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RESEARCH PAPER

Probabilistic analysis of georisk from capping of contaminated sediment in Gunneklev Fjord, Norway

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Abstract

The seabed of the Gunneklev Fjord in south-eastern Norway is covered with exceptionally soft contaminated sediment containing mercury and dioxins. The sediments have an undrained shear strength less than 1 kPa and a variable thickness of up to 2.5 m. To reduce the potential for leaching of contaminants from these sediments, Hydro Energy AS developed a remediation plan involving capping of the sediment. Significant uncertainties are associated with the sediment's undrained shear strength and the cap thickness. An unfortunate combination of low shear strength and thick cap could cause slope failure (e.g. translational sliding of the sediment and/or bearing capacity failure (e.g. punching through). Failure, particularly slope failure, can cause spreading of the contaminants in the fjord causing serious consequences. This paper presents an assessment of the probability of slope failure associated with the contaminated sediment before and after the cap placement. Probability of bearing capacity failure is considered to have more local effect and is also discussed briefly. The study used the Monte Carlo method with random undrained shear strength and sediment thickness. The simulations show high slope failure probability when the seabed inclination was $\geq$ 1:50 combined with an average undrained shear strength $\leq$ 0.4 kPa. Based on the probabilistic analyses, a pilot field testing campaign was carried out (after this study) with 20-cm thickness cap in the "gentle" sloping area and 5-cm thickness in the "steep" areas to avoid causing a failure. The analyses were used to support evidence-based decision-making on the cap design and implementation for further field testing.

Keywords Capping · Monte Carlo analysis · Sediments · Slope stability · Uncertainty

List of symbols

a	Level of confidence
β	Inclination of the slope (rad)
Δ	Desired accuracy of the mean
γ_s	Unit weight of the sediment
γ_t	Unit weight of the cap
μ	Mean
σ	Standard deviation
$1{:}L$	Slope inclination ratio
F	Factor of safety
G	Weight of the sediment per metre length (kN/m)
N	Number of iterations
P_f	Probability of failure
$P(S)$	Probability of slope failure
$P(B)$	Probability of bearing capacity failure
q	The bearing capacity (kPa) of the contaminated sediment
s^2	Sample variance
s_u	Undrained shear strength
s_{ur}	Remolded undrained shear strength
t	Thickness of the cap
t_s	Thickness of the sediment
w	Water content
W	The load due to the capping material
z	z-Score

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1 Introduction

1.1 Background and motivation for the study

In situ capping of contaminated materials is a popular remediation method for quick risk reduction and is cost-effective. In feasibility study to evaluate in situ capping for a site, the engineers must consider geotechnical condition such as consolidation, bearing capacity and slope stability and potential interactions between the cap and the sediment. In many cases, the thickness and property of the contaminated sediments vary over the site (i.e. random/uncertain) and thus require the use of methods that can deal with these randomness or uncertainties.

The application of probabilistic methods to deal with uncertainties in geotechnical engineering problems has gained popularity significantly on the research front in the last forty years. Methods, tools and strategies for dealing with different sources of uncertainties have been proposed [1, 2, 4, 20, 22, 33, 45, 54, 56, 60, 61]. Geotechnical researchers and engineers are aware of the value of extensive amount of geotechnical data that are becoming increasingly available with modern technology. Statistical and probabilistic methods will play an increasingly central role in exploiting these data in the future [52]. There is increasing expectation that engineers take into account geotechnical uncertainty explicitly in design, model and decision-making [53].

Several engineering codes, standards and guidelines for design have promoted the use of probabilistic/statistical methods for dealing with soil data and uncertainties in parameters, e.g. Eurocode 7 [8], Recommended Practice DNV-RP-C207 [12], and International Organization for Standardization [28]. The concept of acceptable risk or acceptable probability of failure (P_f) for engineering projects has also been suggested for different engineering projects as a complement to the traditional deterministic factor of safety (F) to evaluate reliability [31, 32, 57]. In practice, the application of statistical and probabilistic methods is, however, still rather limited. This limitation is due to various factors including a lack of understanding of probabilistic methods for a number of practicing geotechnical engineers. Codes and regulations, though have promoted the use of reliability methods, are still based primarily on deterministic factor of safety. Limited amount of data can be used to fully characterise probabilistic model parameters such as mean, standard deviation and spatial correlations. Many engineers still regard probabilistic methods as of solely theoretical research interest. There are, however, a number of research publications which demonstrate the applicability and usefulness of probabilistic methods in practical engineering projects [9, 10, 14–17, 29, 48, 56, 61]. Many studies demonstrate applicability of probabilistic method to real-world problems including natural hazard assessment, dam safety, open pit mine and landfill, to name a few. For example, Javankhoshdel et al. [29] used probabilistic slope stability analysis on a case study of James Bay hydrolelectric project reported in El Ramly [14]. The Jame Bays project was also analysed probabilistically by Christian et al.[9]. Obregon and Mitri [48] reported a probabilistic analysis of an open pit mine located at the Andean region of Peru. Cuomo et al. [10] presented a recent study on multiseasonal probabilistic slope stability analysis of unsaturated pyroclastic soils in Cercinara region in southern Italy. Falamaki and Shafiee [17] applied probabilistic stability to a landfill site for municipal solid waste in Shiraz city in Iran under and post-construction. The study concluded that the slope of the landfill has high risk of failure with excessive infiltration of water under construction. These authors, based on the results of their study, gave recommendations for the construction of open landfill in Iran.

Though there are increasing of number of real case studies where probabilistic methods are applied, there is a need for more examples, particularly outside the traditional cut/fill slopes to encourage its application by practicing engineers. The current study will contribute to address this need by demonstrating the use of probabilistic analyses to assess the probability of slope failure in contaminated sediments in Gunneklev Fjord in Norway. Slope failure mode at this site can be expected to occur as translational gliding of the contaminated sediment dominated in areas with steeper terrain. Another mode of failure, bearing capacity failure due to the shear stresses in the sediment exceed the shear strength, can occur in both flat and inclined terrain. Both slope failure and bearing capacity failure can cause spreading of contaminants in the fjord though bearing capacity failure in many cases will cause only local spreading, while slope failure will cause spreading of contaminants in larger areas with more serious consequence. This study focuses therefore on slope stability failure. Bearing capacity failure is also discussed briefly in relation to how acceptable probability of failure should be selected depending on failure modes. The engineering solution in this project focuses on the in situ capping of the subaqueous contaminated sediments. Such an application of probabilistic method for designing of in situ cap for remediation seems to be lacking in the literature. This remediation method for contaminated sediments involves isolating the sediments from the environment by placing layers of selected good-quality materials over the contaminated sediment [51]. Due to the complexity of the subaqueous environment and sediments, the engineering team encountered multiple uncertainties associated with the sediment properties and the cap thickness [44]. It was therefore necessary to find an effective method to handle these uncertainties in a systematic and optimal way.

Various methods have been developed for geotechnical reliability analysis from simple analytical solutions to advanced machine learning techniques [59]. One popular method which is capable of dealing with complex problems and multiple sources of uncertainty is the Monte Carlo method [6, 26, 27, 30, 58]. The underlying concept of the Monte Carlo method is to model probability of different outcomes for a problem that is not easily predicted by using random variables. The main purpose is to understand the impact of uncertainty and risk. Monte Carlo simulations rely on multiple random sampling of parameters to produce numerical result in probabilistic distributions which can be interpreted further in terms of the occurrence probability of a certain event. The Monte Carlo method has been used to analyse various geotechnical problems, e.g. [7, 18, 19, 21, 23–25, 35–41].

The Monte Carlo method was employed in this study because the problem to analyse could be expressed explicitly and its ability to include multiple uncertainties. The current analysis calculated the probability of failure by using Monte Carlo simulations and included the uncertainty of input parameters that have the most significant influence on the uncertainty in predicting the stability of the sediments specifically at Gunneklev fjord. Detailed discussion on the selection of model parameters is presented later in Sect. 2.4. The assessment of probability of slope failure and bearing capacity failure was used further for designing the pilot field test which was carried out after this study. The consequence of failure is discussed qualitatively, and the risk is not quantitatively estimated in this study. The probability of failure contributes to form the evidence-based reliability decision-making related to implementation of the cap.

1.2 Studied area and the project

Gunneklev Fjord is a landlocked fjord (i.e. a long, narrow, deep sea inlet between cliffs) located in a densely populated area about 2 km southwest of the city of Porsgrunn in south-eastern Norway. The contaminated area extends over about 770,000 m^2 and lies close to the Herøya Industrial Park in Telemark (Fig. 1). The sludge from industrial activities, started in 1928–1929 by the industrial company Hydro Energy AS, has contaminated the bottom sediments in the fjord. The contaminants include mercury (Hg), dioxins (CH_4O_2) and other pollutants such as TBT and PAH (tributyltin products and polycyclic aromatic hydrocarbons).

