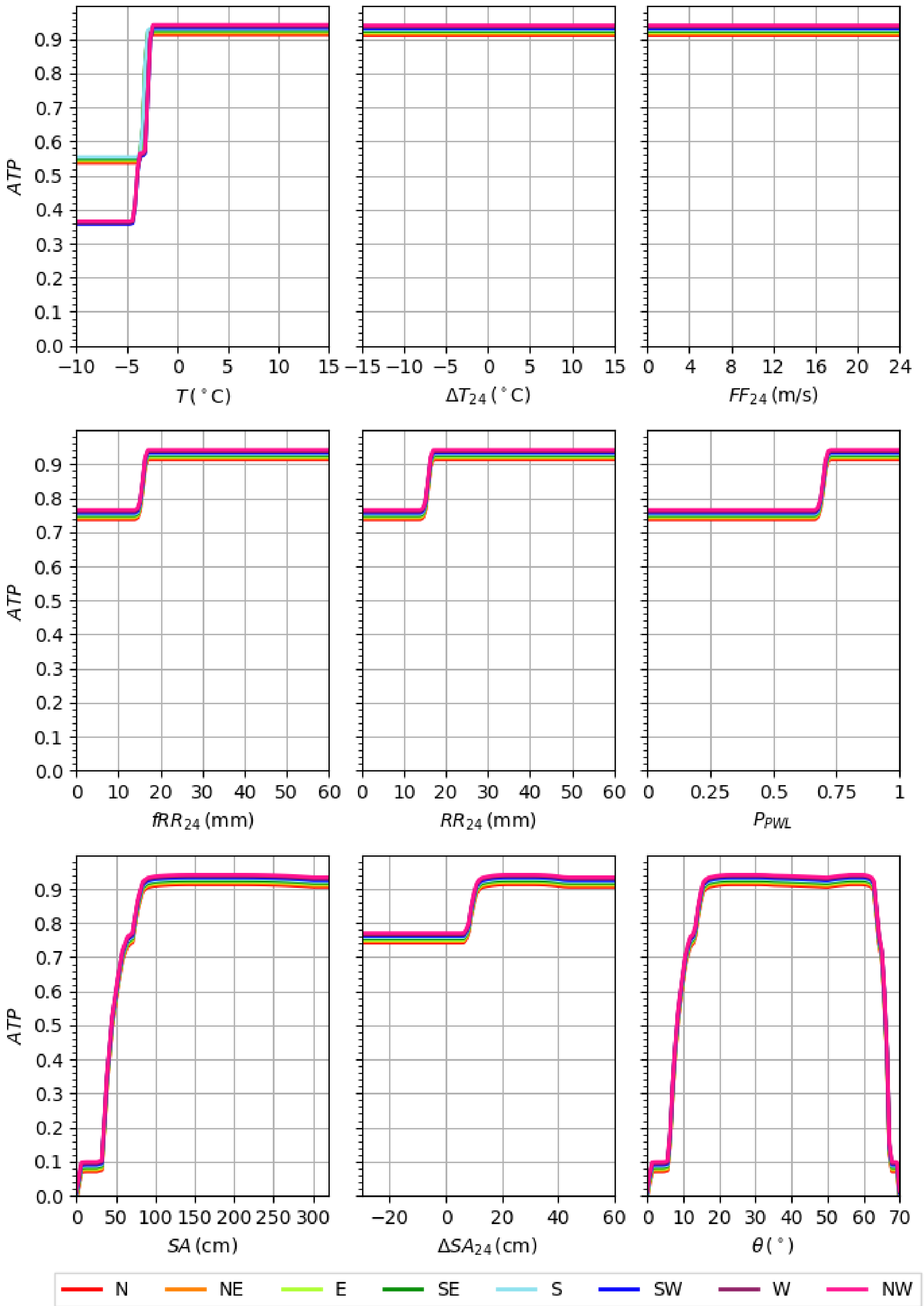

Figure 12 Sensitivity analysis for Scenario 2 ("Very probable avalanche release")

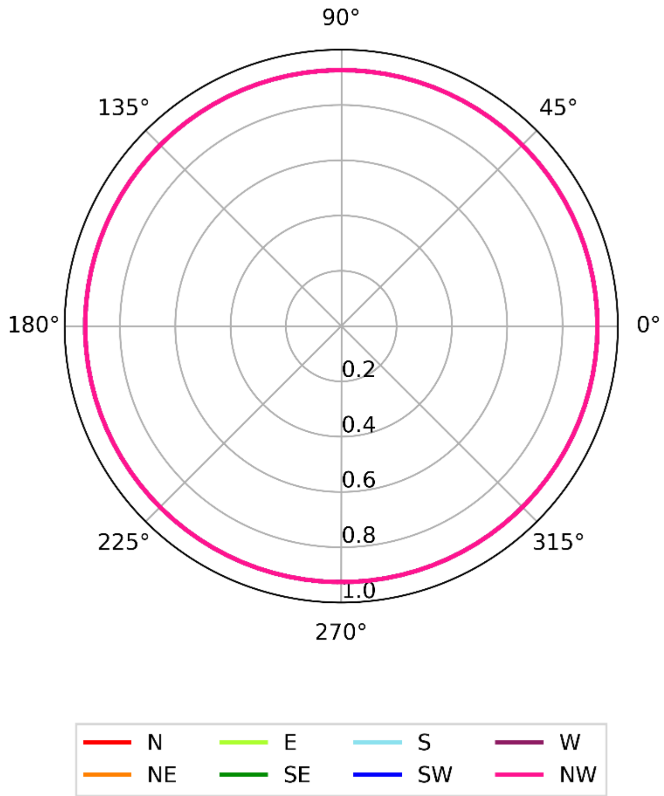

Figure 13 Sensitivity towards the forecasted wind direction during next 24h, $\boldsymbol{DD_{24}}$, in the Scenario 2

Even though in Scenario 2 (*"Very probable avalanche release"*) all the parameters are set to their "highly dangerous" values for an avalanche release, a decrease in value of almost each parameter separately will result in lower predicted ATP, demonstrating that the model is sensitive to every parameter even at the high values of ATP (Figure 12). Only wind direction, DD_{24}, wind velocity, FF_{24}, and forecasted change in air temperature, ΔT_{24}, show no effect on ATP in this scenario (Figure 13). This is possibly because the forecasted air temperature, T, in this scenario is high (+7°C). When $T < -3$°C given all the other parameter values in this scenario, leeward and parallel slopes (with respect to the wind direction) will have a higher ATP due to possible snow transport / snow drift.