The water depth in the fjord varies from 1.5 to 7 m. The soils beneath the contaminated sediment are soft, and in some parts sensitive, clay. The contamination is contained in a top sediment layer with a thickness of up to 2.5 m. This sediment is exceptionally soft with an undrained shear strength less than 1 kPa. Disturbance of this sediment will pose a significant threat of spreading contaminants over large areas of the fjord and may even re-expose more severely contaminated layers underlying the top layer.

The Norwegian Environment Agency (NEA) imposed on Hydro Energy AS to undertake a clean-up to reduce the potential for leaching of mercury and dioxin from the sediments in Gunneklev Fjord [42]. Hydro Energy AS plans to remediate Gunneklev Fjord to reduce the spreading of contaminated mud. One of the planned remediation measures is to cover the contaminated sediment with an isolating cap consisting of either sand or a mixture of sand and active charcoal [42]. Sand and active carbon (charcoal) are popular capping materials which provide a physical and geochemical barrier between the underlying contaminated material and the overlying water. A sand cap can stabilise the underlying sediment to prevent re-suspension of contaminated particles and reduce chemical exposure under certain conditions. Sand primarily provides a passive barrier to the downward penetration of bioturbating organisms and the upward movement of sediment or contaminants. Upward movement of contaminants due to consolidation of the contaminated sediment under the weight of the cap must be considered during the design phase. It is considered that sand cap combined with active charcoal is possibly suitable for the condition at Gunneklev fjord [43]. The proportion of active charcoal was not determined at the time of this study, thus only sand cap will be assumed further in the subsequent analyses in this study.

The contaminated sediment at the bottom of the fjord has gravimetric water content of 400–900% and a texture like “yogurt”. It was not possible to either conduct in situ measurements or take undisturbed samples of the sediment due to the low shear strength. There is therefore substantial uncertainty associated with the undisturbed undrained shear strength of this sediment. The uncertainties related to undisturbed shear strength of the contaminated sediments in Gunneklev fjord arise primarily from measurement uncertainty due to the challenge associated with both in situ measurement and with taking undisturbed samples in these very soft sediments. There is also contribution from spatial variability and transformation uncertainty to the total uncertainty, but these factors are expected to be less dominant than measurement uncertainty in this case. The

Fig. 1 Map of Gunneklev Fjord at Herøya Industry Park, Porsgrunn, and (corner map) location in a map of south Norway

sediment layer is thickest in the area with steep sea bed in the north and northwest areas of the fjord.

Due to the limitation of the capping equipment and the implementation technique, the thickness of the cap cannot be implemented precisely as designed. The cap thickness will vary spatially, and this introduces an additional uncertain factor that needs to be accounted for in the remediation project. Given the low undrained shear strength of the top clay, there is a risk that the cap can initiate sliding and/or bearing capacity failure in the sediment layer if the cap becomes too thick for the soft sediment. This would result in re-exposure of even more severely contaminated layers underneath and further spreading of the contaminants. Therefore, it is crucial to control the thickness during the implementation of the cap.

1.3 Research and development challenges

To ensure a safe implementation of the remediation plan, Hydro Energy, together with NEA, and with assistance from the Norwegian Geotechnical Institute (NGI), performed a research and development study to evaluate the probability of slope failure and/or bearing capacity failure associated with the implementation of the cap. The main research questions for the project are:

- How to deal with the uncertainties in the undrained shear strength, thickness of the sediment and thickness of the cap?
- How to deal with the variation in the sloping ground over the fjord?
- What is the acceptable probability of slope failure and bearing capacity failure of the sediment?

As the main challenges were associated with several uncertainties, probabilistic methods were identified as a suitable approach to examine these research questions. Probabilistic analyses of stability and bearing capacity of the top sediment in Gunneklev Fjord were made for the existing conditions (before capping) and for different cap thicknesses (after capping). This paper presents the analysis approach and discusses the choice of input parameters and the results from the probabilistic analyses. The use of the probabilistic results to assist with decision-making on the capping solution is also illustrated.

2 Methods

2.1 Field investigation campaign

A ground investigation campaign was carried out between March 2018 and May 2019 to survey the topography and layer thickness and take samples for geotechnical analyses. The field was scanned with multibeam radar and light seismic equipment to survey the water depth and thickness of the contaminated sediments. The sediment is a fine-grained sludge with very soft, almost ‘yogurt-like’ texture. It was not possible to apply common in situ investigation methods (e.g. piezocone penetration tests, CPTU) on this sediment due to its extremely low shear strength. Samples were taken with the Niemistoe core sampler at point 1 to 4, 7 and 9 and with the hand-operated piston sampler at point 5, 6 and 8 (Fig. 2). The Niemistoe core sampler was developed during the 70 s to retrieve potentially undisturbed samples with good recovery [46]. The hand-operated piston sampler [11] is simple to operate and provided good samples in areas with thick sediment. Due to the extremely soft characteristic of the sediment, it was not possible to retrieve undisturbed samples. Disturbed samples were transferred to plastic containers and transported to the laboratory for testing.

2.2 Calculation models

2.2.1 Infinite slope model

The contaminated sediments and the cap in the Gunneklev Fjord can be assimilated to an “infinite slope”, with a much larger length than height (or sediment thickness). Placement of the cap over the sediment will lead to an increase in pore water pressure in the sediment, which is unlikely to dissipate quickly due to the fine-grained characteristic and low permeability of the sludge sediment. The resistance under undrained conditions is therefore critical for stability and bearing capacity. The clay layer underneath the contaminated sediment has considerably higher undrained shear strength than the sediment layer. Thus sliding due to the cap placement is likely to occur within the soft contaminated sediment layer or at the interface between the sediment layer and the underlying stronger clay layer. Given the low shear strength of the contaminated sediment and the high ratio between the length and height of the slope in Gunneklev fjord, it is likely that the 3D-effect and the influence from assuming ’infinite’ sliding surface are relatively small. An infinite slope model was therefore considered appropriate to estimate the stability of the sediment before and after cap placement. Model uncertainty in the simulation process is not addressed explicitly in this study. Quantification of model uncertainty is challenging and often relies on databases such as centrifuge models and/or physical field models which are not available in this case. This will be a topic for future research.

The factor of safety is governed by the undrained shear strength of the contaminated sediment. Figure 3a shows the slope model and the soil parameters in each layer used in the analysis. Figure 3b shows a photograph of a remoulded sample.

The factor of safety (F) for the infinite slope was estimated using Eq. 1 below:

$$F = \frac{L \times s_u}{G \times \cos\beta \times \sin\beta} = \frac{s_u}{((\gamma_s - 10) \times t_s + (\gamma_t - 10) \times t) \times \cos\beta \times \sin\beta} \qquad (1)$$

where L: Length of the slopes (m), s_u: Undrained shear strength of the contaminated sediment (kPa), G: Weight of the sediment per meter length (kN/m), β: Inclination of the slope (rad), γ_s: Total unit weight of the sediment (kN/m^3), t_s: Thickness of the contaminated sediment layer (m), t:

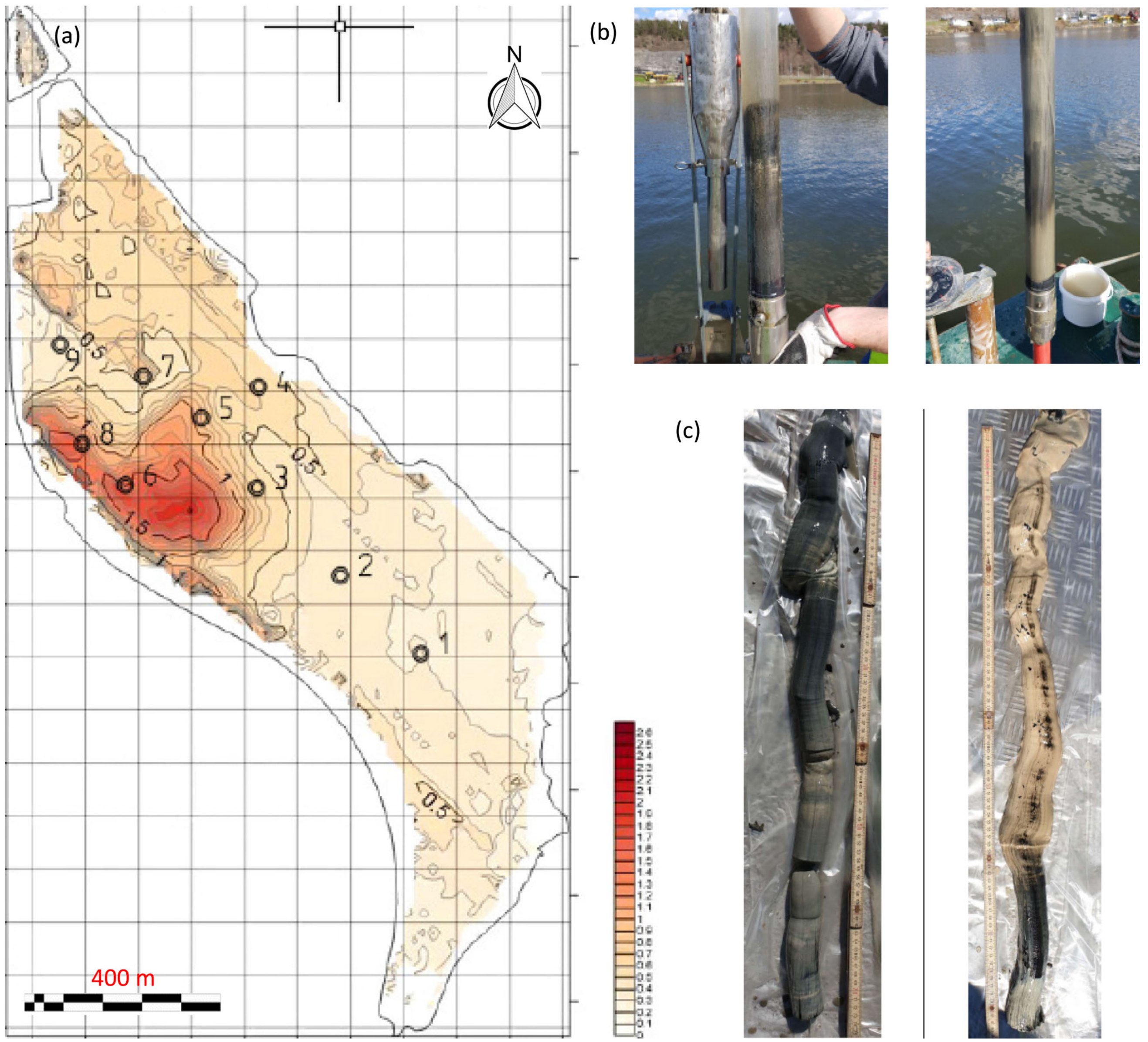

Fig. 2 **a** Thickness of contaminated sediment (contours) in metre and location of sampling points, **b** Niemistoe core sampler (left) was used at point 1 to 4, 7 and 9, while hand-operated piston sampler (right) was used at point 5, 6 and 8, **c** examples of cylinder samples retrieved

Thickness of the cap (m), γ_t: Total unit weight of the capping material (kN/m^3).

Note that, for the range of slope inclination relevant to this study (from 1:25 to 1:100), $\cos\beta$ in Eq. 1 becomes almost equal to 1.

2.2.2 Bearing capacity failure

Local failure can occur near the edge of the cap due to the weight of the capping material. The safety factor for bearing capacity failure can be estimated from the following simplified equation:

$$F = \frac{q}{W} = \frac{5.14.s_{\mathrm{u}}}{\gamma_t \times t} \qquad (2)$$

where q is the bearing capacity (kPa) of the contaminated sediment, W is the stress imposed by the weight of the capping material (kPa).

The constant 5.14 is the bearing capacity factor for the case where the angle of friction is zero corresponding to undrained condition [5].

Equation 2 assumes that the placement of the cap is carried out over a short period, with the bearing capacity and/or slope stability mechanisms governed by undrained behaviour.

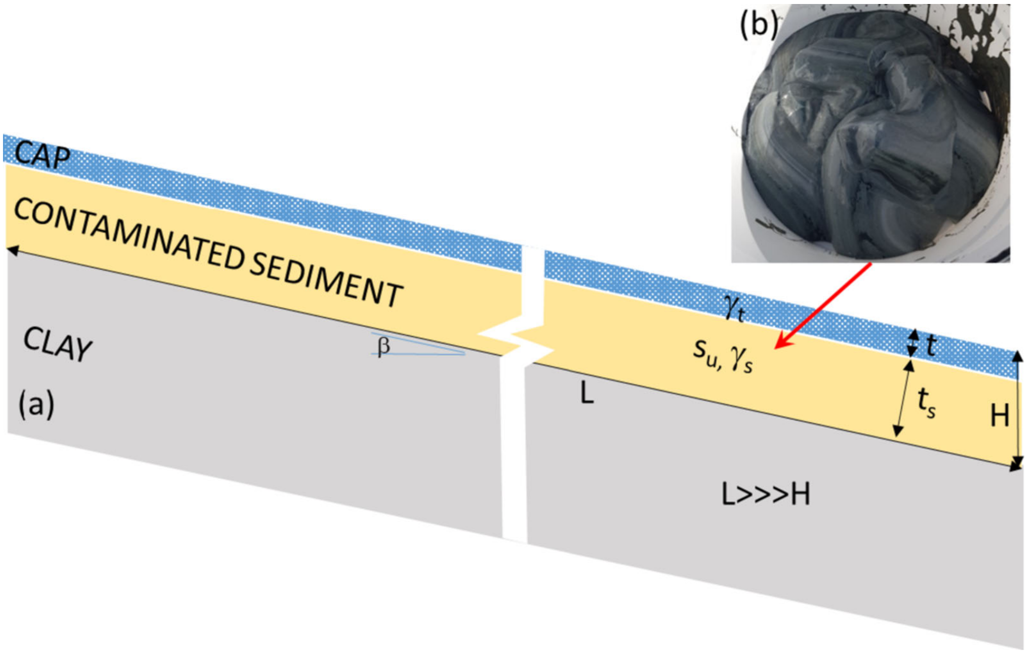

Fig. 3 **a** Simplified model of infinite slope used to calculate stability of contaminated sediment in Gunneklev Fjord; **b** a remoulded sample of the contaminated sediment

Equation 2 is a simplification of Hansen (1970) in order to focus on the failure mode in which the cap causes sliding or bearing capacity failure in the contaminated sediment. This is the critical failure mode which can lead to spreading of the contaminants. The soft clay underneath the sediment has likely much higher undrained shear strength than the contaminated sediment (i.e. minimum 10–20 kPa). Thus, it is not likely that the placement of the cap within 20 cm will cause bearing capacity or slope failure in this soil.

2.3 Probabilistic approach

The Monte Carlo simulation approach was used for the probabilistic analyses in this study. The probabilistic analyses included the uncertainty in the input variables (i.e. the undrained shear strength s_u, the thickness of the contaminated sediment t_s, and the thickness of the cap t, by assuming that they varied randomly from one calculation to the other. Each random variable in this study was assumed, for simplicity, to vary independently from one another and follow the lognormal distribution function. In addition, this study does not take into account spatial variability, normally characterised by an additional parameter such as a scale of fluctuation (also referred as a correlation length). Each random variable was defined, in this study, by a set of statistical parameters: mean μ and standard deviation σ. This is a simplification which was introduced due to the challenge with characterising the value for correlation length and the interdependency relationship (if any) between parameters.

The random variables were modelled by generating random values from its statistical parameters and using these random values for the calculation of factor of safety. This means, for each calculation in a Monte Carlo simulation, a set of random undrained shear strength s_u, thickness of the cap t, and thickness of the sediment layer t_s was generated from their μ, σ and the probability density function and used to estimate the factor of safety.

Monte Carlo simulation requires a large number of calculations, each with a different set of input values for s_u, t, and t_s. The infinite slope model was considered to “fail” if the calculated factor of safety from the analytical solution was less than 1. A lognormal was fitted to the calculated factors of safety for each analysis (i.e. each combination of the parameters μ and σ presented in this paper). The probability of failure (P_f) was estimated as the probability of the calculated factor of safety being less than 1 in each iteration.

There are different methods to estimate the minimum number of iterations required to achieve a target significance level including, for example, the Wald method [55], the Wilson score method [13] and the Central Limit Theorem [47]. Each method requires certain assumptions (for example, the distribution of the estimated quantity) and has their own advantages and drawbacks. The number of iterations in this study is iteratively decided using the following equation based on the Central Limit Theorem [47]:

$$N = \frac{z_{\alpha/2}^{2} s^{2}}{\Delta^{2}} \quad (3)$$

where N is the number of iterations required, z is the z-score associated with α level of confidence and Δ is the desired accuracy of the mean of the estimated factor of safety. The variance s^2 will vary with the sample size and not known in advance in the equation above. An iterative process was performed initially to estimate s^2 using a small sample size to refine N. The result of this iterative process gives an approximate of 1000 to 10 000 samples required to achieve a desired accuracy within 0.01 for the estimated mean of the safety factor at 95% level of confidence for almost all cases considered. In a combination in which both undrained shear strength and thickness of the sediments were varied randomly (for example, case 2), 10 000 random values of shear strength and 10 000 random values of thickness of contaminated sediment were generated and used together requiring 100 million iterations.