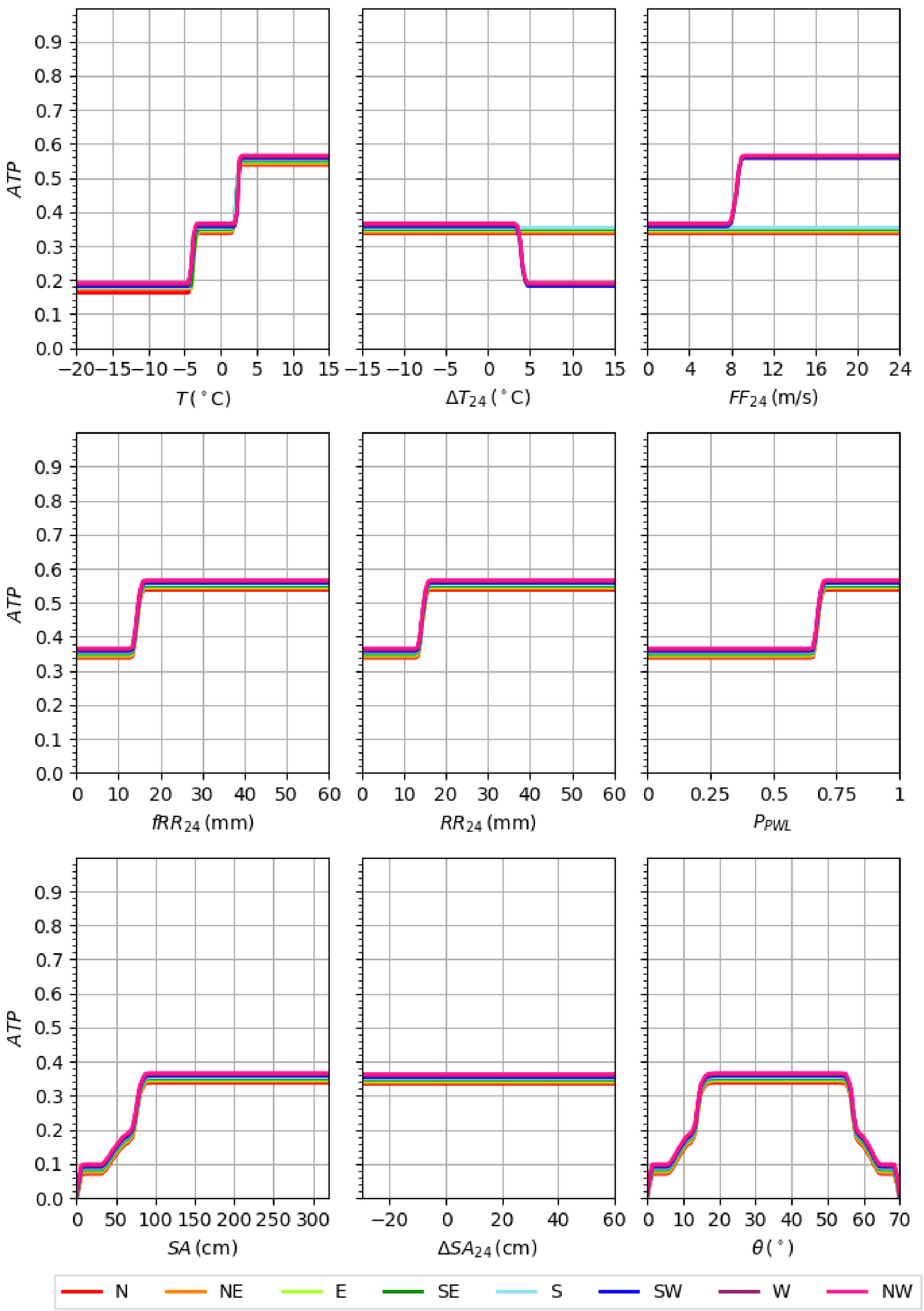

Figure 14 Sensitivity analysis for Scenario 3 ("Moderately possible avalanche release")

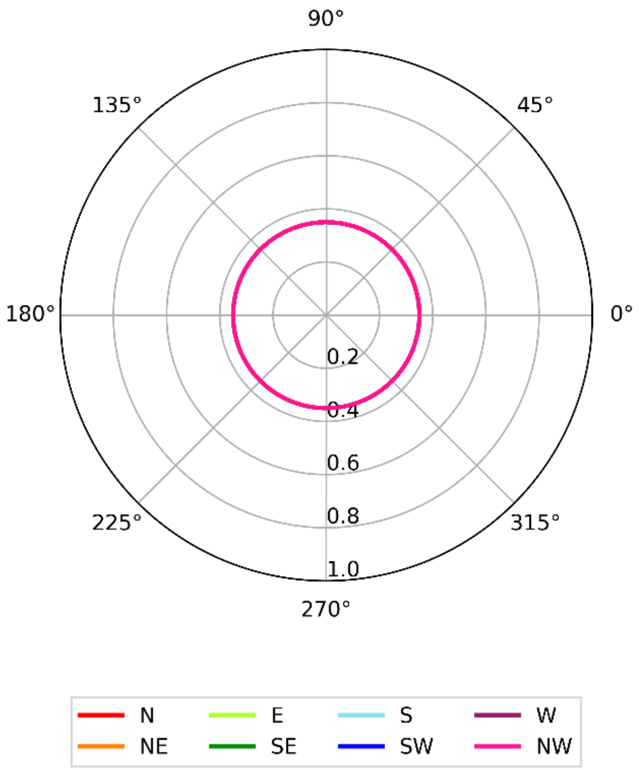

Figure 15 Sensitivity towards the forecasted wind direction during next 24h, $\boldsymbol{DD_{24}}$, in the Scenario 3

Scenario 3 and Scenario 4 are rather similar in the parameter values. This is reflected in the similar results of sensitivity testing for these scenarios (Figure 14 – Figure 17). In Scenario 3 ("Moderately possible avalanche release"), ATP values range below 57%. There is a clear separation in ATP values between windward and leeward slopes, where the latter display up to about 15% higher ATP when $\Delta T_{24} > +3°\text{C}$, and the opposite, i.e. about 15% higher ATP when $FF_{24} > 9\text{m/s}$. The same pattern can be observed in Scenario 4 but with bigger separation of about 20%. The higher ATP values conditioned on positive forecasted change in air temperature, ΔT_{24}, i.e. stronger than 3°C, are explained by possible snow transport since the air temperature, T, would start raising from below -3°C and up to 0°C in these scenarios. The lower ATP values conditioned on strong wind suggest a possible stronger energy flux to the windward slopes when $T \geq 0°\text{C}$ and $\Delta T_{24} \leq +3°\text{C}$ as this is the case in both scenarios.

In Scenario 4 ("Spring avalanche") ATP values reach up to approximately 75%, (Figure 16 and Figure 17; note that wind blows from south in this scenario). It is interesting to note that in this scenario the model is invariant to the amount of observed and forecasted precipitation (RR_{24} and fRR_{24}) and to changes in snow height, ΔSA_{24}. Moreover, cells with terrain inclination of as low as 10° display relatively high ATP showing that the model is able to address wet snow avalanches and slush flows as well.

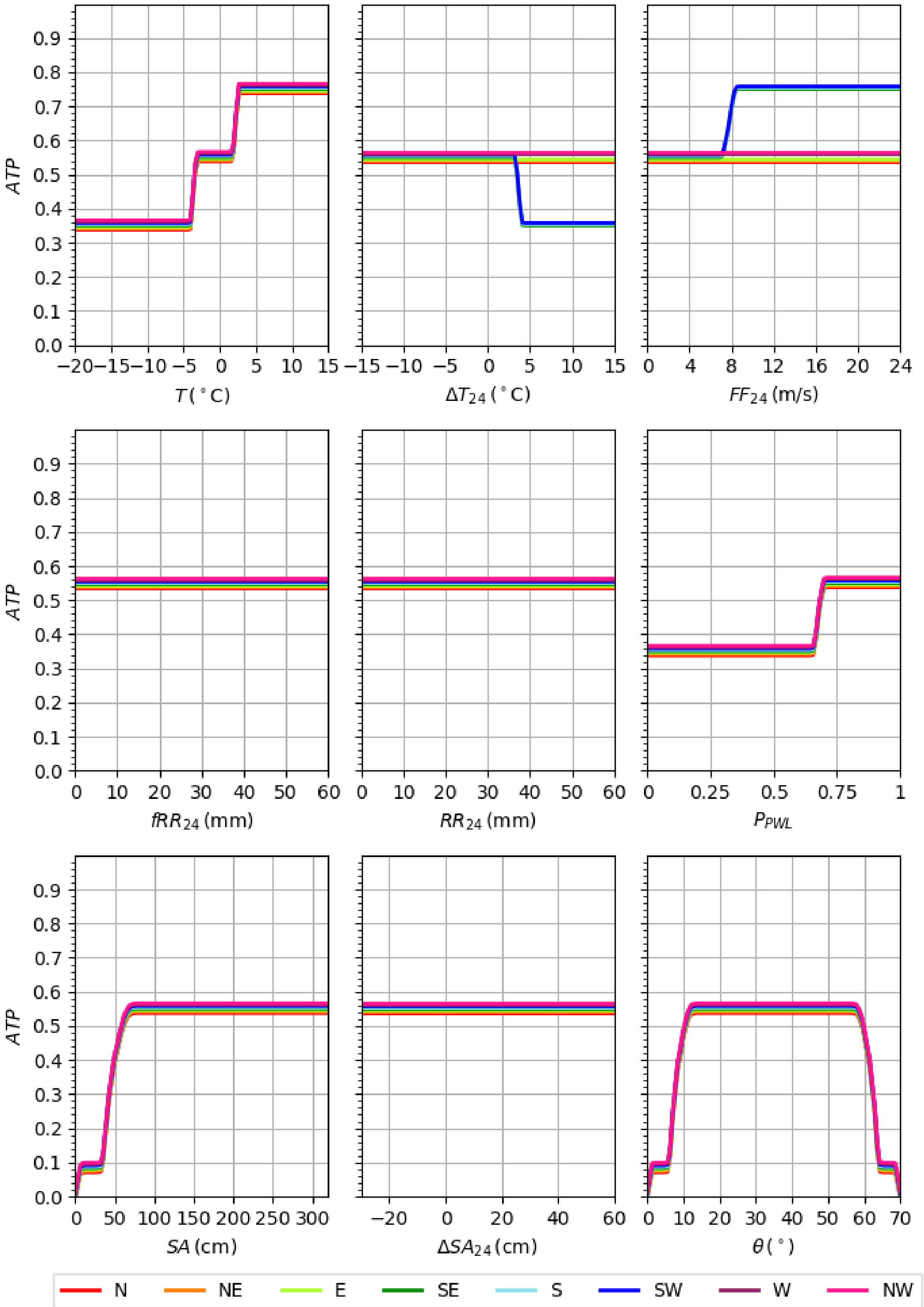

Figure 16 Sensitivity analysis for Scenario 4 ("Spring avalanche")