Between 10 and 100 million iterations were performed for each Monte Carlo analysis presented in this paper. These large numbers of simulation were feasible thanks to the use of infinite slope model which enables the use of analytical solution. The numbers of iterations were chosen to make sure that the results are statistically stable while still computationally feasible, particularly when more than one parameter were varied randomly. The analyses were performed with MATLAB (MathWorks). In this study, single random variables were used meaning that the random parameter was assumed to be spatially uniform in each calculation. The main advantage of single random variable approach is low computational expense compared with other more advanced approaches such as random field approach. Since the "infinite slope" model with analytical solution is considered satisfactory to model the field condition at Gunneklev fjord, the single random variable approach saves computational cost, particularly for case in which several parameters are assumed to vary randomly.

The undrained shear strength of the contaminated sediment was assumed to have infinite horizontal and vertical correlation lengths. Thus, only one random value of shear strength was used for the entire layer of contaminated sediment in each simulation. The main reason for adopting this assumption is the challenge in conducting in situ measurement and taking samples at the Gunneklev fjord leading to insufficient number of samples for characterising correlation lengths. An earlier study with probabilistic slope has shown that the single random variable approach produces conservative predictions compared with approaches in which equivalent spatial variation is considered (Griffiths and Fenton, 2000). Other studies (e.g. Burgess et al. [7]) have shown that the influence of correlation length depends on the slope angle. For gentle slope (i.e. slope angle less than 40–45°), long horizontal correlation length leads to higher probability of failure [7], while the opposite is true for steep slopes. As the slopes at Gunneklev fjord has very mild slope angle (less than 3° or 1:25 at the steepest area in Gunneklev), the assumption of infinite correlation length is considered conservative. By assuming infinite vertical correlation, the undrained shear strength was treated to be constant over depth in the contaminated sediment layer. This assumption is considered acceptable in this case as the contaminated sediment layer is within 1 m thick for most part, and less than 2,5 m thick at the site. The contaminated sediment was also relatively newly deposited and has not undergone geological compression and consolidation processes which can considerably alter the undrained shear strength with depth. The variation of undrained shear strength over this limited depth is likely to be rather small.

For each set of mean values for the input parameters, deterministic analyses were also performed to compare the deterministic factors of safety with the probabilistic probability of failure. In the deterministic analyses, the factor of safety was calculated assuming that all parameters were constant and equal to the mean value. The deterministic analyses were carried out with three deterministic values of s_u, taken as 0.4, 0.5, and 0.6 kPa. The probabilistic analyses were conducted with the same three sets of mean values μ (s_{u}), equal to 0.4, 0.5, and 0.6 kPa, respectively. The deterministic and probabilistic analyses of stability were performed for both the existing situation before capping and after placement of the cap.

For the existing situation before capping, the failure probability P_{f} was estimated for four different slope inclination ratios (1:25, 1:50, 1:75, and 1:100). For the situation after cap placement, probabilistic analyses were performed for a slope inclination ratio of 1:L = 1: 50.

2.4 Model parameters

2.4.1 Undrained shear strength of the sediment

Since the contaminated sediment is fine-grained, stability and bearing capacity right after capping are most likely to be governed by undrained behaviour. Undrained shear strength (s_{u}) is therefore a critical parameter. As it was not possible to retrieve undisturbed samples, the remoulded undrained shear strength (s_{ur}) was measured with fall cone tests in the laboratory. The results are shown in Table 1. The values of s_{ur} are very low. Seven out of nine samples show s_{ur} = 0.1 kPa, which was the lowest value that could be measured by the fall cone apparatus used. This means that the true value of s_{ur} might be even lower than 0.1 kPa. Two samples show s_{ur} values 0.2 and 0.8 kPa. These s_{ur} values are consistent with the s_{ur} for industrial sludge with

Table 1 Samples taken in Gunneklev Fjord for geotechnical investigation and measured water content and remoulded undrained shear strength

Borehole	Sampling method	Depth (m)	*w* (%)	s_{ur} (kPa)
1	Niemistoe	0.3	681	0.1
2		0.4		0.1
3		0.4		0.1
4		0.4		0.1
5	Piston	0.4	491	0.1
6		0.4	470	0.2
7	Niemistoe	0.4		0.1
8	Piston	0.4		0.8
9	Niemistoe	0.4		0.1

high water content (*w*) between 400 and 700%, based on other studies (e.g. [49, 50]).

The contaminated sediment over the fjord originated from industrial processes and has not undergone geological transformation or other processes that could have altered the undrained shear strength significantly. Observations from the site investigation indicated relatively consistent types of sediment over the entire fjord. It was therefore decided to model the undrained shear strength of the sediment with a single random variable with a lognormal distribution function. The lognormal distribution has the advantage of generating only positive values for the undrained shear strength, for example, compared to some other types such as the normal distribution. The lognormal distribution has been proposed as appropriate for a number of geotechnical parameters [18, 33].

The mean $\mu(s_u)$ and standard deviation $\sigma(s_u)$ of the undisturbed undrained shear strength were chosen based on expert knowledge and taking into consideration the very low measured remoulded shear strength. The largest uncertainty in this case is due to "lack of information" as it was neither possible to obtain any in situ measurements of undisturbed s_u nor to retrieve undisturbed samples for laboratory testing. Due to the very high water content of the sediment, the undrained shear strength, even in the undisturbed state, is likely to be very low. The probabilistic analyses were therefore conducted with conservatively chosen mean values: $\mu(s_u)$ equal to 0.4, 0.5, and 0.6 kPa, respectively. A standard deviation, $\sigma(s_u)$ equal to 0.2 kPa was used in all probabilistic analyses. The coefficients of variation (σ/μ) for the three mean values of s_u were correspondingly equal to 50%, 40%, and 33%, respectively. An example of the distribution of generated random s_u is shown in Fig. 4. In each Monte Carlo simulation, a random s_u value generated from the lognormal distribution was assumed for the entire contaminated sediment layer. In reality, the undrained shear strength tends to increase slightly with the confinement (i.e. depth). This is not taken into account in this study; thus, the s_u is assumed to be constant with depth.

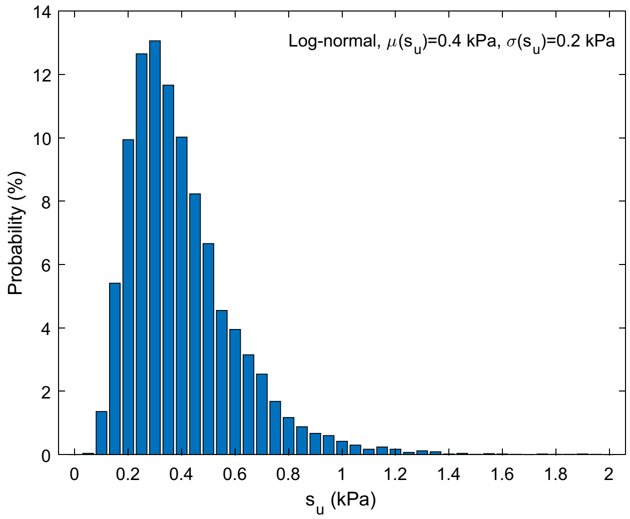

Fig. 4 Example of randomly generated s_u values in Monte Carlo simulations ($\mu(s_u)$ = 0.4 kPa and $\sigma(s_u)$ = 0.2 kPa). The number of samples for this plot is 10,000 samples

2.4.2 Thickness of the sediment

The thickness of the contaminated sediment was surveyed by multibeam radar and calibrated with light seismic measurement and sampling at selected points (Fig. 2a).

The measured thickness of the contaminated sediments in this area lies between 0.5 and 2.5 m. The sediment thickness varies over the entire fjord. The measured sediment thickness was up to 2.5 m in the west and northwest parts. Over many parts of the fjord in the south and in the east, the thickness of the sediment is between 0.1 and 0.5 m. Based on the measured sediment thickness, the mean value $\mu(t_s)$ was estimated as 1 m with a standard deviation $\sigma(t_s)$ equal to 0.2 m for the probabilistic analyses. A lognormal distribution was assumed for the sediment thickness.

2.4.3 Capping procedure and thickness of the cap

The cap is to be placed in a few consecutive layers in a standard implementation process. Capping materials are unloaded to a pumping barge from a sand boat (Fig. 5). The material is then mixed with seawater and loaded directly into a pipeline and transported to the area for capping. At the end of the pipeline, the laying head reduces velocity of the material and sprinkles it out in a controlled way onto the seabed. However, various factors can influence the precision of the cap thickness including waves,

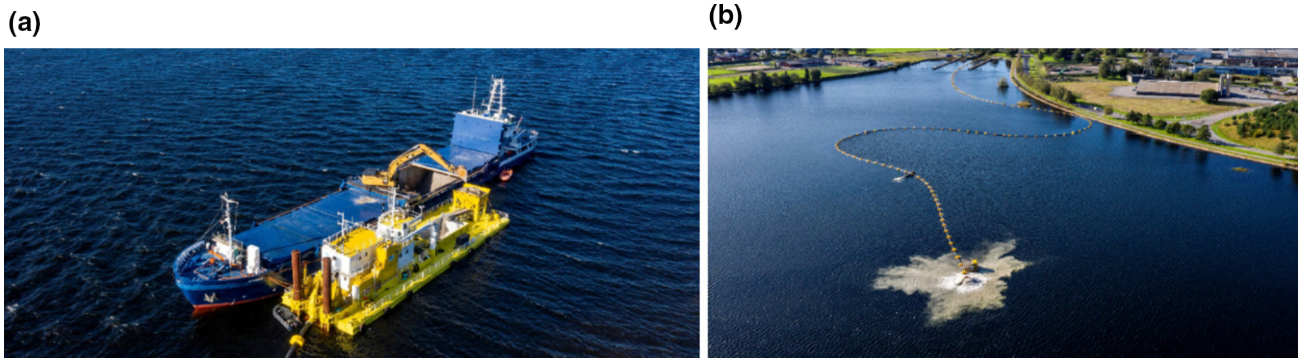

Fig. 5 **a** Barge and sand boat for laying out capping material, **b** pipeline for transporting and placing capping materials (Photos: Hydro Energy AS)

currents, and limitation of equipment and methods. The thickness of each layer (*t*) can therefore vary from the design thickness and impose another "uncertain" input into the risk picture. In this study, the thickness *t* of the cap was therefore also treated as a random variable. The mean, ($\mu(t)$, and standard deviation, $\sigma(t)$, were estimated based on experience from similar projects in Norway, e.g. Asplan viak, DNV-GL [3]. In the probabilistic analyses, it was assumed that each layer had an average thickness of 5 cm with a standard deviation of 2 cm. A lognormal distribution function was assumed for the layer thickness. Figure 6 shows the probability density functions of the cap thickness with 1, 2, 3, and 4 layers of 5 cm-average capping material. The combined mean thicknesses became 5, 10, 15, and 20 cm, while the standard deviations of the combined thickness were 2, 2.8, 3.5, and 4 cm, respectively.

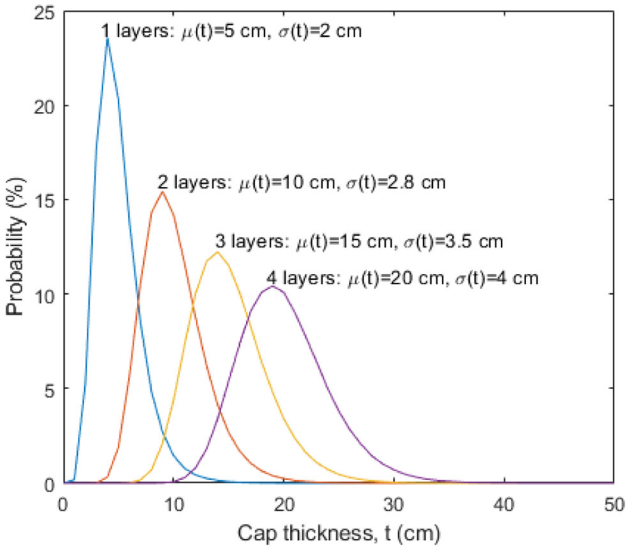

Fig. 6 Probability density function of the thickness of the cap for 1, 2, 3, and 4 layers of 5 cm ($\mu(t)$ = 5 cm and $\sigma(t)$ = 2 cm). The $\sigma(t)$ shown on each curve is the combined standard deviation of all the layers

2.4.4 Other input parameters

The seabed inclination in the fjord is steepest in the west and northwest parts and gentler in the south and southeast parts. As the seabed inclination was surveyed for the entire fjord, it was not treated as a random variable. Variation in seabed topography causes the slope variation of the sediment layer. Preliminary deterministic calculations showed that the areas with an inclination gradient less than 1:100 had very high factor of safety (> 20). The probabilistic analysis focuses therefore on the areas with slope inclination ratio steeper than 1:100. The influence of the slope inclination was dealt through sensitivity analyses. Four slope inclination ratios were analysed: 1:*L* = 1:25, 1:50, 1:75, and 1:100.

The unit weights of the contaminated sediment (γ_s) and the capping material (γ_t) also varied to some degree, but their variation was relatively small. To reduce computation time, deterministic (non-random) values were used for the two unit weights: γ_s = 13 kN/m^3 for the sediment and γ_t = 17 kN/m^3 for the capping material. The sediment total unit weight (γ_s) was calculated from 10 disturbed samples of the sediment taken by cylinder sampler. The total unit weight of the capping materials (γ_t) was based on experience with this type of material.

Sensitivity analyses were performed with the infinite slope model to investigate the possible influence of uncertainty of different parameters on the estimated factor of safety. Each parameter was varied within a range that are considered realistic for their variation based on above-mentioned field data and experiment results together with values reported in the literature. Specifically, the variation ranges were: 0.05–1.2 kPa for s_u—the undrained shear strength of the contaminated sediment, 0.2–1.6 m for t_s—the thickness of the contaminated sediment, 11–15 kN/m^3 for γ_s—the unit weight of the contaminated sediment, 0–20 cm for *t*—the thickness of the cap, 15 and 19 kN/m^3 for γ_t the unit weight of the sand cap and between 1:25 and

Table 2 Input parameters for the deterministic and probabilistic Monte Carlo analyses

Parameter	Notation	Unit	Mean (μ)	Standard deviation (σ)	Probability density function
Thickness of sediment	t_s	m	1	0.2	Lognormal
Unit weight of sediment	γ_s	kN/m^3	13	–	Deterministic
Thickness of the cap	T	cm	0–20	0–4	Lognormal
Unit weight of the cap	γ_t	kN/m^3	17	–	Deterministic
Slope inclination ratio	1:L	–	1:25; 1:50; 1:75; 1:100	–	Sensitivity analysis

1:100 for the slope angle. The results indicate that the uncertainty in undrained shear strength, thickness of the contaminated sediment and thickness of the cap have significant influence on the estimated factor of safety; thus, these three parameters are modelled as random parameters. The unit weight of sediment and unit weight of the cap are modelled as deterministic parameters to reduce computational efforts. Table 2 summarises the input parameters used in the analyses.

3 Results

3.1 Slope stability

Four cases were studied to illustrate the influence of the variation in parameters. The cases differentiate from each other in terms of which parameters were modelled randomly. The aim of the analyses is to highlight the impact of each variable separately. Unless otherwise specified, the input parameters are as shown in Table 2. The four cases were:

- Case 1: Random s_u, deterministic $t_s = 1$ m
- Case 2: Random s_u, random t_s
- Case 3: Random s_u, deterministic but increased t_s to 1.1 m, which gives a P_f close to Case 2.
- Case 4: Random s_u, deterministic t_s from Case 3, and random t

Case 1–3 stimulate condition before placement of the cap, while case 4 consider the changes in probability of failure with the placement of the cap. In comparison with case 1 and case 3, the computational time increases significantly in case 2 and case 4 during which two parameters were varied randomly. This study therefore did not conduct analyses with all three variables varied simultaneously in order to keep computational expense at the manageable level. This would, however, be an interesting exercise in future study.

3.1.1 Case 1. Random s_u

Case 1 is for the existing conditions (i.e. before placement of the cap) and was done with both deterministic and