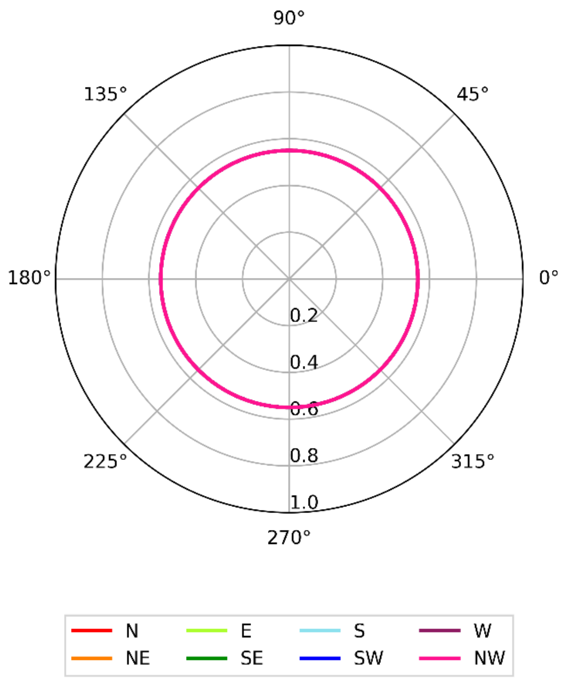

Figure 17 Sensitivity towards the forecasted wind direction during next 24h, $\boldsymbol{DD_{24}}$, in the Scenario 4

In Scenario 5 ("Wind-slab avalanche") (Figure 18 and Figure 19), three input parameters (T, FF_{24} and DD_{24}) influence the ATP when $SA > 50$cm and slope $\theta \in (30°; 65°)$. The insensitivity to observed precipitation, RR_{24} can be explained by the high value of the forecasted precipitation, fRR_{24}, which provides significant volumes of snow available for wind transport, and vice versa: insensitivity to fRR_{24} can be explained by the high value of RR_{24}.

Leeward slopes are clearly more prone to avalanche release in this scenario in all cases with negative air temperature and steep terrain. This result may account for a possible snow transport/ snow drift which is also reflected in the sensitivity to wind direction, DD_{24} (Figure 18 and Figure 19).

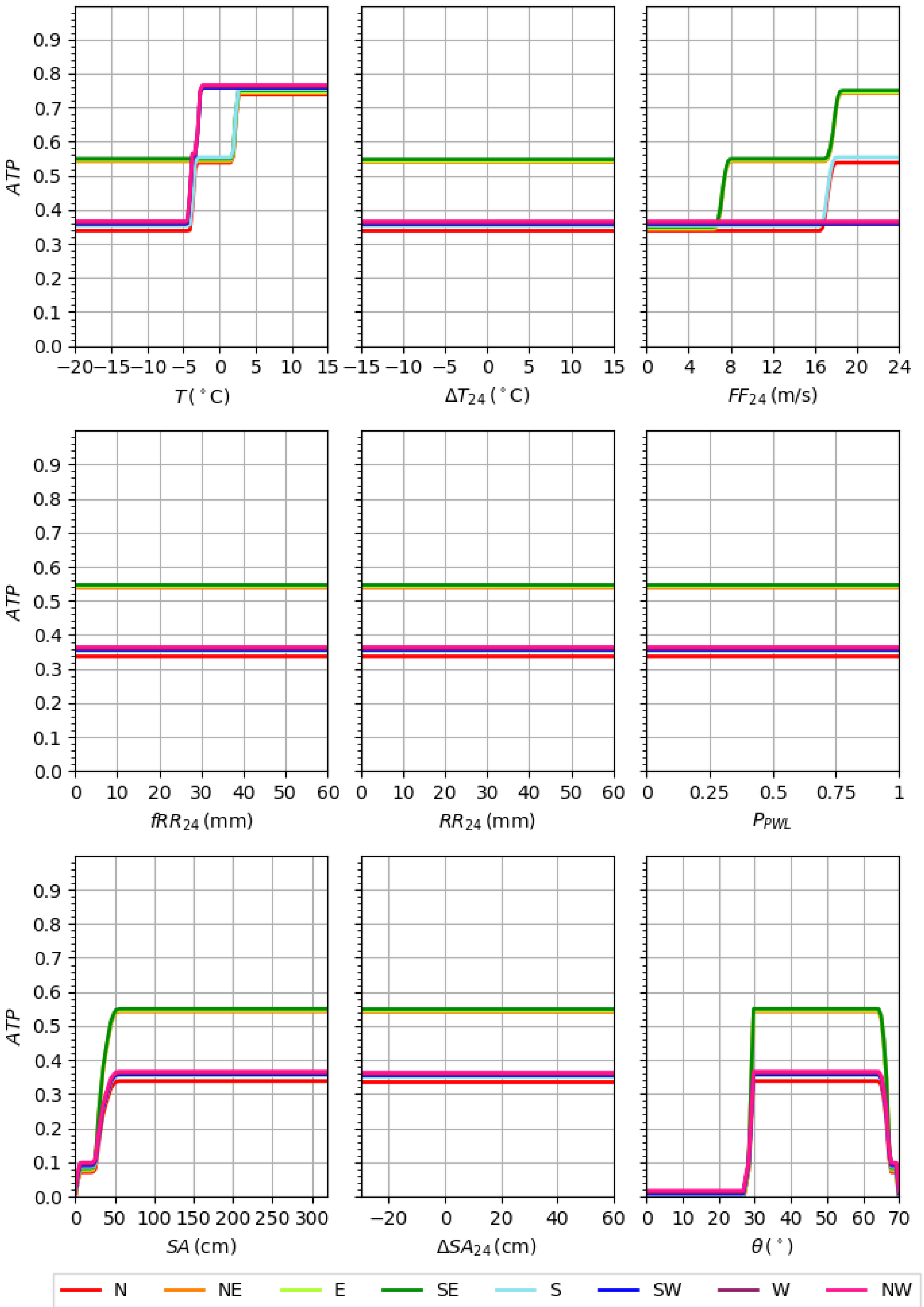

Figure 18 Sensitivity analysis for Scenario 5 ("Wind-slab avalanche")

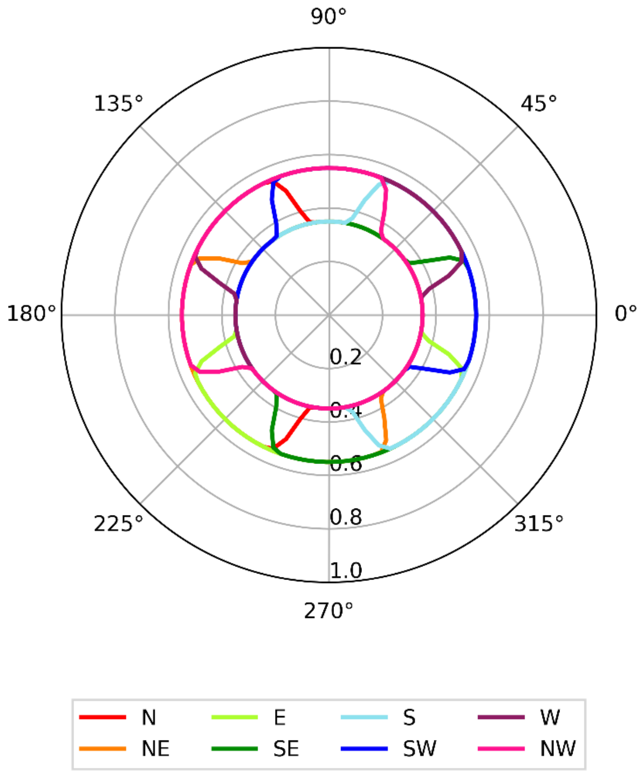

Figure 19 Sensitivity towards the forecasted wind direction during next 24h, $\boldsymbol{DD_{24}}$, in the Scenario 5

The sensitivity analysis conducted for all five scenarios showed that forecasted air temperature has the strongest influence on ATP. ATP is quasi-constant for all cells with terrain inclination between 30° and 55° as well as for all snow heights over 100cm, independently of the values of other parameters.