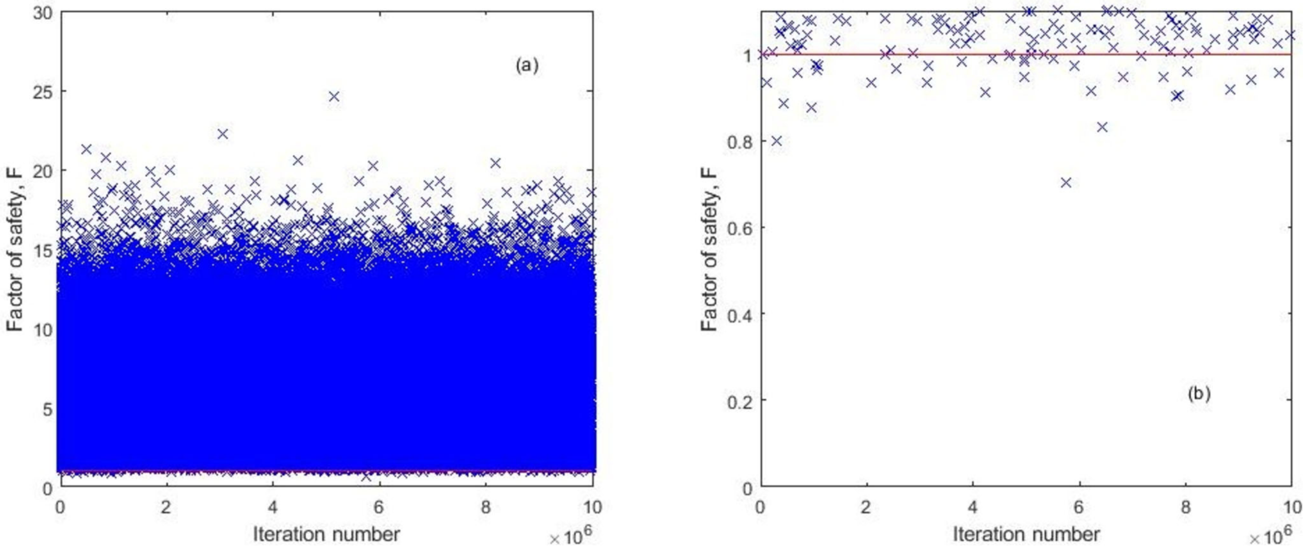

Fig. 7 Example of the factors of safety against iteration number in an analysis with random s_u with $\mu(s_u) = 0.6$ kPa and 1:L = 1:25 **a** F range between 0 and 30 **b** F range between 0 and 1, 2

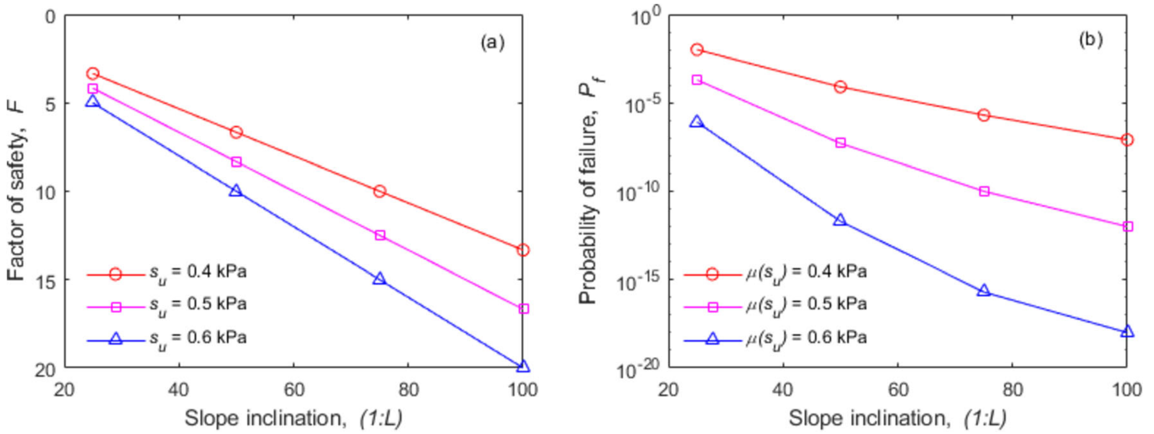

Fig. 8 **a** Deterministic factor of safety (F) and **b** failure probability (P_f) versus slope inclination (1:L) before cap placement. The thickness of sediment t_s = 1 m

probabilistic analyses. Figure 7 shows an example of calculated factors of safety against iteration number, at different scale for vertical axis. The data points lying below the red line (representing $F = 1$) are the iteration with "failed" F. Figure 8 shows that the deterministic F reduces with increasing slope inclination and/or decreasing s_u. The deterministic F is relatively large, with $F > 3$ for all cases analysed. The probabilistic results show that the P_f increases as the slope becomes steeper and/or as the mean undrained shear strength decreases. For the lowest mean undrained shear strength (μ (s_u) = 0.4 kPa), the failure probability becomes relatively high ($P_f = 10^{-2}$ or 0.01) for a slope with an inclination of 1:25 or steeper. The failure probability reduces to approximately 10^{-4} (or 0.0001) for a slope of with an inclination of 1:50. Notably, the deterministic values of F for these cases can be high, giving the perception of a very stable condition. But the failure probability P_f can be relatively high (1% for 1:$L \leq$ 1:25) when the uncertainty in s_u is taken into account as a random variable in the Monte Carlo analyses. The results highlight the importance of accounting for the uncertainty in the undrained shear strength of the contaminated sediment in the evaluation of slope safety.

3.1.2 Case 2. Random s_u and t_s

In Case 2 (before cap placement), both the s_u and the t_s were modelled as random variables in the probabilistic analyses. Figure 9 shows that the P_f increases when the uncertainties in both s_u and t_s are taken accounted for, compared to the case where only s_u was a random variable. For example, P_f increases slightly, from 10^{-2} to 1.5×10^{-2} for the lowest $\mu(s_u)$ = 0.4 kPa and for the steepest slope (1:L = 1:25). The differences in failure probability are relatively larger for the other cases analysed. These results show that ignoring the uncertainty in sediment layer thickness can lead to unconservative reliability assessment. The underlying reason for this is potentially because varying t_s together with varying s_u will allow more possible "dangerous" combination of sufficiently thick contaminated sediment layer with sufficiently low s_u leading to a factor of safety lower than 1. Statistically, the probability distribution of factors of safety from Monte Carlo simulation will have higher standard deviation leading to more values at the tails (i.e. very low or very high factor of safety) compared with when t_s is assumed a constant value.

3.1.3 Case 3. Random s_u, and deterministic, but increased t_s

An analysis was performed to find a deterministic t_s value that would give P_f values that are close to the P_f from Case 2. This deterministic value of t_s is to be used in further probabilistic analyses in order to reduce calculation time. This is particularly useful for cases when the variability of the cap is also be taken into account in Case 4. Figure 10 shows that a constant sediment thickness of 1.1 m leads to P_f-*values* that are close to the P_f from Case 2, particularly for the most critical cases with 1:$L <$ 1:50 and μ (s_u) = 0.4 kPa. The analyses with different thickness of capping materials in Case 4 were then conducted with a deterministic value of t_s = 1.1 m, to in order to reduce computational effort while still account for the increased P_f due to the uncertainty in t_s.

3.1.4 Case 4. Random s_u and random t and constant increased t_s

Deterministic and probabilistic analyses were performed of the placement of the cap in four successive layers. Each layer had a mean thickness of 5 cm and a standard

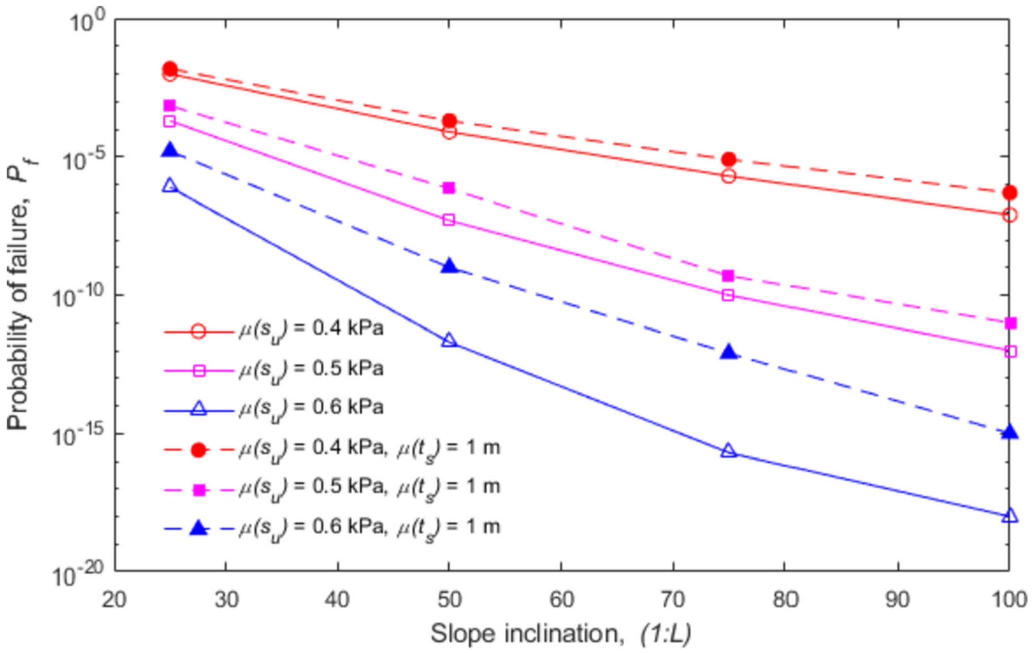

Fig. 9 Failure probability (P_f) versus slope inclination (1:L) before cap placement. Comparison between probabilistic analyses with random s_u and deterministic t_s = 1 m (continuous line with hollow symbols) and probabilistic analyses with random s_u and random t_s (dashed line with solid symbols)