The obvious weaknesses of the model at the present stage based on the sensitivity analysis presented above, are:

1. Unnatural step-wise variation of ATP
2. Rather high values of about 10% ATP for cases with stable snowpack conditions
3. Absent or limited sensitivity to precipitation in some scenarios

7 Validation

Visual inspection presented in Section 5.2, allows the qualitative expert assessment of the consistency of the fuzzy inference outputs to expert knowledge, and the sensitivity test performed in Section 6, provides an idea of how each input parameter influences the system. A further assessment can be conducted through validation of the fuzzy inference system using real data sets.

7.1 Data sets

Two different data sets were used for validation of the model.

The first data set (hereinafter referred to as "LAWS-data set") represents an extract from the Norwegian Local Avalanche Warning Service log for 2015-2018, 632 entries. The LAWS data set includes only data from areas which contain a meteorological station in close proximity to the area of interest to minimise the uncertainty with respect to weather conditions, as well as expert judgement about the Avalanche Danger Level (ADL) during the coming 24h assigned according to the EAWS Avalanche Danger Scale (http://www.avalanches.org/eaws/en/main_layer.php?layer=basics&id=1). However, the data set does not include information neither about P_{PWL}, since it is not usually observed at the Norwegian stations, nor about the terrain inclination, because the warnings are meant for mountain areas with naturally varying slope. Information about observed height of new snow during the last 24h was used instead of ΔSA_{24}, because the latter is not among the recorded parameters in the warning system. This limits the parameter to exclusively positive values in the LAWS data set, whereas in the reality the parameter may take negative values as well.

The second data set (hereinafter referred to as “Stillberg data set”) was compiled from the data kindly provided for this study by Peter Bebi (WSL Institute for Snow and Avalanche Research, SLF). It represents daily snow avalanche observational data from the Stillberg research experimental field, Switzerland, over 17 consecutive years from autumn 1975 (i.e., over 6000 days), for a total of 298 days with overall 1332 avalanche events. Meteorological data from the meteorological station situated in the field, for the same period were downloaded from https://www.envidat.ch/dataset/stillberg-climate (Bebi, 2016) and processed for use as an input to the model. The Stillberg data set does not include information about P_{PWL} since this parameter is not usually observed at meteorological stations. The Stillberg research field has a general *N-NE-E* exposition, with local variations. Even though the exposition of the slopes where each avalanche was released was provided in the initial data set, this information was not used for this study when the avalanche events were aggregated/combined by the days they were observed.

7.2 Results

7.2.1 LAWS-data set (so-called classification conformity-based validation)

Classification conformity-based validation relies on the statistical characterization of the uniformity of avalanche danger classification between model outputs and expert-based assessment for an extensive database of case-histories. In other words, ADL is compared to a classification parameter C_{ATP} derived from the ATP output by the fuzzy ensemble for the LAWS data set cases calculated for the same weather conditions:

$$C_{ATP} = \frac{\sum \mu_i C_i}{\sum \mu_i} \tag{5}$$

where μ_i represents the membership value described in equation (1), and $C_i \in \{1, 2, 3, 4, 5\}$ is the respective class for ATP from Table 7, where 5 corresponds to “Very high”, 4 to “High”, 3 to “Considerable”, 2 to “Moderate”, and 1 includes “Low”, “Very low” and “Null”. The C_{ATP} parameter, thus, acts as a classification parameter of ATP weighted according to fuzzy membership.

It is important to remark that ADL accounts for both number of avalanches and their size/volume in addition to the probability of avalanche triggering, whereas ATP is only concerned with the latter. Notwithstanding this difference, the two parameters are compared for validation purposes, with ADL representing the “ground truth” and ATP representing model output.

The values of the expert forecasted ADL, C_{ADL}, were compared with the respective values of C_{ATP} in form of frequency histograms of the difference $C_{ATP} - C_{ADL}$ (Figure 20 and Figure 21). A good performance of the model minimizes the absolute value of such difference.

Since ADL is meant to include all slope expositions, and due to the necessity to specify slope exposition for ATP estimating, C_{ATP} was calculated separately for eight expositions. Due to the lack of precise information about the terrain inclination and P_{PWL} and based on the sensitivity test presented in section 5.3, two different scenarios for C_{ATP} were used when it comes to these lacking parameter values:

1. $\theta = 37°$, $P_{PWL} = 50\%$
2. $\theta = 37°$, $P_{PWL} = 90\%$

The value $\theta = 37°$ was chosen as the steepness which is correspondent with the highest number of observed dry slab avalanches according to McClung&Schaerer (2006).

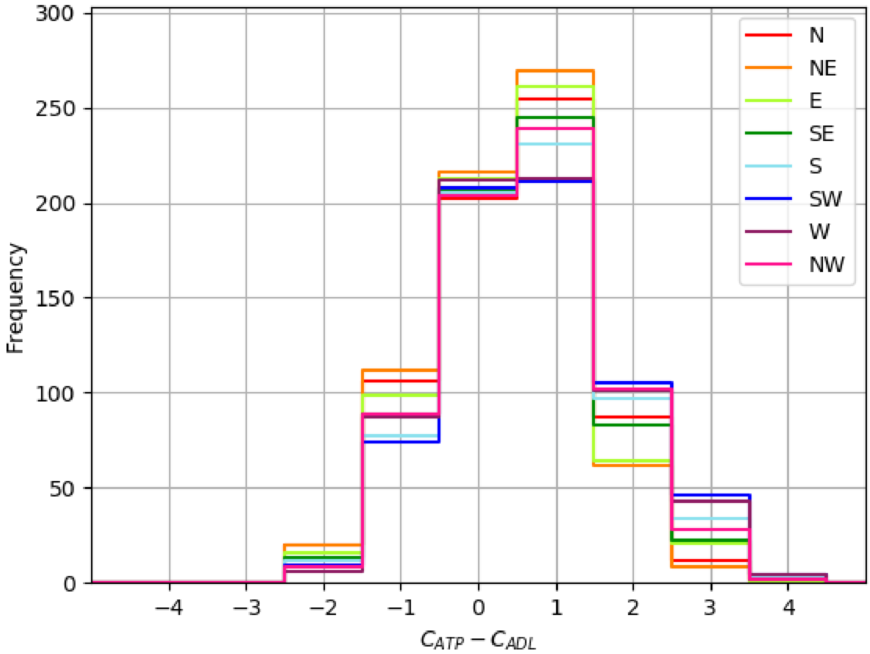

Figure 20 Frequency histogram of the difference between C_{ADL} and C_{ATP} when $\theta = 37°$and $P_{PWL} = 50\%$

Figure 20 shows that for $P_{PWL} = 50\%$, the difference between the forecasted ADL and calculated ATP is primarily within 1 class having a slightly bigger weight on over-estimating ATP. The difference lays in the range between -2.5 through roughly +3.5 indicating that overestimating ATP is a somewhat general tendency.

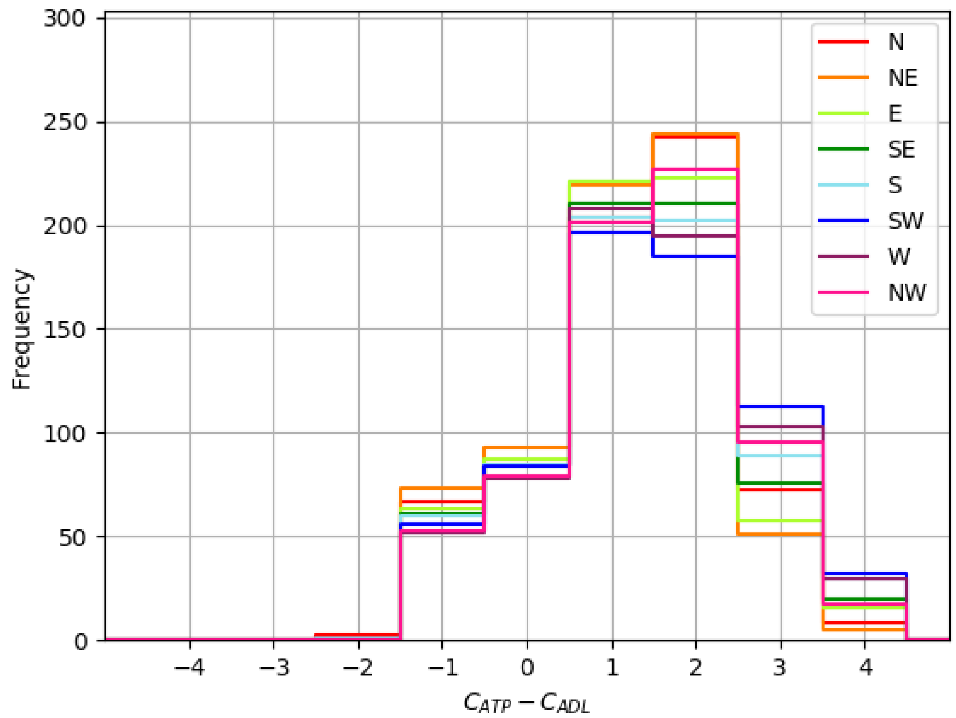

Figure 21 Frequency diagram of the difference between C_{ADL} and C_{ATP} when $\theta = 37°$and $P_{PWL} = 90\%$

When $P_{PWL} = 90\%$, the tendency of overestimating ATP compared to ADL becomes even more clear: the slightly left-skewed bell-shape is centred at approximately +1.5 classes (Figure 21). This pattern of overestimating the ATP is expected since the "ground truth" does not change, however increasing P_{PWL} logically leads to increasing ATP.

The effect of slope exposition on the frequency distribution is also investigated. The difference in frequency between the expositions for the cases with the same class difference can be as large as up to about 65 occurrences (out of the total 632 cases). While there is no consistent regularity in this observation, ATP is more often under-estimated on the *S-SW-W* slopes, whereas ATP is more often overestimated on the *NW-N-NE* slopes. This is not a general result but related to the specific study areas.

7.2.2 Stillberg data set (categorization-based validation)

Testing our model towards observed avalanches has a great value for understanding of the model's actual performance. In pursuing model validation through the Stillberg data set, the following conditions should be remarked, though:

- The model was developed for the Norwegian conditions, not Swiss, i.e. ATP may be considerably dependent on other parameters which were not included in the Norwegian model (e.g. solar radiation), and/or the fuzzy rules-setting may be different for Swiss conditions

- An observed avalanche does not necessarily mean that the ATP was 100%
- Not observed avalanches do not necessarily mean that the ATP was 0%
- Number of the observed avalanches may not be linearly associated with ATP

As mentioned above, the Stillberg research field has a general *N-NE-E* exposition with local variations. This poses a question about which representative value of ATP should be compared with the "ground truth". A parametric analysis was conducted by plotting the cumulative amount of observed avalanches (normalized by the total amount of observed avalanches) versus: (1) the mean; (2) the minimum; and (3) the maximum of the three estimates of ATP for the *N*, *NE* and *E* expositions, namely ATP_N, ATP_{NE} and ATP_E, respectively. Figure 22 shows that the "mean value" displays a smoother relationship with the number of daily observed avalanches compared to both the "minimum value" and the "maximum value" cases. All of the types present a pronounced step-wise relationship with the number of avalanches. There is almost no difference between the expositions when ATP is below about 17% and above about 55%.

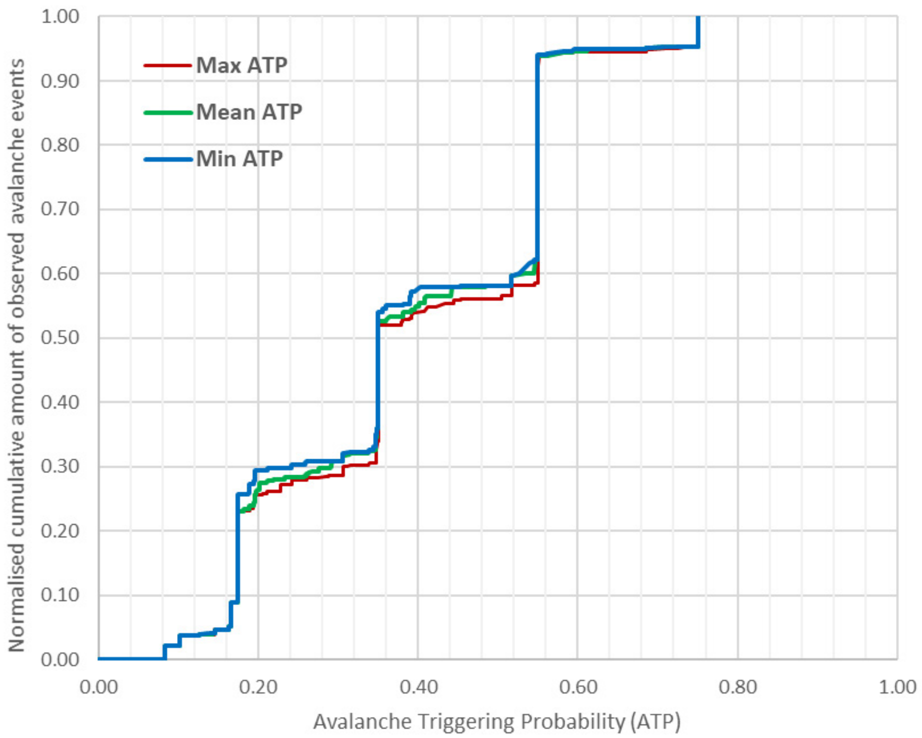

Figure 22 Relationship between the normalized cumulative number of daily observed avalanches and ATP (for P_{PWL} = 50%).

The model developed herein requires information about the terrain inclination, θ, and the probability of the persistent weak layer, P_{PWL}, which were not provided in the data set. To this purpose, it was decided to set θ=37° which is the terrain inclination corresponding to the highest number of observed dry slab avalanches according to McClung & Schaerer (2006).

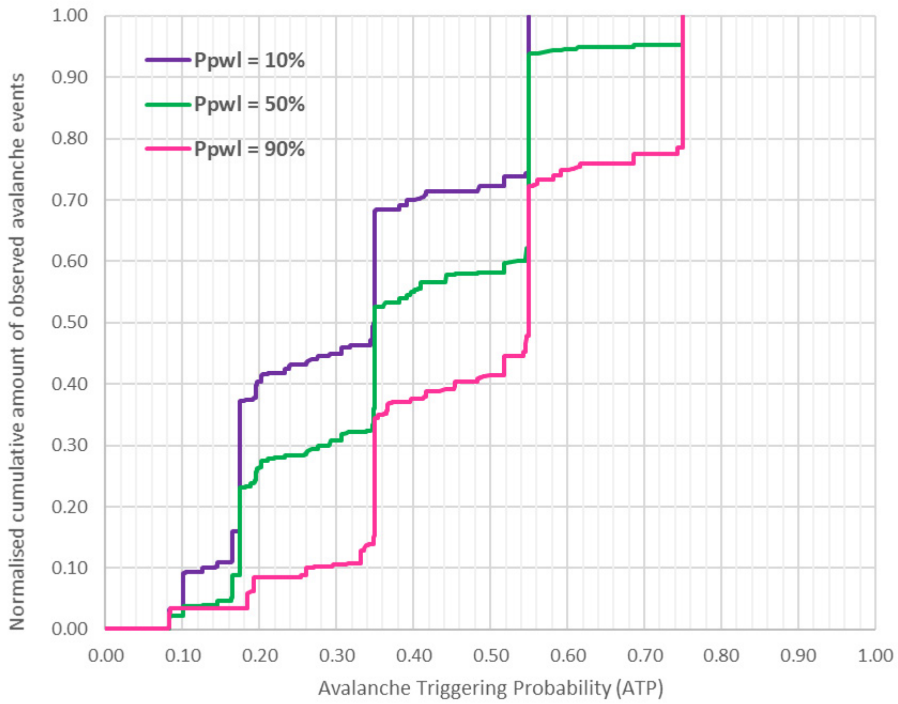

Figure 23 Relationship between the normalized number of daily observed avalanches and the mean ATP for $P_{PWL} = 10\%$, $P_{PWL} = 50\%$ and $P_{PWL} = 90\%$.

Figure 23 plots the correlation between the mean ATP for the three expositions and the normalized cumulative amount of observed avalanches for three values of P_{PWL}; namely: $P_{PWL} = 10\%$, $P_{PWL} = 50\%$ and $P_{PWL} = 90\%$. The plot reflects the increase in ATP with increasing P_{PWL}. The correlations converge for $ATP < 8\%$, $ATP \approx 35\%$, $ATP \approx 55\%$ and for $ATP > 75\%$, indicating that P_{PWL} may have a limited influence on ATP compared to other input parameters in these intervals and that occurrence of the majority of the observed avalanches may be associated with the ATP values of about 35%, 55% and 75%, as well as with $ATP \approx 17\%$ when $P_{PWL} \leq 50\%$.