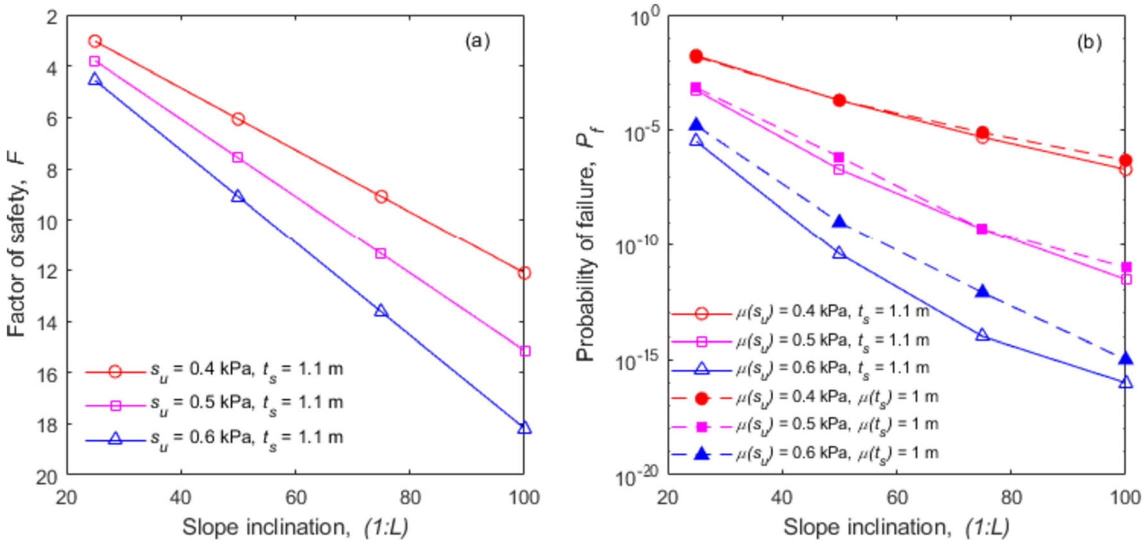

Fig. 10 **a** Deterministic factor of safety (F) with deterministic thickness of sediment t_s = 1.1 m and **b** failure probability (P_f) versus slope inclination (1:L) before cap placement

deviation of 2 cm. Table 3 lists the input parameters for the slope stability analyses under the placement of the cap. The analyses were performed with a constant slope inclination equal 1:L = 1:50 as this is considered the most representative value for the seabed terrain in the Gunneklev Fjord.

Figure 11 presents the results of the deterministic and probabilistic analyses. The deterministic F decreases with increasing cap thickness, but the values of F remains higher than 4 even for the lowest s_u = 0.4 kPa combined with the thickest cap t = 20 cm. The failure probability P_f, on the other hand, increases with cap thickness, and P_f becomes larger than 10^{-3} (or 0.001) when the cap thickness reaches 10 cm for μ (s_u) = 0.4 kPa. If the μ (s_u) is equal or larger than 0.5 kPa, P_f is smaller than 10^{-5} even when the cap is 20 cm thick. The results highlight the importance of controlling the thickness of the cap to compensate for the uncertainty in undrained shear strength.

Table 3 Input parameters for stability analysis after placement of cap

Parameter	Unit	Mean, μ	Standard deviation, σ	Probability density function
Undrained shear strength (s_u)	kPa	0.4; 0.5; 0.6	0.2	Lognormal
Thickness of sediment (t_s)	m	1.1	–	Deterministic
Thickness of the cap (t)	cm	0; 5; 10; 15; 20	0; 2; 2.8; 3.5; 4	Lognormal
Slope inclination (1:L)		1:50	–	Deterministic

3.2 Bearing capacity failure

Bearing capacity failure was calculated with random values of s_u and t. The input parameters for analyses of bearing capacity are the same as in Table 3. Figure 12 shows that the probability of bearing capacity failure increases considerably with the thickness of the cap, t. Probability of failure P_f increases when the uncertainty in t is taken into account compared with when it was assumed as deterministic (Fig. 12b). Figure 12 shows that $P_f \geq 10^{-3}$ once $t \geq 10$ cm for all three values of μ (s_u), even though the deterministic factor of safety is larger than 1.4 for all cases.

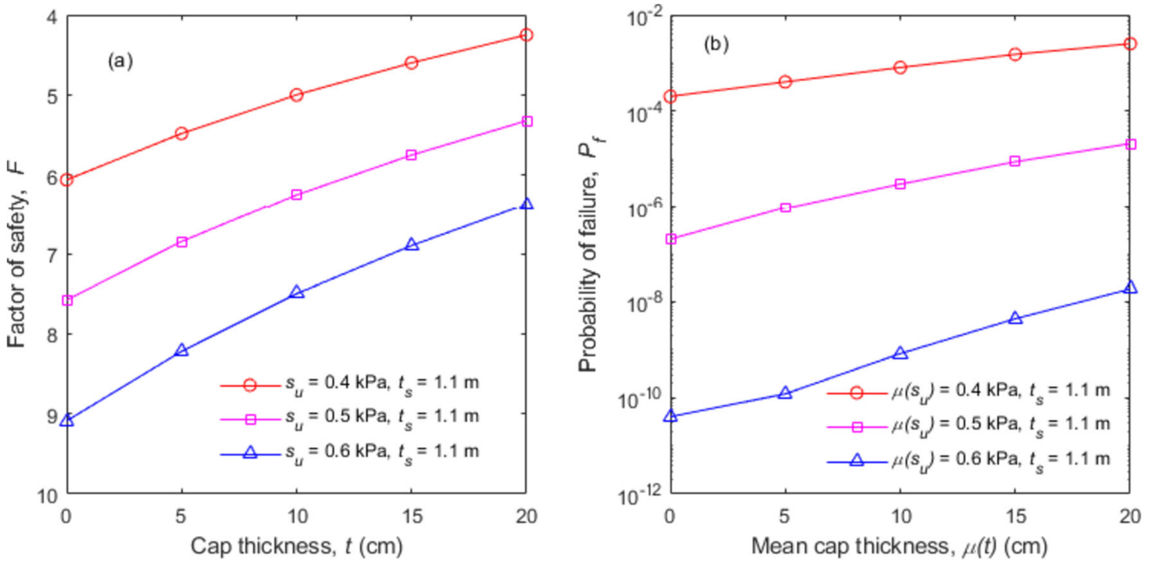

Fig. 11 **a** Deterministic factor of safety for different cap thicknesses and **b** probability of failure (P_f) for different mean cap thicknesses μ(t) for slope inclination (1:L) = 1:50. The thickness of the contaminated sediment is taken as deterministic and t_s = 1.1 m for all analyses

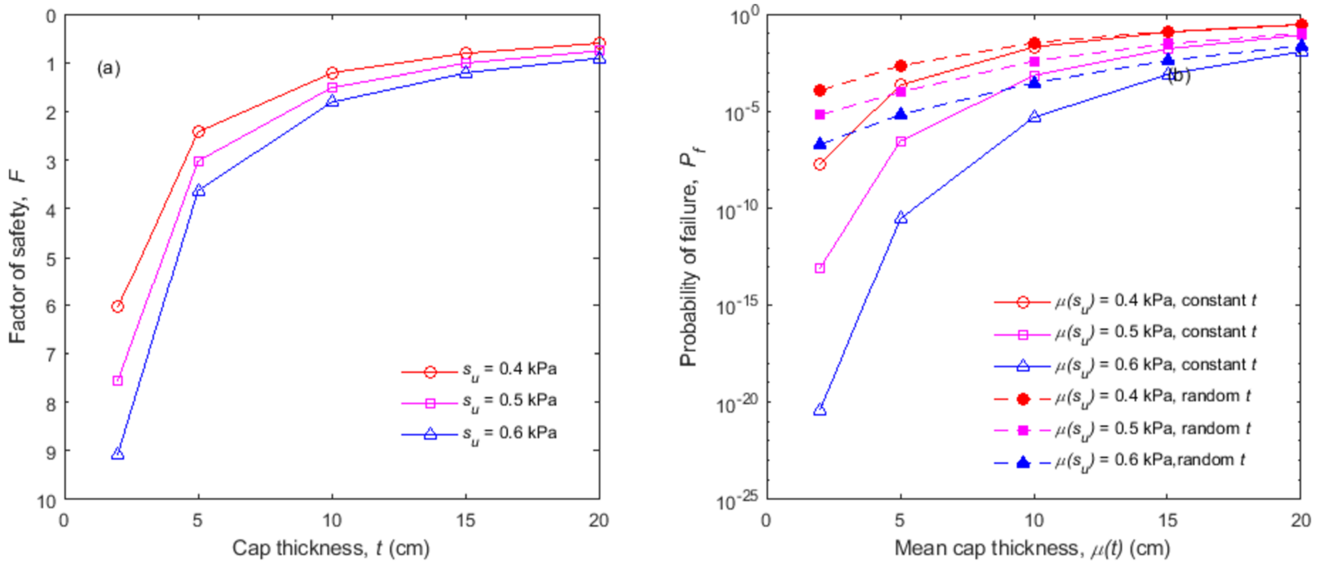

Fig. 12 **a** Deterministic factor of safety and **b** probability of bearing capacity failure for different means of undrained shear strength (s_u)

For μ (s_u) $\leq$ 0.4 kPa, $P_f \geq 10^{-3}$ even at t = 5 cm, and $P_f \geq 10^{-2}$ once t $\geq$ 15 cm for $\mu(s_u) \leq$ 0.5 kPa.