Just plotting the days with avalanche number observed and the respective values of ATP for these days (Figure 24, Figure 25 and Figure 26) does not provide a clear understanding of how well the model is associated with the "ground truth". However, these figures provide a good overview of the data set used for the model testing. We can see, among other things, that the period 1975-1981 had days with much higher number of observed avalanches compared to the later seasons.

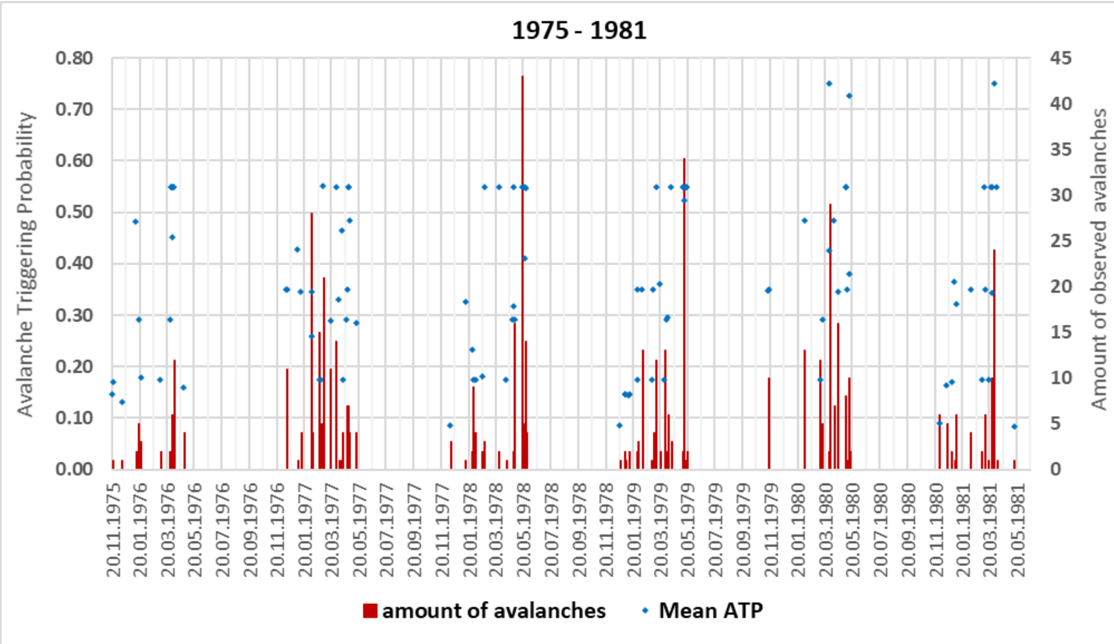

Figure 24 Number of daily observed avalanches in 1975-1981 with the respective values of ATP. $P_{PWL} = 50\%$, $ATP = mean(ATP_N, ATP_{NE}, ATP_E)$.

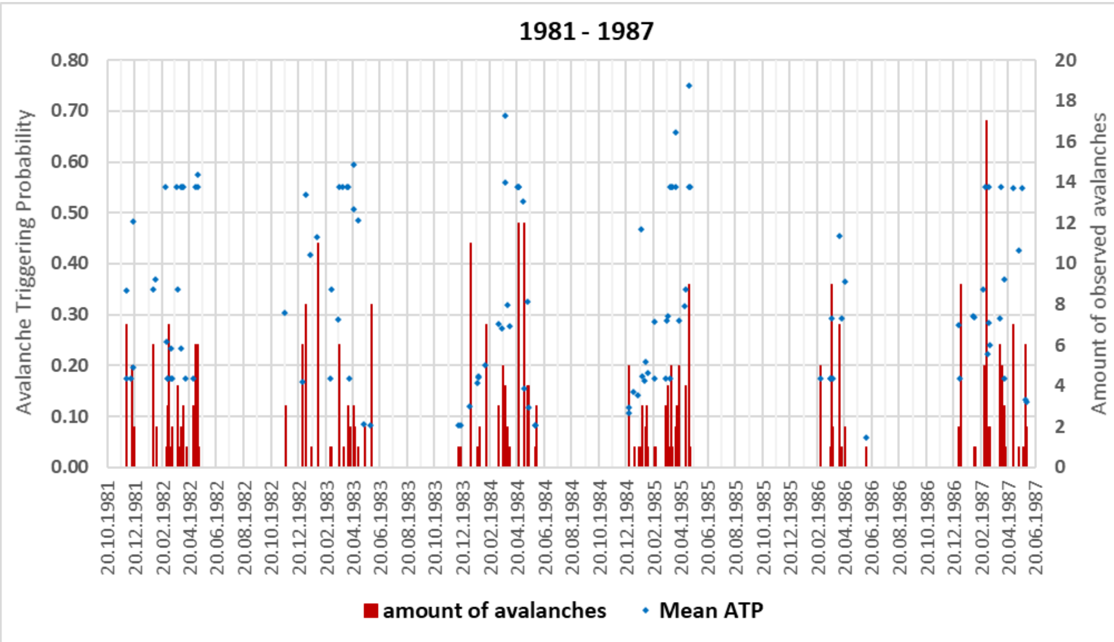

Figure 25 Number of daily observed avalanches in 1981-1987 with the respective values of ATP. $P_{PWL} = 50\%$, $ATP = mean(ATP_N, ATP_{NE}, ATP_E)$.

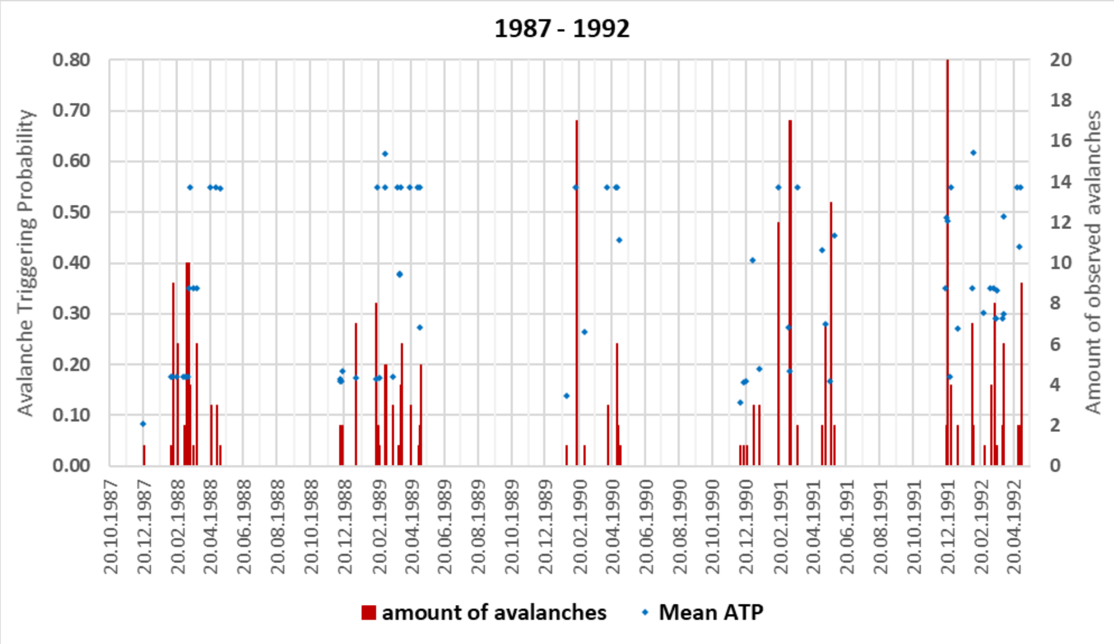

Figure 26 Number of daily observed avalanches in 1987-1992 with the respective values of ATP. $P_{PWL} = 50\%$, $ATP = mean(ATP_N, ATP_{NE}, ATP_E)$.

In order to validate the model against the Stillberg data set, exposition-averaged ATP values were compared with database assessments of avalanche occurrence or non-occurrence. More specifically, for the binary "avalanche – no avalanche" mode and for a more refined quantitative classification mode in which the number of avalanches observed on a given day at the site is provided.