It is important to note that bearing capacity failure in areas with low inclination is likely to limit to local failures and does not cause substantial spreading of contaminants. A bearing capacity failure occurs in a steep slope might initiate sliding and consequently large spreading of contaminants. Risk assessment should therefore take into account also the consequence of bearing capacity failure, though it is not the focus of this paper. Threshold for acceptable probability of failure for bearing capacity can be higher than that for a slope instability due to its less severe and possibly local consequence.

For the Gunneklev site, if we define the probability as follows:

$P(S)$ = Probability of slope failure (i.e. Failure caused by translational gliding of the contaminated sediment and is expected to be dominated in areas with inclined terrain.)

$P(B)$ = Probability of bearing capacity failure (i.e. Failure occurs due to the shear stresses in the sediment exceed the shear strength and can occur in both flat and inclined terrain.)

Slope failure and bearing capacity failure are not mutually exclusive event in Gunneklev. The probability of either slope or bearing capacity failure or both failure occurring at the same time can be estimated as:

$$P(S \cup B) = P(S) + P(B) - P(S \cap B) \tag{4}$$

In using Eq. (4), it is important to acknowledge that it does not take into account cascading effect. A slope failure can, for example, cause redistribution of contaminated sediment mass which lead to increased loads to certain areas in the runout area of the slopes. This increasing load can subsequently cause bearing capacity failure.

Assume conservatively that the total probability of failure of the sediments in Gunneklev Fjord is the sum of slope instability and bearing capacity failure. For the most critical case, where the slope inclination ratio is 1:L = 1:25 and the mean undrained shear strength is μ (s_u) = 0.4 kPa, the total failure probability is approximately ($1.5 \times 10^{-2} + 10^{-3}$, or 1.6×10^{-2}) for t = 5 cm, and the total failure probability increases to almost 10^{-1} or more for $t \geq$ 15 cm.

4 Evidence-based decision-making for pilot field testing of mitigation measures

There are serious consequences with the leaching and spreading of the contaminated sediment, should a failure occur in Gunneklev Fjord under the placement of the rehabilitating cap. Whitman [57] recommended different acceptable annual probabilities of "failure" for various engineering projects depending on the number of lives lost and financial costs. Whitman [57] suggested limiting an acceptable annual probability of failure between 10^{-3} and 10^{-2} for slope and foundation failure with estimated lives lost limited to 1 and a financial lost within 1 million US dollar.

For the present study, we suggested to set the threshold for an upper acceptable P_f for slope instability in Gunneklev Fjord to 10^{-3} (i.e. a probability of slope failure less than 1 in 1000). This acceptable threshold is selected based on recommended in the literature (e.g. Whitman [57]) and serves as example for using probabilistic methods in decision-making in engineering. A detailed study of consequence of slope failure should be performed in order to provide sufficient basis for deciding on an acceptable probability of failure for each specific case.

For slopes with inclination equal or larger than 1:50, the thickness of the cap must then be less than 10 cm if the mean undrained shear strength is 0.4 kPa or less to avoid an unacceptable probability for slope failure ($P_f \geq 10^{-3}$). In other words. the slopes at Gunneklev Fjord with inclination less than 1:50 or mean shear strength larger than 0.4 kPa ($\mu(s_u) > 0.4$ kPa) have an acceptable nominal failure probability ($P_f < 10^{-3}$) even after implementation of a 10-cm cap. The placement of a cap thicker than 10 cm would result in an unacceptably high probability of sliding in the areas where the slope inclination is steeper than 1:50 and the mean undrained shear strength is equal to or less than 0.4 kPa.

We recommend an upper acceptable P_f of 10^{-2} for bearing capacity failure (i.e. probability of bearing capacity failure less than 1 in 100) based on recommended value from Whitman [57]. This acceptable threshold is lower than that for slope failure because bearing capacity failure tends to influence locally in smaller area. For bearing capacity failure, an acceptable threshold P_f equal to 10^{-2} also indicates that the thickness of the cap must be less than 10 cm for slope inclination $1{:}L \geq 1{:}50$ and $\mu(s_u) \leq 0.4$ kPa.

In view of the results of the probabilistic analyses and the potential consequences, and high failure probability numbers and consequences, Hydro Energy AS decided, in agreement with NEA, to carry out pilot field tests with capping in three test areas in the Gunneklev Fjord. The pilot field tests were designed to check the available soil resistance to sliding (slope instability) and punching (bearing capacity failure). An important target with the pilot field tests is to develop an effective and sustainable method for placing the capping materials and to reduce the uncertainty with respect to the thickness of the cap and the resistance of the soil.

The pilot cap was tested with 20-cm thickness cap in the "gentle" sloping area, and 5-cm thickness cap in the steep areas to avoid causing a failure. For the field tests in the area with a gentle slope, the allowable variation in the cap thickness was allowed between the range 15–30 cm. Due to the variability in the properties of the contaminated sediment, larger deviations from these allowable tolerances could potentially lead to bearing capacity failure (i.e. capping material punching-through the contaminated sediment, with $P_f \geq 10^{-2}$). In the steeper slope areas, it was decided to test the mitigation measure with a tolerance of ± 2.5 cm. The small tolerance value is to minimise the potential of the capping material causing slope instability leading to further spreading of the contaminant in the fjord. The results of the pilot tests are to be presented in a separate publication pending permission from project owner because this study stems from an industry project.

5 Summary and conclusions

This study demonstrated the use of a probabilistic approach to deal with uncertainty and to assist decision-making for the planning and design of a remediating cap over contaminated sediments in Gunneklev Fjord. The probabilistic analyses showed that the failure probability can be quite high when the uncertainties in the key analysis parameters are accounting for, even though the calculated deterministic factor of safety is very high. This highlights the importance of including the uncertainty in the analyses of slope stability and bearing capacity. It is especially crucial in the case where the input parameters have high uncertainty, as exemplified herein with the undrained shear strength of the contaminated sediment and the thickness of the remediation cap. Thus, there is real danger of underestimating the potential for failure if the assessment is based solely on a deterministic calculation of the factor of safety. The results also show that the combination of the uncertainties in several key parameters can increase the probability of failure of a slope significantly. The combination the key parameters in geotechnical reliability analysis should be selected taking into account the influence of the parameters on the actual problem, the basis for statistical characterisation of input parameters and the feasibility of the analysis (e.g. numerical expense). The selected parameters should have significant influence on analysis of the actual problem; otherwise, it is not necessary to use time and effort in including their uncertainty. Another factor to be taken into account is the availability of measurements. Engineers frequently encounter the situation in which there is insufficient measurements to characterise the uncertainty properly (i.e. to estimate mean, standard deviation, spatial variability). This is one of the main motivations for using probabilistic method instead of deterministic method. In such situation, an acceptable solution is to select values based on relevant literature and engineering judgement, combined with parametric study to understand the effect of uncertainty in the selected values. The last important factor is computational effort. Methods such as Monte Carlo simulations can handle multiple variability of complex problem but also requires considerable computational capability. If including uncertainties for all the key parameters leads to infeasible computational time, a strategy can be to use an increased (or decreased) constant value for some parameters to achieve approximately similar effects as including their uncertainty, as demonstrated in Case 3 in this study.

For the Gunneklev Fjord remediation, the results from the probabilistic analyses instigate the need to reduce uncertainty before final design and implementation of the cap. A pilot field testing program was designed based on the results of the probabilistic analyses, in order to refine the implementation method and reduce the uncertainty in the thickness of the cap. The pilot capping was tested with 20-cm cap thickness in the "gentler" sloping area and 5-cm cap thickness in the steeper sloping areas. The study also demonstrated that probabilistic analyses can be used effectively to assist evidence-based and risk-informed decision-making in cases where the parameters have high uncertainty.

Further studies should consider including spatial variability of different variables, for example shear strength. Particularly, a significant improvement can be made in the future with more field or laboratory measurements of different random parameters to provide stronger basis for the selection of input statistical parameters (mean/standard deviation).

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