The box-whisker plots in Figure 28 provide relevant descriptive sample statistics (i.e., minimum, 25th percentile, median, mean, 75th percentile and maximum) of exposition-averaged ATP model-calculated for P_{PWL}=50% with respect to dataset-based avalanche observation classification modes (Figure 28a for the binary classification mode and Figure 28b for the quantitative classification mode). Both plots display a strong association between the exposition-averaged ATP and avalanche observations. Inter-quantile ranges (i.e., the difference between the 75th and 25th percentiles) are almost disjoint between the "no avalanche" and "avalanche" categories, with only a few outliers with high values of ATP for the "no avalanche" category. Some of the outliers may be explained by lacking snowpack-availability, i.e., when an "avalanche" weather continues but all avalanches have already been triggered and there is no more snow available for producing new avalanches. Aside from the outliers, the maximum model-calculated ATP values for no-avalanche cases is 48%. The minimum model-calculated ATP values for cases with observed avalanches is 8%. In assessing the outputs of the validation process critically, it should be remarked that the weather data comes from a weather station and not from the exact locations of avalanche triggering. In the quantitative mode, there is a non-decreasing progression of 25th, 50th (median) and 75th percentiles with increasing number of observed avalanches, as well as a progressive decrease in the inter-quartile range. Considering the amount of analysed data (over 6000 days), the results used in the validation process can be deemed statistically significant.

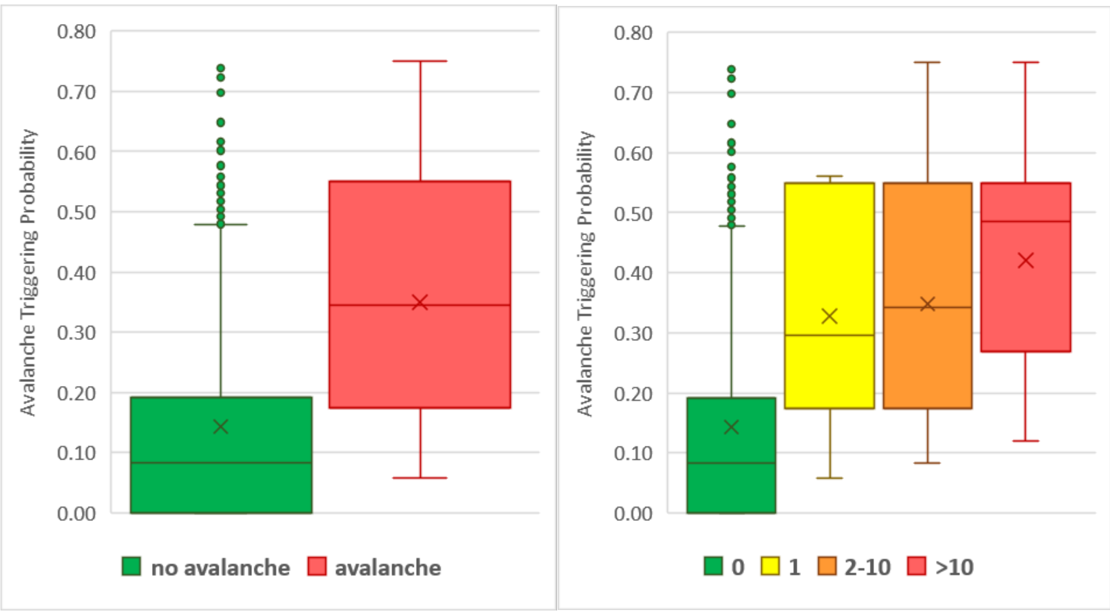

Figure 27 Association between the ATP and the number of daily observed avalanches divided in classes: "no avalanche" vs "avalanche" (on the left side) and "0", "1", "2-10", ">10" (on the right side). $P_{PWL} = 50\%$, $ATP = mean(ATP_N, ATP_{NE}, ATP_E)$.

8 Discussion and concluding remarks

The sensitivity tests presented in section 5.3, as well as the validation of the model presented in section 6, provide a solid ground for understanding of the model performance.

The use of fuzzy logic in building the model allowed the incorporation of qualitative or semi-quantitative expert knowledge and judgement of the snow avalanche release process by operating with parameter classes and *"if – then"* rules.

The sensitivity analysis showed that the model reflects expert belief about the process in a consistent manner. The model is sensitive to the conditions that enable snow transport—negative air temperature and sufficient wind velocity. Moreover, the model recognises conditions conducive to wet snow avalanches and slush flows when the air temperature is positive, even if the terrain inclination is as low as 10°.

Statistical validation of the model against observational data is not a trivial task. The main challenge is the impossibility of directly measuring ATP. To calculate ATP trustworthily from observational data, we would need to bin each of nine input variables into sufficiently fine classes, collect at least several hundred observations for every possible combination of these value classes and record whether they resulted in avalanche triggering or not. It is impossible in practice.

The classification-based validation approach implemented for the LAWS data set showed very good correspondence between the model-calculated ATP and the expert-estimated ADL, with a tendency of C_{ATP} being larger than ADL. This is a rather natural tendency considering that ADL includes both the number of avalanches and their size/volume in addition to ATP. In other words, ADL would be assessed as being lower if there are many small avalanches expected compared to many large avalanches, whereas ATP would stay the same.

The categorization-based validation approach implemented for the Stillberg research field data set showed strong correlation between avalanche (non-)observations and model-calculated ATP values. A dual categorization was attempted, with both binary and quantitative categorization approaches providing meaningful quantitative results and opportunities for further refinement through further formal statistical testing. The presence of outliers in this validation approach may be explained by the lack of snowpack available for an avalanche after avalanches have already been triggered. Snow-avalanche forecasters are usually updated regarding the places where avalanches have recently occurred and can rely on this information when choosing input parameters for the model (e.g. setting SA to a small value). This would allow to decrease the amount of “false-alarms”.

Even though the model shows an overall good correspondence with expert knowledge and with the data sets for which it was tested and validated, there is still a considerable potential for improvement. In addition to the improvement at the points listed in Section 6, validation against further data sets with good daily observations may provide a better ground for further model development.

We are pleased to report that even not being perfect the model provides valuable ATP estimates for the daily activities of the local avalanche warning service in Norway.

9 Acknowledgements

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Appendix A

THE FUZZY RULES

Contents

A1 Introduction

The fuzzy rules used in the ensemble are tabulated in the following for *FS1, FS2, FS3* and *FST* respectively.

A2 Fuzzy rules for *FS1*

	Antecedent (*"If-and-and-and"*)				Consequent ("Then")
Rule No.	T	ΔT_{24}	FF_{24}	DD_{24}	$TF1$
001	very low	low	low	leeward	low
002	very low	low	low	parallel	low
003	very low	low	low	windward	low
004	very low	low	medium	leeward	low
005	very low	low	medium	parallel	low
006	very low	low	medium	windward	low
007	very low	low	high	leeward	high
008	very low	low	high	parallel	medium
009	very low	low	high	windward	low
010	very low	medium	low	leeward	low
011	very low	medium	low	parallel	low
012	very low	medium	low	windward	low
013	very low	medium	medium	leeward	medium
014	very low	medium	medium	parallel	low
015	very low	medium	medium	windward	low
016	very low	medium	high	leeward	high
017	very low	medium	high	parallel	medium
018	very low	medium	high	windward	low
019	very low	high	low	leeward	medium
020	very low	high	low	parallel	low
021	very low	high	low	windward	low
022	very low	high	medium	leeward	medium
023	very low	high	medium	parallel	low
024	very low	high	medium	windward	low
025	very low	high	high	leeward	medium
026	very low	high	high	parallel	medium
027	very low	high	high	windward	low
028	low	low	low	leeward	low
029	low	low	low	parallel	low
030	low	low	low	windward	low
031	low	low	medium	leeward	medium
032	low	low	medium	parallel	low

033	low	low	medium	windward	low
034	low	low	high	leeward	high
035	low	low	high	parallel	medium
036	low	low	high	windward	low
037	low	medium	low	leeward	low
038	low	medium	low	parallel	low
039	low	medium	low	windward	low
040	low	medium	medium	leeward	medium
041	low	medium	medium	parallel	low
042	low	medium	medium	windward	low
043	low	medium	high	leeward	high
044	low	medium	high	parallel	medium
045	low	medium	high	windward	low
046	low	high	low	leeward	medium
047	low	high	low	parallel	low
048	low	high	low	windward	low
049	low	high	medium	leeward	medium
050	low	high	medium	parallel	low
051	low	high	medium	windward	low
052	low	high	high	leeward	medium
053	low	high	high	parallel	medium
054	low	high	high	windward	low
055	medium	low	low	leeward	medium
056	medium	low	low	parallel	medium
057	medium	low	low	windward	medium
058	medium	low	medium	leeward	medium
059	medium	low	medium	parallel	medium
060	medium	low	medium	windward	medium
061	medium	low	high	leeward	medium
062	medium	low	high	parallel	medium
063	medium	low	high	windward	high
064	medium	medium	low	leeward	medium
065	medium	medium	low	parallel	medium
066	medium	medium	low	windward	medium
067	medium	medium	medium	leeward	medium
068	medium	medium	medium	parallel	medium
069	medium	medium	medium	windward	high
070	medium	medium	high	leeward	medium
071	medium	medium	high	parallel	medium
072	medium	medium	high	windward	high
073	medium	high	low	leeward	medium

074	medium	high	low	parallel	medium
075	medium	high	low	windward	medium
076	medium	high	medium	leeward	medium
077	medium	high	medium	parallel	high
078	medium	high	medium	windward	high
079	medium	high	high	leeward	high
080	medium	high	high	parallel	high
081	medium	high	high	windward	high
082	high	low	low	leeward	high
083	high	low	low	parallel	high
084	high	low	low	windward	high
085	high	low	medium	leeward	high
086	high	low	medium	parallel	high
087	high	low	medium	windward	high
088	high	low	high	leeward	high
089	high	low	high	parallel	high
090	high	low	high	windward	high
091	high	medium	low	leeward	high
092	high	medium	low	parallel	high
093	high	medium	low	windward	high
094	high	medium	medium	leeward	high
095	high	medium	medium	parallel	high
096	high	medium	medium	windward	high
097	high	medium	high	leeward	high
098	high	medium	high	parallel	high
099	high	medium	high	windward	high
100	high	high	low	leeward	high
101	high	high	low	parallel	high
102	high	high	low	windward	high
103	high	high	medium	leeward	high
104	high	high	medium	parallel	high
105	high	high	medium	windward	high
106	high	high	high	leeward	high
107	high	high	high	parallel	high
108	high	high	high	windward	high

A3 Fuzzy rules for *FS2*

	Antecedent (*"If-and-and"*)			Consequent ("Then")
Rule No.	fRR_{24}	RR_{24}	PWL	$TF2$
001	low	low	low	low
002	low	low	medium	low
003	low	low	high	medium
004	low	medium	low	low
005	low	medium	medium	medium
006	low	medium	high	medium
007	low	high	low	medium
008	low	high	medium	medium
009	low	high	high	medium
010	medium	low	low	low
011	medium	low	medium	medium
012	medium	low	high	medium
013	medium	medium	low	medium
014	medium	medium	medium	medium
015	medium	medium	high	high
016	medium	high	low	medium
017	medium	high	medium	high
018	medium	high	high	high
019	high	low	low	medium
020	high	low	medium	medium
021	high	low	high	high
022	high	medium	low	medium
023	high	medium	medium	high
024	high	medium	high	high
025	high	high	low	high
026	high	high	medium	high
027	high	high	high	high

A4 Fuzzy rules for *FS3*

	Antecedent (“If-and-and”)			Consequent (“Then”)
Rule No.	SA	ΔSA_{24}	θ	$TF3$
001	null	very low	null	null
002	null	very low	extremely low	null
003	null	very low	very low	null
004	null	very low	low	null
005	null	very low	medium	null
006	null	very low	high	null
007	null	very low	very high	null
008	null	low	null	null
009	null	low	extremely low	null
010	null	low	very low	null
011	null	low	low	null
012	null	low	medium	null
013	null	low	high	null
014	null	low	very high	null
015	null	medium	null	null
016	null	medium	extremely low	null
017	null	medium	very low	null
018	null	medium	low	null
019	null	medium	medium	null
020	null	medium	high	null
021	null	medium	very high	null
022	null	high	null	null
023	null	high	extremely low	null
024	null	high	very low	null
025	null	high	low	null
026	null	high	medium	null
027	null	high	high	null
028	null	high	very high	null
029	very low	very low	null	null
030	very low	very low	extremely low	very low
031	very low	very low	very low	low
032	very low	very low	low	low
033	very low	very low	medium	low
034	very low	very low	high	low
035	very low	very low	very high	null
036	very low	low	null	null

037	very low	low	extremely low	very low
038	very low	low	very low	low
039	very low	low	low	low
040	very low	low	medium	low
041	very low	low	high	low
042	very low	low	very high	null
043	very low	medium	null	null
044	very low	medium	extremely low	very low
045	very low	medium	very low	low
046	very low	medium	low	low
047	very low	medium	medium	low
048	very low	medium	high	low
049	very low	medium	very high	null
050	very low	high	null	null
051	very low	high	extremely low	very low
052	very low	high	very low	low
053	very low	high	low	low
054	very low	high	medium	low
055	very low	high	high	high
056	very low	high	very high	null
057	low	very low	null	null
058	low	very low	extremely low	very low
059	low	very low	very low	low
060	low	very low	low	low
061	low	very low	medium	low
062	low	very low	high	low
063	low	very low	very high	null
064	low	low	null	null
065	low	low	extremely low	very low
066	low	low	very low	low
067	low	low	low	low
068	low	low	medium	low
069	low	low	high	high
070	low	low	very high	null
071	low	medium	null	null
072	low	medium	extremely low	very low
073	low	medium	very low	low
074	low	medium	low	low
075	low	medium	medium	low
076	low	medium	high	medium
077	low	medium	very high	null

078	low	high	null	null
079	low	high	extremely low	very low
080	low	high	very low	low
081	low	high	low	low
082	low	high	medium	low
083	low	high	high	medium
084	low	high	very high	null
085	medium	very low	null	null
086	medium	very low	extremely low	very low
087	medium	very low	very low	low
088	medium	very low	low	low
089	medium	very low	medium	low
090	medium	very low	high	medium
091	medium	very low	very high	null
092	medium	low	null	null
093	medium	low	extremely low	very low
094	medium	low	very low	low
095	medium	low	low	low
096	medium	low	medium	low
097	medium	low	high	medium
098	medium	low	very high	null
099	medium	medium	null	null
100	medium	medium	extremely low	very low
101	medium	medium	very low	low
102	medium	medium	low	medium
103	medium	medium	medium	medium
104	medium	medium	high	high
105	medium	medium	very high	null
106	medium	high	null	null
107	medium	high	extremely low	very low
108	medium	high	very low	low
109	medium	high	low	medium
110	medium	high	medium	high
111	medium	high	high	high
112	medium	high	very high	null
113	high	very low	null	null
114	high	very low	extremely low	very low
115	high	very low	very low	low
116	high	very low	low	low
117	high	very low	medium	medium
118	high	very low	high	medium

119	high	very low	very high	null
120	high	low	null	null
121	high	low	extremely low	very low
122	high	low	very low	low
123	high	low	low	medium
124	high	low	medium	medium
125	high	low	high	medium
126	high	low	very high	null
127	high	medium	null	null
128	high	medium	extremely low	very low
129	high	medium	very low	low
130	high	medium	low	medium
131	high	medium	medium	high
132	high	medium	high	high
133	high	medium	very high	null
134	high	high	null	null
135	high	high	extremely low	very low
136	high	high	very low	low
137	high	high	low	medium
138	high	high	medium	medium
139	high	high	high	high
140	high	high	very high	null

A5 Fuzzy rules for *FST*

	Antecedent ("If-and-and")			Consequent ("Then")
Rule No.	$TF1$	$TF2$	$TF3$	ATP
001	low	low	null	null
002	low	low	very low	very low
003	low	low	low	low
004	low	low	medium	low
005	low	low	high	moderate
006	low	medium	null	null
007	low	medium	very low	very low
008	low	medium	low	moderate
009	low	medium	medium	moderate
010	low	medium	high	moderate
011	low	high	null	null
012	low	high	very low	very low
013	low	high	low	moderate
014	low	high	medium	moderate
015	low	high	high	moderate
016	medium	low	null	null
017	medium	low	very low	very low
018	medium	low	low	low
019	medium	low	medium	moderate
020	medium	low	high	moderate
021	medium	medium	null	null
022	medium	medium	very low	very low
023	medium	medium	low	considerable
024	medium	medium	medium	considerable
025	medium	medium	high	considerable
026	medium	high	null	null
027	medium	high	very low	very low
028	medium	high	low	considerable
029	medium	high	medium	considerable
030	medium	high	high	high
031	high	low	null	null
032	high	low	very low	very low
033	high	low	low	considerable
034	high	low	medium	considerable
035	high	low	high	high
036	high	medium	null	null

037	high	medium	very low	very low
038	high	medium	low	considerable
039	high	medium	medium	high
040	high	medium	high	very high
041	high	high	null	null
042	high	high	very low	very low
043	high	high	low	high
044	high	high	medium	very high
045	high	high	high	very high

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